CHAPTER 4

ALGORITHM ANALYSIS AND ASYMPTOTIC NOTATION

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Linear Search

```
def LS(A, x):
"""" Return index i, x == A[i].
     Otherwise, return -1 """"
    i = 0                      ☺
    while i < len(A):          ☺ ☺ ☺
        if A[i] == x:          ☺ ☺ ☺
            return i               ☺
        i = i + 1              ☺ ☺
    return -1
```

What is the runtime of `LS(A, x)`?

if the first index where x is found is k
i.e., A[k] == x

$t_{LS}(A, x) = 1 + 3(k+1)$
$= 3k + 4$

$t_{LS}([2, 4, 6, 8], 6) = 10$

Today's Outline

- Formal definition of O, Ω, Θ

FORMAL DEFINITIONS OF O, Ω, Θ

Recap $O(n^2)$

Set of functions that **grow no faster** than $\mathbf{n^2}$

- count the number of steps
- constant factors don’t matter
- only highest-order term matter

The following functions are in $\mathbf{O(n^2)}$

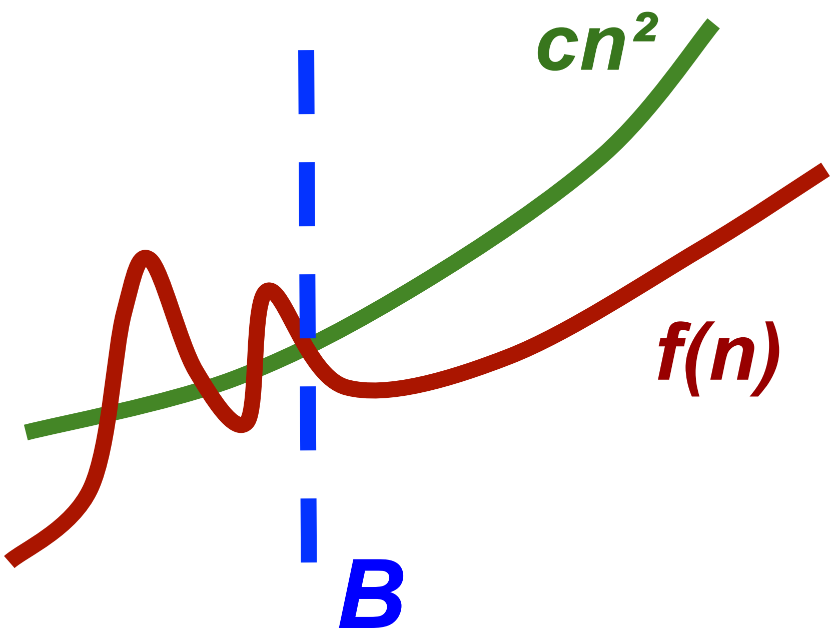
$$n^2 \qquad 2n^2 + 3n \qquad \frac{n^2}{165} + 1130n + 3.14159$$

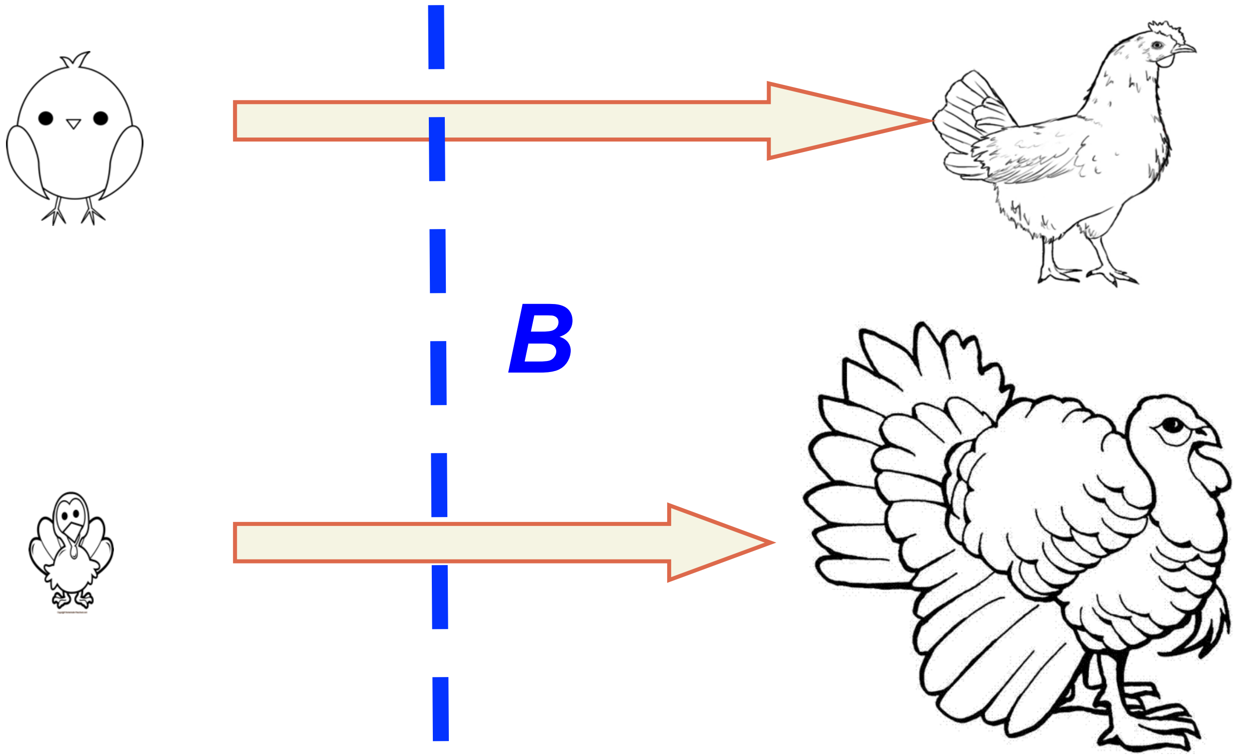
Formal definition of O(n²)

