Chapter 2

Logical Notation

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Announcements

- **Tutorials**:
  - Locations and times are posted on the course web page.
  - Tutorial exercises are posted on the course web page. Work on the exercises before the tutorial.
  - Each **quiz** covers all topics that you have learned **during the week** prior to the quiz.

Today's Topics

- **Review: Negation, Conjunction, Disjunction**
- **Exercise: Translation between Logical Notation and English**
- **Exercise: Problem Solving**

Chapter 2

Logical Notation

Review: Negation, Conjunction, Disjunction

Review: Negation

Negation Symbol

For the sake of brevity we will write:
$\mathbf{P(x_1, ..., x_n)}$ when $P(x_1, ..., x_n) = \textbf{True}$
$\mathbf{\neg P(x_1, ..., x_n)}$ when $P(x_1, ..., x_n) = \textbf{False}$

- "$\neg$" is called the **negation symbol**.
- $\neg P(x_1, ..., x_n)$ is the negation of predicate $P(x_1, ..., x_n)$.

Example #1

$F(x)$: x feels good.
Translate the following logical sentence to English

- $\neg F(Betty)$: Betty does **not** feel good.

Review: Conjunction (Logical AND)

Conjunctive Sentences

- A **conjunction** is a sentence that joins two other sentences and claims that **both** of the original sentences are true.
  - Al makes more than $25,000 **and** less than $75,000.
- **Conjunct Symbol**: $\wedge$
- **Conjunction in logical notation**: $\mathbf{P} \wedge \mathbf{Q}$, where **P** and **Q** are **logical sentences**.

$L(x)$: x earns less than $75,000.
$K(x)$: x earns more than $25,000.

- Al makes more than $25,000 and less than $75,000.
  $K(Al) \wedge L(AL)$.
- All employees make more than $25,000 and less than $75,000.
  $\forall x \in E, K(x) \wedge L(x)$.

Review: Evaluating Conjunctions

$P \wedge Q$ is **True** if P is **True** and Q is **True**.
$P \wedge Q$ is **False** if P is **False** or Q is **False**.

Evaluating Conjunctions

- To **prove**, verify that **both** P and Q are **True**.
- To **disprove**, show that **at least one** of P and Q is **False**.

Review: Disjunction (Logical OR)

Disjunctive Sentences

- A **disjunction** is a sentence that joins two other sentences and claims that **at least on** of the original sentences are true.
  - The employee is female **or** makes less than \$75,000.
- **Disjunct Symbol**: $\vee$
- **Disjunction in logical notation**: $\mathbf{P} \vee \mathbf{Q}$, where **P** and **Q** are **logical sentences**.

$L(x)$: x earns less than \$75,000.
$F(x)$: x is female.

- The employee is female or makes less than \$75,000.
  $x \in E, F(x) \vee L(x).$
- All employees are female or make less than \$75,000.
  $\forall x \in E, F(x) \vee L(x).$

Review: Evaluating Disjunctions

$P \vee Q$ is **True** if P is **True** or Q is **True**.
$P \vee Q$ is **False** if P is **False** and Q is **False**.

Evaluating Disjunctions

- To **prove**, verify that **at least one** of P and Q is **True**.
- To **disprove**, show that **both** P and Q are **False**.

Disjunction

Logical OR vs. Everyday-English OR

- **Logical disjunction** are true when **more than one** of the properties is true. $\rightarrow$ INCLUSIVE OR
- In **everyday English** we sometime use *OR* to indicate that **exactly one** of the properties is true. $\rightarrow$ EXCLUSIVE OR (XOR):
  - Example: Either we play the game my way, or I'm taking my ball and going home.

In this course, we do not use a symbol to denote XOR! Instead, we express exclusive-or sentences using $\neg$, $\wedge$, and $\vee$ symbols.[a]

[a] See Exercises #1 in the following slides.

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Exercises

Exercise #1: Exclusive OR Sentences

Translate the following sentence to logical notation using $\neg$, $\wedge$, and $\vee$ symbols.

- Exactly one of P and Q is true.
  $(P \wedge \neg Q) \vee (Q \wedge \neg P)$.

Exercise #2: Translation between English and Logical Notation

E: set of all employees.
M: set of all males.
F: set of all females.
$L(x)$: x earns less than \$55,000.
$S(x, y)$: x supervises y.
$C(x)$: x is a car.
$R(x)$: x is red.

Translate the following sentences to English or logical notation:

1. John is not an employee.
2. $\exists x, x \in E, x \in F, \neg L(x)$.
3. All employees are supervised by Al or they earn more than or equal to \$55,000.
4. The car is red.

Exercise #2: Solution

E: set of all employees.
M: set of all males.
F: set of all females.
$L(x)$: x earns less than \$55,000.
$S(x, y)$: x supervises y.
$C(x)$: x is a car.
$R(x)$: x is red.

Translate the following sentences to English or logical notation:

1. John is not an employee.
   $John \notin E$
2. $\exists x, x \in E, x \in F, \neg L(x)$.
   Exists a female employee who earns not less than \$55,000.
3. All employees are supervised by Al or they earn more than or equal to \$55,000.
   $\forall x \in E, S(Al, x) \vee \neg L(x)$
4. The cars is red.
   $C(x) \wedge R(x)$

Exercise #3: Translation to Logical Notation

Translate the following sentences to logical notation:

1. There is no prerequisite for CSC108.
2. Every course has a prerequisite.
3. Some course is not a prerequisite for any course.
4. No course is a prerequisite for itself.
5. Some courses have several prerequisites.

Exercise #3: Solution

C: the set of all courses.
$P(x, y)$: x is a prerequisite for y.

1. There is no prerequisite for CSC108.
   $\forall x \in C, \neg P(x, \text{CSC108})$
2. Every course has a prerequisite.
   $\forall x \in C, \exists y \in C, P(y, x)$
3. Some course is not a prerequisite for any course.
   $\exists x \in C, \forall y \in C, \neg P(x, y)$
4. No course is a prerequisite for itself.
   $\forall x \in C, \neg P(x, x)$
5. Some courses have several prerequisites.
   $\exists x \in C, \exists y \in C, \exists z \in C, P(y, x) \wedge P(z, x) \wedge y \neq z$