Chapter 2

Logical Notation

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Announcements

- **Assignment 1**:
  - **Assignment 1** is posted on the course web page.
  - **Due date**: Jan 30, before midnight on MarkUs.
  - You **won't** be able to log into MarkUs and submit the assignment **before Jan 24**.
  - Assignments may be submitted in groups of up to **two** students. You may choose your group-mate from students in the other section.
  - Submissions must be **typed**. LaTeX is strongly recommended.
  - There are some useful links for LaTeX on the web page.

Today's Topics

- **Truth Tables, Tautology, Satisfiability, Unsatisfiability**
- **Application of Negation to Logical Sentences**

Chapter 2

Logical Notation

Truth Tables, Tautology, Satisfiability, Unsatisfiability

Truth Tables

- Logical statements evaluate either to **True** or **False**.
- It's not easy to **evaluate complex** statements:

$$(P \Rightarrow (Q \Rightarrow R)) \Leftrightarrow ((P \wedge Q) \Rightarrow R)$$

Truth Tables

In a **truth table**, we write **all** possible truth values for the **predicates** in a statement and compute the truth value of the statement under **each** of these truth assignments.

- **Question**: if there are **n** predicates in a statement, how many rows do you need in a truth table to evaluate the statement? $\mathbf{2^n}$

Truth Tables

Truth Tables for Logical Symbols

P	Q	$\neg P$	$\neg Q$	$P \wedge Q$	$P \vee Q$	$P \Rightarrow Q$	$P \Leftrightarrow Q$
T	T	F	F	T	T	T	T
T	F	F	T	F	T	F	F
F	T	T	F	F	T	T	F
F	F	T	T	F	F	T	T

Evaluate S_2

$\mathbf{S_2} : (P \Rightarrow (Q \Rightarrow R)) \Leftrightarrow ((P \wedge Q) \Rightarrow R)$

P	Q	R	$Q \Rightarrow R$	$P \Rightarrow (Q \Rightarrow R)$	$P \wedge Q$	$(P \wedge Q) \Rightarrow R$	$\mathbf{S_2}$
T	T	T					
T	T	F					
T	F	T					
T	F	F					
F	T	T					
F	T	F					
F	F	T					
F	F	F					

Truth Tables

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T	**T**	**T**	**T**				
T	**T**	**F**	**F**				
T	**F**	**T**	**T**				
T	**F**	**F**	**T**				
F	**T**	**T**	**T**				
F	**T**	**F**	**F**				
F	**F**	**T**	**T**				
F	**F**	**F**	**T**				

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T	T	F	**F**	**F**			
T	F	T	**T**	**T**			
T	F	F	**T**	**T**			
F	T	T	**T**	**T**			
F	T	F	**F**	**T**			
F	F	T	**T**	**T**			
F	F	F	**T**	**T**			

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T	**T**	F	F	F	**T**		
T	**F**	T	T	T	**F**		
T	**F**	F	T	T	**F**		
F	**T**	T	T	T	**F**		
F	**T**	F	F	T	**F**		
F	**F**	T	T	T	**F**		
F	**F**	F	T	T	**F**		

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T	T	**F**	F	F	**T**	**F**	
T	F	**T**	T	T	**F**	**T**	
T	F	**F**	T	T	**F**	**T**	
F	T	**T**	T	T	**F**	**T**	
F	T	**F**	F	T	**F**	**T**	
F	F	**T**	T	T	**F**	**T**	
F	F	**F**	T	T	**F**	**T**	

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T	T	T	T	**T**	T	**T**	**T**
T	T	F	F	**F**	T	**F**	**T**
T	F	T	T	**T**	F	**T**	**T**
T	F	F	T	**T**	F	**T**	**T**
F	T	T	T	**T**	F	**T**	**T**
F	T	F	F	**T**	F	**T**	**T**
F	F	T	T	**T**	F	**T**	**T**
F	F	F	T	**T**	F	**T**	**T**

New Terms

Tautology

A **tautology** is a sentence that is **always True** in any domain.

Satisfiability

A statement is **satisfiable** if it is **True in some** domain.

Unsatisfiability (Contradiction)

A statement is **unsatisfiable** if it is **always False** in any domain domains.

Exercise

- Use **truth tables** to evaluate the following claims. Indicate which one is a **tautology**, which one is **satisfiable** and which one is **unsatisfiable**.
    1. $P \Rightarrow Q$ is equivalent to its contrapositive.
    2. $P \Rightarrow Q$ is equivalent to its converse.
    3. $P \Leftrightarrow Q$ is equivalent to $(P \Rightarrow Q) \wedge (Q \Rightarrow P)$.
    4. $P \wedge \neg P$.
    5. $P \vee \neg P$.

Chapter 2

Logical Notation

Application of Negation to Logical Sentences

Negation

Negation of a Sentence

- The negation of a sentence inverts its **truth value**.
- The negation of a sentence P is written as $\neg P$,
- $\neg P$ is **True** if P was **False**, $\neg P$ is **False** if P was **True**.
- $\neg\neg P$ is equal to P. (why?)

Example:

Claim: All employees making over 80,000 are female.

The negation is: Not all employees making over 80,000 are female.

Negation over Conjunction and Disjunction

DeMorgan's Law

- Sentence $S_1 \wedge S_2$ is **False** exactly when at least one of S_1 or S_2 is **False**.

$$\neg(S_1 \wedge S_2) \Leftrightarrow (\neg S_1 \vee \neg S_2)$$

- Sentence $S_1 \vee S_2$ is **False** exactly when both S_1 and S_2 are **False**.

$$\neg(S_1 \vee S_2) \Leftrightarrow (\neg S_1 \wedge \neg S_2)$$

These laws can be **verified** either by a truth table, or by representing the sentences as Venn diagrams and taking the complement.

Negation over Conjunction and Disjunction

Exercise:

Recall that

- $(P \Rightarrow Q) \Leftrightarrow (\neg P \vee Q)$.
- $(P \Leftrightarrow Q) \Leftrightarrow ((P \Rightarrow Q) \wedge (Q \Rightarrow P))$

Now use DeMorgan's law to simplify the following sentences so that only P and Q are negated.

1. $\neg(P \Rightarrow Q)$.
2. $\neg(P \Leftrightarrow Q)$