CSC165 Mathematical Expression and Reasoning for Computer Science

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Department of Computer Science
University of Toronto

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General Info

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- **Bahar Aameri** (L0101: MWF 11-noon)

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Lisa: Monday afternoon, Wednesday morning & afternoon sessions.

Bahar: Monday morning, Friday morning & afternoon sessions.

Course Info

- Course web page:

  www.cdf.toronto.edu/~csc165h/winter

- Course info sheet (important):

  www.cdf.toronto.edu/~csc165h/winter/165infosheet.pdf

- Course Notes:

  www.cdf.toronto.edu/~csc165h/winter/165notes.pdf

Ambiguity

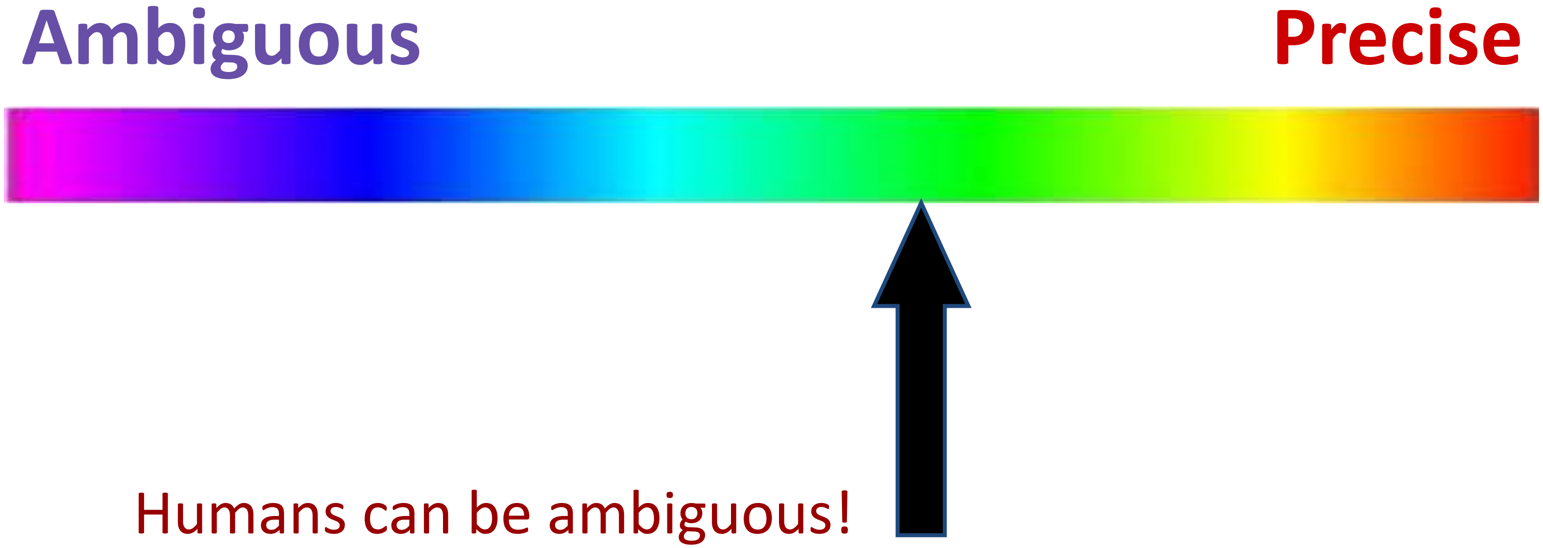
A woman asks her husband to peel half the potatoes and put them on to boil, and then leaves the house.

Ambiguity

A woman asks her husband to peel half the potatoes and put them on to boil, and then leaves the house.

Here's what she finds when she is back!

Natural language precise?



Ambiguity

When you use a natural language (English, Chinese) you can make it as precise or ambiguous as you need.

- For some purposes (jokes, gossip) rich ambiguity is essential.
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We're all equipped to work in both modes. Work out the double meanings of these headlines:

- Two sisters reunite after 18 years at checkout counter
- Iraqi head seeks arms

Precision

How to be precise?

➔ Restrict the meanings of words.

Being a profession means learning the vocabulary (words with restricted meanings) in the filed.

➔ i.e., for mathematicians:

- continuous, open, closed
- group, ring, field
- for all, for each: $\forall$
- there is (exists): $\exists$

Balance

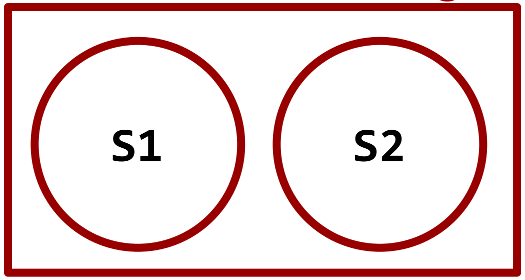
- computers are precise: in identical environments they execute identical instructions identically
- humans are as precise as necessary, and different human audiences require different levels of precision
- difficult job is finding the right level of precision:
  - too much precision introduces unbearable tedium;
  - too little introduces unavoidable ambiguity.

