CSC165, Winter 2015
Assignment 3
Due Apr 07, 11:59 p.m.

- You may work in groups of no more than **two** students, and you should produce a single solution in a PDF file named `a3.pdf`, submitted to MarkUs. Submissions must be **typed**.
- The size of the PDF file **must** be less than **1MB**.
- **Late submission policy**: Each **group** can benefit from **one** time 24-hour grace period with no penalty.

  If you use your grace period for one assignment, then change your group for the next one, and your new group-mate has not use his/her grace period, the new group **can** submit late by 24 hours. However, if both students in the group have used their grace period, the new group **cannot** submit late.

IMPORTANT: You **must** use the proof structures and format of this course. Otherwise, you won't get full mark even if your answers are correct.

1. **Prove** or **disprove** each of the following claims.

   You may assume that $f : \mathbb{N} \to \mathbb{R}^{\geq 0}$ and $g : \mathbb{N} \to \mathbb{R}^{\geq 0}$.

   (a) Let $f(n) = n\lfloor \frac{n}{2} \rfloor$, and $g(n) = n^2 - 2n + 1$.
   Then $f \in \Theta(g)$.

   (b) Let $f(n) = n^4 + 3n^3 + n^2 - 1$, and $g(n) = n^5 - 8n^3 - n$.
   Then $f \in \Theta(g)$.

   (c) Let $f(n) = n^n$, and $g(n) = n^{n-5}$.
   Then $f \in \Theta(g)$.

2. Prove a **tight bound** on the worst-case running time of each of the following algorithms

   (a)
```
def mystery1(L):
""" L is a non-empty list of length len(L) = n. """
    if L[0] is even:
       i=0
       while i <n^2:
          L[0] = L[0] + L[i/n]
          i=i+1
    else:
       i=0
       while i < n-1:
          L[0] = L[0] - L[i]
          i=i+1
```

   (b)
```
def mystery2(L):
""" L is a non-empty list of length len(L) = n. """
    step = 1
    index = 0
    while index < len(L):
       index = index + step
       step = step + 1
```

3. Prove each of the following statements by induction (in all parts, assume that $n \in \mathbb{N}$)

   (a) $1 + 6 + 11 + 16 + ... + (5n - 4) = \frac{n(5n-3)}{2}$, $n \geq 1$.

   (b) For all natural numbers $n \geq 3$,
   $4^3 + 4^4 + 4^5 + ... + 4^n = \frac{4(4^n - 16)}{3}$

   (c) $\sqrt{n} < \frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + ... + \frac{1}{\sqrt{n}}$, $n \geq 2$.