Note Title 2015-04-01

For all natural numbers a, b, n

$(a \equiv b \mod n) \Rightarrow (a^2 \equiv b^2 \mod n)$

Restate the definition:

$(a \equiv b \mod n)$ <=> n divides $(a - b)$ <=> exists an integer k such that $(a - b) = kn$

Restate the claim:

if exists an integer k such that $(a - b) = kn$ then exists an integer k' s.t $(a^2 - b^2) = k'n$

Scratch Work:

$a - b = kn$ ==> $(a - b)(a + b) = kn(a + b)$ ===> $(a^2 - b^2) = kn(a + b)$ === > $k' = k(a + b)$

===> then exists an integer k' such that $(a^2 - b^2) = k'n$

For any integer n, n^2 2 is not divisible by 4.

Restate the claim:

for all integers **n**, there is no integer **k** such that $n^2 - 2 = 4k$

Scratch Work (Proof by Contradiction):

exists an integer **n** such that $n^2 - 2$ is divisible by **4** === >
exists an integer **n,** exists an integer **k** such that $n^2 - 2 = 4k$ ===>

$n^2 = 4k + 2 = 2(2k+1)$ **2** is not a factor of **2k+1** because **2k+1** is an odd number

|n| = sqrt(2) * sqrt(2k+1)

sqrt(2) is not an integer, but by the assumption |n| is an integer ===> **Contradiction!**

If a, b, and c are integers and $a^2 + b^2 = c^2$, then at least one of a and b is even.

$\forall a \in \mathbb{Z}, \forall b \in \mathbb{Z}, \forall c \in \mathbb{Z}, a^2 + b^2 = c^2 \Rightarrow E(a) \vee E(b)$

Scratch Work (Proof by Contradiction):

exists **a, b, c** such that **odd(a)** and **odd(b)** and **$a^2 + b^2 = c^2$** ===>

a = 2k+1 and **b = 2k'+1** ==> $a^2 + b^2 = (2k+1)^2 + (2k'+1)^2 = 4k^2 + 4k + 1 + 4k'^2 + 4k' + 1$

$a^2 + b^2 = 4(k^2+k'^2+k+k')+2$ == > $4(k^2+k'^2+k+k')+2$ is not a perfect square by the previous example ==

$a^2 + b^2$ is not a perfect square. ===> Contradiction because $a^2 + b^2 = c^2$

Prove that $f(n)=n^3 + 2n^2 + 1$ is in Big-O of $g(n)=n^4-3n^2-5$

$\lim f(n)/g(n) = 0$ ===> $c=1$

Scratch Work:

Assume $n^3 + 2n^2 + 1 =< c(n^4-3n^2-5)$, $c = 1$

$6 =< n^4 - n^3 - 5n^2$

$6 =< n^2(n^2 - n - 5)$ ===> $(n^2 - n - 5) >= 1$ and $n^2 >= 6$ ===>
$n >= 3$ ===> $B = 3$

Let $c=1$, $B=4$
(go backward through the scratch work to complete the proof)