Computer language => human language

S1 and S2 are two sets

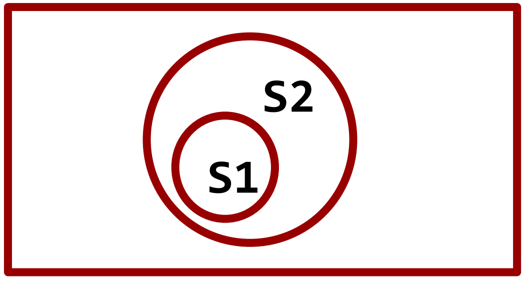
```
def q1(S1, S2):
    '''Return whether S1 and S2 have NO
       intersection
    '''
    for x in S1:
        if x in S2 : return False
    return True
```

Venn Diagram



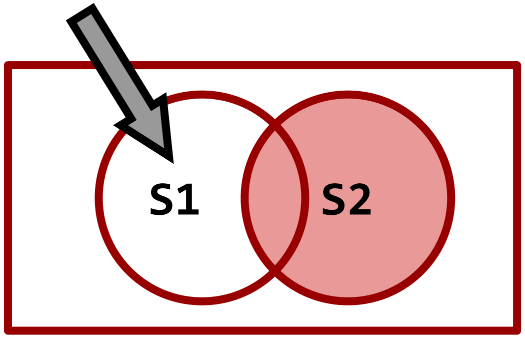
Computer language => human language

```
def q2(S1, S2):
    '''Return whether all elements in S1
       are in S2
    '''
    for x in S1:
        if x not in S2 : return False
    return True
```



Computer language => human language

```
def q3(S1, S2):
    '''Return whether there exists an element
       in S1 which is not in S2
    '''
    for x in S1:
        if x not in S2 : return True
    return False
```



Verify

Check your comments for q1-q3 in various ways (checking isn't proving, but it increases our confidence or reveals flaws).

- try out particular values for S1 and S2; see whether the results are consistent with your comments. Check “corner” values, e.g. when one or both lists are empty.
- Draw a Venn diagram.

Chapter 2
Logical Notation

Example

Consider the following table that associates employees w properties:

Employee	Gender	Salary
Al	male	60,000
Betty	female	500
Carlos	male	40,000
Doug	male	30,000
Ellen	female	50,000
Flo	female	20,000

Claims:

a. Every employee earns less than 70,000.
b. Each employee makes at least 10,000.
c. All female employees make less than 55,000.

How to evaluate these claims are true or false?

Domain

Employee	Gender	Salary
Al	male	60,000
Betty	female	500
Carlos	male	40,000
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Ellen	female	50,000
Flo	female	20,000

Claims:

a. Every employee earns less than 70,000.
b. Each employee makes at least 10,000.
c. All female employees make less than 55,000.

Analyze and evaluate these claims:

- Claims a,b&c are called statements.
- claims a&b are about the entire database to be considered.

Domain: entire database of all the objects/elements being considered. i.e. here are the six employees.

Sets

Employee	Gender	Salary
Al	male	60,000
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- Claims a,b & c are statements.
- These claims are about the entire domain to be considered.

Sets: i.e. define set of employee and denote it as symbol E; the set of female employee as F; the set of male employee as M, the set of employees who earn less than 70,000 as L, etc.

Properties and Sets

- To describe a domain, we write **statements** that specify **properties** of objects within the domain and their **relationships**.
  Recall that we want the **statements** to be in **symbolic** notation.
  To achieve that, **properties** and **relationships** are represented as **sets**.

Example

Emp.	Gender	Supervisor
Al	male	-
Betty	female	Doug
Carlos	male	Ellen
Doug	male	Ellen
Ellen	female	Al
Flo	female	Ellen

- **Property**:
  $\mathbf{M} = \{x \mid x \text{ is } \mathbf{male}\}$.
  $\mathbf{M} = \{Al, Carlos, Doug\}$.
- **Relationship**:
  $\mathbf{S} = \{\langle x, y\rangle \mid x \ \mathbf{supervises}\ y\}$.
  $\mathbf{S} = \{\langle Al, Ellen\rangle, \langle Ellen, Carlos\rangle, \langle Ellen, Doug\rangle, \langle Ellen, Flo\rangle, \langle Doug, Betty\rangle\}$.

