Examples for Slide #14

Note Title 2015-03-31

Prove $2^0 + 2^1 + 2^2 + \ldots + 2^n = 2^{n+1} - 1$ for $n >= 0$

Base Case: $n=0$

P(0): $2^n=2^0 =1=2-1=2^{0+1}-1=2^{n+1}-1$

Induction Hypothesis: P(n): $2^0 + 2^1 + 2^2 + \ldots + 2^n = 2^{n+1} - 1$

Assume P(n), prove P(n+1)

P(n+1) : $2^0 + 2^1 + 2^2 + \ldots + 2^n + 2^{n+1} = 2^{(n+1)+1} - 1$

Scratc hwork:

P(n) ==>

$2^0 + 2^1 + 2^2 + \ldots + 2^n = 2^{n+1} - 1$ ==>

$2^0 + 2^1 + 2^2 + \ldots + 2^n + 2^{n+1} = 2^{n+1} + 2^{n+1} - 1 = 2*2^{n+1} -1 = 2^{n+2} -1$

==> P(n+1)

Prove $2 + 4 + 6 + \ldots + 2n = n^2 + n$ for $n >= 1$

Base Case: $n=1$

P(1): $2n = 2*1 = 1 + 1$, $n^2=1$, $n=1$ ==> $2n = n^2 + n$ for $n=1$

Induction Hypothesis: P(n) : $2 + 4 + 6 + \ldots + 2n = n^2 + n$

Assume P(n), prove P(n+1)

P(n+1): $2 + 4 + 6 + \ldots + 2n + 2(n+1) = (n+1)^2 + (n+1)$

Scratc hwork:

P(n) ==> $2 + 4 + 6 + \ldots + 2n + 2(n+1) = n^2 + n + 2(n+1)$

$= n^2 + n + 2n + 2$

$= (n^2 + 2n + 1) + (n+1)$

$= (n+1)^2 + (n+1)$

==> P(n+1)