Fall 2003	CSC2423	Finite model theory and descriptive complexity

Assignment 1

Exercises (due date: November 2).

(a) Consider sets of the form $X_\Phi = \{n \in \mathbb{N} \mid L_n \models \Phi\}$ where Φ is FO, and L_n is a linear order with n elements. Describe these sets.

(b) From the text: exercises 3.1, 3.5 (choose any 2), 3.6, 3.8, 4.6, 4.8, 5.4, 6.1 (any two), 6.2

(c) Provide a complete proof of the game characterization of MSO.

Problems. (due date: December 1). Choose one or two you want to work on. It's ok to work in groups. It's better to have a complete solution for one problem rather than make progress towards solving two.

(1) Prove that $+$ and $\times$ are expressible in FO($<$, BIT).

(2) Show how to express $|X| \leq \log_2 |Y|$ in FO($<$, BIT).

(3) Prove Gaifman's theorem, and establish the bound $O(4^{\mathsf{qr}(\varphi)})$ on r (rather than $7^{\mathsf{qr}(\varphi)}$ given in the text).

(4) Consider MSO over strings and trees (ranked or unranked). Show how to transform a given sentence Φ into a structure (again, string or tree) M_Φ such that $\mathcal{A} \models \Phi$ iff there is a homomorphism $h : \mathcal{A} \to M_\Phi$. *Note*: you may have to modify – slightly – the vocabulary for coding strings and trees as structures. Starts with strings, where the construction is the simplest.

(5) Prove that FO on planar graphs is fixed-parameter tractable.

Challenges.

(α) Use Ehrenfeucht-Fraïssé games to prove that parity is not definable in FO(All).

(β) Prove that every invariant FO($<, +$) query is Gaifman-local (you may find it convenient to start with the case of one free variable, and prove weak locality first).