Review Chapter 4

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Mathematical Expression and Reasoning

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Announcements

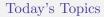
• Additional Review Session: Tues, Mar 31, 2-3:45pm in MP203.

• Additional Instructor Office Hours (Bahar):

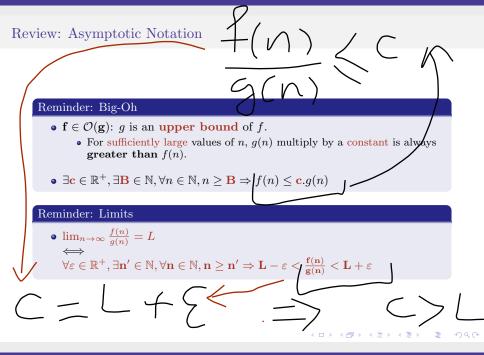
- Mar 30, 2-4pm, BA3201
- Apr 01, 12-1pm, BA3201
- Apr 08, Time and location TBA.

• Additional TA Office Hours:

- Mar 30, 4-6pm, BA3201
- Mar 31, 4-6pm, BA3201
- More to be announced!



• Proving Bounds for Functions



Proving \mathcal{O} using Limits

Prove
$$f \in \mathcal{O}(g)$$

• $\lim_{n \to \infty} \frac{f(n)}{g(n)} = L$
• $\lim_{n \to \infty} \frac{f(n)}{g(n)} = 0$
• $\lim_{n \to \infty} \frac{f(n)}{g(n)}$ does NOT exist

Proving \mathcal{O} using Limits

- Suppose $\lim_{n\to\infty} \frac{f(n)}{g(n)} = L$.
- Prove $f \in \mathcal{O}(g)$:
 - **(**) Choose any value larger than L for c.
 - 2 Assume $f(n) \leq cg(n)$. Find a value for n such that the inequality holds.
 - **8** must be larger than or equal to that value.

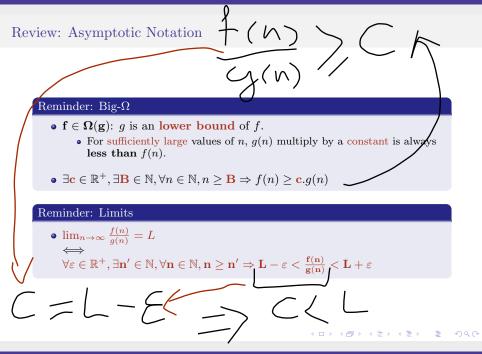
Proving \mathcal{O} using Limits

- Suppose $\lim_{n\to\infty} \frac{f(n)}{g(n)} = 0.$
- Prove $f \in \mathcal{O}(g)$: • Assume c = 1.
 - 2 Assume $f(n) \leq cg(n)$. Find a value for n such that the inequality holds.
 - **8** must be larger than or equal to that value.

Proving \mathcal{O} using Limits

- Suppose $\lim_{n\to\infty} \frac{f(n)}{g(n)}$ does NOT exist.
- Prove $f \in \mathcal{O}(g)$:
 - Find a function h(n) such that $\lim_{n\to\infty} h(n)$ exists, and $\frac{f(n)}{g(n)} \leq h(n)$ for a sufficiently large value n_1 of n.
 - 2 Choose a value for c such that $c > \lim_{n \to \infty} h(n)$.
 - (a) Assume $f(n) \leq cg(n)$. Find a value n_2 for n such that the inequality holds.
 - **(4)** B must be larger than or equal to $max(n_1, n_2)$.

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Proving Ω using Limits

Prove $f \in \Omega(g)$ **1** $\lim_{n \to \infty} \frac{f(n)}{g(n)} = L$ **2** $\lim_{n \to \infty} \frac{f(n)}{g(n)} = \infty$ **3** $\lim_{n \to \infty} \frac{f(n)}{g(n)}$ does NOT exist.

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Proving Ω using Limits

- Suppose $\lim_{n\to\infty} \frac{f(n)}{g(n)} = L$.
- Prove $f \in \Omega(g)$:

1 Choose any positive value less than L for c.

2 Assume $f(n) \ge cg(n)$. Find a value for n such that the inequality holds.

8 must be larger than or equal to that value.

Proving Ω using Limits

- Suppose $\lim_{n\to\infty} \frac{f(n)}{g(n)} = \infty$.
- Prove $f \in \Omega(g)$:
 - Assume c = 1.

2 Assume $f(n) \ge cg(n)$. Find a value for n such that the inequality holds.

8 must be larger than or equal to that value.

Proving Ω using Limits

- Suppose $\lim_{n\to\infty} \frac{f(n)}{g(n)}$ does NOT exist.
- Prove $f \in \Omega(g)$:
 - Find a function h(n) such that $\lim_{n\to\infty} h(n)$ exists, and $h(n) \leq \frac{f(n)}{g(n)}$ for a sufficiently large value n_1 of n.
 - 2 Choose a value for c such that $c < \lim_{n \to \infty} h(n)$.
 - (a) Assume $f(n) \ge cg(n)$. Find a value n_2 for n such that the inequality holds.
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Review: Asymptotic Notation

Big-Theta

- $\mathbf{f} \in \Theta(\mathbf{g})$: g is a **tight bound** of f.
 - For sufficiently large values of n, g(n) is both an upper bound and a lower bound for f(n).
- $\exists \mathbf{c_1} \in \mathbb{R}^+, \exists \mathbf{c_2} \in \mathbb{R}^+, \exists \mathbf{B} \in \mathbb{N}, \forall n \in \mathbb{N}, n \geq \mathbf{B} \Rightarrow \mathbf{c_1}.g(n) \leq f(n) \leq \mathbf{c_2}.g(n)$

Proving Big-Theta

• Find c_1 and B_1 such that

$$\exists \mathbf{c_1} \in \mathbb{R}^+, \mathbf{B_1} \in \mathbb{N}, \forall n \in \mathbb{N}, n \ge \mathbf{B_1} \Rightarrow \mathbf{c_1}.g(n) \le f(n)$$

• Find c_2 and B_2 such that

 $\exists \mathbf{c_2} \in \mathbb{R}^+, \mathbf{B_2} \in \mathbb{N}, \forall n \in \mathbb{N}, n \ge \mathbf{B_2} \Rightarrow f(n) \le \mathbf{c_2}.g(n)$

• Then $B = max(B_1, B_2)$.

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