a function $f(n)$ is in $O(n^2)$ iff

$$\exists c \in \mathbb{R}^+, \exists B \in \mathbb{N}, \text{such that } \forall n \in \mathbb{N}, n \geq B \Rightarrow f(n) \leq cn^2$$

Beyond ***breakpoint B****,* ***f(n)*** *is upper-bounded by* ***cn²****, where* ***c*** *is some wisely chosen constant multiplier.*





“chicken size” is in O(“turkey size”)

A chicken **grows slower** than a turkey in the sense that, after a certain **breakpoint**, a chicken will always be smaller than a turkey.

Formal Definition O(n²)

a function $f(n)$ is in $O(n^2)$ iff

$$\exists c \in \mathbb{R}^+, \exists B \in \mathbb{N}, \text{such that } \forall n \in \mathbb{N}, n \geq B \Rightarrow f(n) \leq cn^2$$

Simple example: prove 700n² ∈ O(n²)

Pick $c = 711$, or any real number ≥ 700

Pick $B = 0$, or any natural number ≥ 0

$$\text{then } \forall n \in \mathbb{N}, n \geq 0 \Rightarrow 700n^2 \leq 711n^2$$

$$\text{then } \exists c \in \mathbb{R}^+, \exists B \in \mathbb{N}, \forall n \in \mathbb{N}, n \geq B \Rightarrow 700n^2 \leq cn^2$$

then $700n^2 \in O(n^2)$

Formal Definition Ω(n²)

a function $f(n)$ is in $O(n^2)$ iff

upper-bound

$$\exists c \in \mathbb{R}^+, \exists B \in \mathbb{N}, \text{ such that } \forall n \in \mathbb{N}, n \geq B \Rightarrow f(n) \leq cn^2$$

a function $f(n)$ is in $\Omega(n^2)$ iff

lower-bound

$$\exists c \in \mathbb{R}^+, \exists B \in \mathbb{N}, \text{ such that } \forall n \in \mathbb{N}, n \geq B \Rightarrow f(n) \geq cn^2$$

$O(n^2)$: set of functions that **grow no faster** than **n^2**

$\Omega(n^2)$: set of functions that **grow no slower** than **n^2**

$\Theta(n^2)$: set of functions that are in **both $O(n^2)$ and $\Omega(n^2)$**

(functions growing **as fast as n^2**)

Growth rate ranking of some common functions

grow fast

$$f(n) = n^n$$

$$f(n) = 2^n$$

$$f(n) = n^3$$

$$f(n) = n^2$$

$$f(n) = n \log n$$

$$f(n) = n$$

$$f(n) = \sqrt{n}$$

$$f(n) = \log n$$

$$f(n) = 1$$

grow slowly

Examples

$$7n \in \mathcal{O}(n^2) \qquad 7n \notin \Omega(n^2)$$

$$7n^3 \notin \mathcal{O}(n^2) \qquad 7n^3 \in \Omega(n^2)$$

$$7n^2 \in \mathcal{O}(n^2) \qquad 7n^2 \in \Omega(n^2)$$

$$7n^2 \in \Theta(n^2)$$

Next Week

- Worst-case analysis of two algorithms