CSC165, Winter 2015
Assignment 2
Due Mar 06, 11:59 p.m.

- You may work in groups of no more than **two** students, and you should produce a single solution in a PDF file named `a2.pdf`, submitted to MarkUs. Submissions must be **typed**.
- The size of the PDF file **must** be less than **1MB**.
- **Late submission policy**: Each **group** can benefit from **one** time 24-hour grace period with no penalty.

  If you use your grace period for one assignment, then change your group for the next one, and your new group-mate has not use his/her grace period, the new group **can** submit late by 24 hours. However, if both students in the group have used their grace period, the new group **cannot** submit late.

IMPORTANT: You **must** use the proof structures and format of this course. Otherwise, you won't get full mark even if your answers are correct.

1. **Prove** or **disprove** each of the following claims.

   (a) $\forall x \in \mathbb{R}, \forall y \in \mathbb{R}, \lceil x + y \rceil = \lceil x \rceil + \lceil y \rceil$.
   (Note that $\lceil x \rceil$ is the ceiling function. See Section 1.5 in the course notes for the definition)

   (b) For all integers x, y, and z, if $x \nmid yz$ then $x \nmid y$ and $x \nmid z$.
   (Note that the symbol $\nmid$ denotes "does not divide")

2. Use **proof by contradiction** to prove that for all prime numbers x, y, and z, $x^2 + y^2 \neq z^2$.
   Hint: It might be helpful to recall that $a^2 - b^2 = (a - b)(a + b)$.

3. Consider the definition of the floor function:

$$\mathbf{Def_1} : \forall x \in \mathbb{R}, \forall y \in \mathbb{Z}, (y = \lfloor x \rfloor) \Leftrightarrow (y \leq x) \wedge (\forall z \in \mathbb{Z}, (z \leq x) \Rightarrow (z \leq y)).$$

   Use the proof structures of this course and $\mathbf{Def_1}$ to prove the following claims

   (a) $\mathbf{S_1} : \forall n \in \mathbb{Z}, \forall y \in \mathbb{R}, (0 \leq y) \wedge (y < 1) \Rightarrow (\lfloor n + y \rfloor = n)$.
   Note: In your proof, you may ONLY use those properties of the floor function that are specified by $\mathbf{Def_1}$.

(b) $\mathbf{S_2} : \forall x \in \mathbb{R}, \exists y \in \mathbb{R}, (0 \leq y) \wedge (y < 1) \wedge (x = \lfloor x \rfloor + y)$.

Note: In your proof, you may ONLY use those properties of the floor function that are specified by $\mathbf{Def_1}$.

(c) $\mathbf{S_3} : \forall x \in \mathbb{R}, \forall n \in \mathbb{Z}, (\lfloor x + n \rfloor = \lfloor x \rfloor + n)$.

Note: In your proof, you may ONLY use those properties of the floor function that are specified by $\mathbf{Def_1}$, $\mathbf{S_1}$, and $\mathbf{S_2}$.