Predicates

Employee	Gender	Salary
Al	male	60,000
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Predicates: M(Betty), L(Carlos), $L(x) : x \in L$

Predicates

Predicate

A unary predicate $L(x)$ is a boolean function returning **True** or **False** such that

$$L(x) = \textbf{True} \text{ if } x \in L.$$
$$L(x) = \textbf{False} \text{ if } x \notin L.$$

An n-ary predicate $L(x_1, ..., x_n)$ is a boolean function returning **True** or **False** such that

$$L(x_1, ..., x_n) = \textbf{True} \text{ if } \langle x_1, ..., x_n \rangle \in L.$$
$$L(x_1, ..., x_n) = \textbf{False} \text{ if } \langle x_1, ..., x_n \rangle \notin L.$$

Predicates

Example

- **Property**:
  $\mathbf{M} = \{Al, Carlos, Doug\}$.
  $M(Al), M(Carlos), M(Doug)$ are **True**.
  $M(Betty), M(Ellen), M(Flo)$ are **False**.
- **Relationship**:
  $\mathbf{S} = \{\langle Al, Ellen\rangle, \langle Ellen, Carlos\rangle, \langle Ellen, Doug\rangle, \langle Ellen, Flo\rangle, \langle Doug, Betty\rangle\}$.
  $S(Al, Ellen), S(Ellen, Carlos), S(Ellen, Doug), S(Ellen, Flo)$
  $S(Doug, Betty)$ are **True**.

Quantifiers

Employee	Gender	Salary
Al	male	60,000
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Carlos	male	40,000
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Ellen	female	50,000
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Claims:

a. **Every employee earns less than 70,000.**

b. Each employee makes at least 10,000.

c. **All female employees** make less than 55,000.

Sets: employee E; female employee F; male employee M, employees who earn less than 70,000 as L, etc.

Predicate: M(Betty); $L(x) : x \in L$

Quantifier: an expression that indicates the scope. i.e. all employees…, some male employees…

Universal Quantification∀

- Claim (a): Every employee earns less than 70,000.

  It isn't about one or even several employees; It's about an entire set of employees (employee domain).

- When an statement is about all the objects being considered, the statement is a Universal Quantification.
- Universal quantifier: ∀, 'for all', 'every','any'

  Claim (a): Every employee earns less than 70,000.

  how to use logic notation to represent this?

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  Claim (a): Every employee earns less than 70,000.

$$\forall x, x \in E, L(x)$$

Existential Quantification ∃

Another sort of claim appear to be about some individual, un-named, employee.

- Existential quantifier ∃: ‘there is’, ‘some’,’exsit’
- How to use logic expression to represent:

  Some employee earns less than 70,000 ?

Existential Quantification ∃

Another sort of claim appear to be about some individual, un-named, employee.

- Existential quantifier ∃: ‘there is’, ‘some’,’exist’
- How to use logic expression to represent:

  Some employee earns less than 70,000 ?

  $\exists x, x \in E, L(x)$

Quick Review

- Domain: all the objects/elements ;
- Sets: define set of employee and denote it as symbol E;
- Predicates: M(Betty), L(Carlos)
- Quantifiers: an expression that indicates the scope of a term:
  - $\forall$ universal quantifier
  - $\exists$ existential quantifier

Exercises

- **All employees earn over 42,000**
- **No male employees earn over 42,000**
- **Some female employee earns over 42,000**

- Express these claims in terms of logic notation & set operations (subsets, intersections, unions, complements, etc.).
- Think of quantification in terms of sets: E is the set of employees, M is the set of male employees, F is the set of female employees, and O is the set of employees earning over 42,000.

Set Theory & Notations

$x \in A$: "x is an element of A."

$\bar{A}$: The set of elements in the domain (universe) that are not in A.

$A \subseteq B$: Every element of A is also an element of B.

$A = B$: A and B contain exactly the same elements, in other words $A \subseteq B$ and $B \subseteq A$.

$A \cup B$: The set of elements that are in either A, or B, or both.

$A \cap B$: The set of elements that are in both A and B.

$A \setminus B$: The set of elements that are in A but not in B (the set difference).

$\emptyset$ or $\{\}$: A set that contains no elements.
For *any* set A, $\emptyset \subseteq A$.

$|A|$: The number of elements in A.

Exercises

Employee	Gender	Salary
Al	male	60,000
Betty	female	500
Carlos	male	40,000
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Flo	female	20,000

E: set of employees

M: set of male employees

F: set of female employees

O: set of employees earning over 42,000

All employees earn over 42,000

Logic notations: $\forall x, \quad x \in E, x \in O$

Set operations: $E \subseteq O$

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E: set of employees

M: set of male employees

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O: set of employees earning over 42,000

No male employees earn over 42,000

Logic notations: ∀x, $x \in M, x \in \overline{O}$

Set operations: $M \subseteq \overline{O}$ or $M \cap O = \emptyset$

Exercises

Employee	Gender	Salary
Al	male	60,000
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E: set of employees

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O: set of employees earning over 42,000

Some female employees earn over 42,000

Logic notations: ∃x, $x \in F, x \in O$

Set operations: $F \cap O \neq \varnothing$ or $F \not\subseteq \overline{O}$