Review

Chapter 3

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Announcements

- **Instructor Office Hours (Bahar)**:
  - **Apr 01**, 12-1pm, BA3201
  - **Apr 08**, Time and location TBA.
- **Additional TA Office Hours**:
  - **Apr 01**, 1-2pm, BA3201
  - **Apr 02**, noon-6pm, BA2230
  - **Apr 06**, 10am-6pm, BA2230
  - **Apr 07**, 10am-4pm, BA2230
  - **Apr 08**, 10am-6pm, BA2230

How to Start a Proof

1. Translate the claim into logical notation.
2. Write the outline of the proof based on the logical structure of the claim.
3. Fill in the outline by appropriate derivations/arguments.
    - Try some specific examples to convince yourself and get some intuition.
    - List all the given definitions/assumptions, preferably in logical form.
    - List (in symbolic form) all the (relevant) facts about the domains, functions and predicates.
    - Restate some of the assumptions/definitions/facts.
    - Restating the claim might also help.

How to Approch a Proof

1. Indirect Proof (Contrapositive or Contradiction):
   - If it is easier to derive the negation of the antecedent from the negation of the consequence to:
     Example:
     $\forall x \in \mathbb{R}^+, \forall y \in \mathbb{R}^+, \forall z \in \mathbb{R}^+, ((x.y) > z) \Rightarrow (x > \sqrt{z}) \vee (y > \sqrt{z})$.
   - If the consequence includes more sub-statements than the antecedent:
     Example: $\forall n \in \mathbb{N}, P(n) \implies Q(n) \wedge R(n) \wedge S(n)$
   - When disproving the negation of the claim is easier than proving the claim:
     Example: (P denotes the set of all prime numbers.)
     $$\forall x \in P, \forall y \in P, \forall z \in P, (x^2 + y^2 \neq z^2).$$
     Negation:
     $$\exists x \in P, \exists y \in P, \exists z \in P, (x^2 + y^2 = z^2).$$
2. If you have no idea how to prove a claim, assume the negation of the claim and see if you can contradict a known fact or a given assumption!