Fall 2001	CSC2423	Finite model theory and descriptive complexity

Assignment 2

Exercises (due date: December 7).

(a) Prove that 2-colorability is LFP-definable over unordered graphs (over ordered graphs this is a corollary of 2-colorability being PTIME).

(b) We gave a proof that FO($\mathbf{C}$) is not equal order-invariant FO($\mathbf{C}$)+ $<$, by showing how to express transitive closure on a subset of logarithmic size. There was a gap in the proof given in class. Find the gap, and fix the proof.

(c) We skipped a part of the proof of Fagin's theorem, that dealt with the initial condition (that the input graph is on the tape – this is a conjunct in the first-order part of the big Σ_1^1 formula). Complete the proof, by providing this part of the formula.

(d) Prove the stage comparison theorem for LFP, using the sketch given in class.

(e) Consider the graph $G = \langle \mathbb{N}, E \rangle$ where $(i, j) \in E$ iff either $p_i | j$ (for $j > i$) or $p_j | i$ (for $i > j$). Here p_i is the ith prime, and $p|m$ means $m = 0 \pmod{p}$.
Prove that G is isomorphic to the random graph.

Problems.

(1) Prove that combined complexity of LFP is EXPSPACE-complete.

(2) It is undecidable if a sentence Φ is true in all finite models. Is it decidable if a sentence Φ is true in almost all finite models? If it is, what is the complexity?