ECO426 - Market Design

→ Markets

- Competitive Markets
  - → laissez-faire, free market, money is liquid
- Matching Markets
  - → use prefs (and possibly \$) in allocation
- Auction Markets
  - → Allocation + \$ for 'thin markets'
    - trading volume low
    - assets are illiquid or 'not very liquid'

→ Matching Markets

- matching $\mu : M \cup W \to M \cup W$ with
  - $\forall m, w \in M \times W$, (m,m) or (w,w) or (m,w) or (w,m).
  - $\mu(\mu(x)) = x$
- prefs are assumed strict
- μ is stable iff pareto effecient $\wedge$ no unacceptable matching
  - → No blocking pairs
- Gale-Shapley (Deferred Acceptance)
  - ♂-opt ⇔ ♀-pess
  - ♀-opt ⇔ ♂-pess
  - → always find a stable matching
  - → ♀(/♂) opt matching is weakly preferred by all ♀ (/♂)

⚠ ∃! stable matching iff DA♂ has same outcome as DA♀

- Theorem : $\forall_{\mu, \mu'}$, $[\forall \sigma, \sigma : \mu \succcurlyeq \mu']$ iff $[\forall ♀, ♀ : \mu' \succcurlyeq \mu]$
- Centralized vs decentralised
  - → NRMP for matching of hospital-residents
  - → pb: early contracting (ad-hocness → security ↑) ⇒ matching quality ↓

. Priority matchings

-> form best matchings (mutual) first, etc...

-> still pb of pre-contracting for safety

. Preference elicitation

-> incentive to reveal true prefs

. Formal strategic models for preference elicitation problem:

. strategic game w/ ordinal prefs

-> players: $(a_i)_{i \in N}$

-> actions: $\forall a_i$, ranked ordered list over matches Q

-> outcomes: determined by matching mech:

$\mathcal{H}$: profile = set of reported prefs -> matching

-> prefs: 'true' pref orderings over partners P

game $G = (M \cup W, \mathcal{H}, P)$

. strategies

strategy: $((m, w_i) \geq_m (m, \ldots$

$q = \{[(m_j, w_i) >_m \ldots], [(m_k, w_\ell)$

$q \in Q$

$P \subsetneq Q$?

-> strategy for m is pref ordering over matches $q_m \in Q^M$

-> strategy profile q: collec° of strategies for each player

q_{-x} : all players but x.

-> best response : a strategy q_x is a best resp.

to profile q_{-x} iff ~~(q_x, q_{-x})~~

$$\forall q'_x \in \underbrace{\cancel{Q^M}}_{\text{PrefOrd over}(Q^M)}, x : \mathcal{H}(q_x, q_{-x}) \geq_x \mathcal{H}(q'_x, q_{-x})$$

-> dominant strategy : q_x is a (weakly) dominant strategy for x iff

$$\forall q_{-x} \in Q_{-x}, \; q_x \text{ is a best resp. to } q_{-x}$$

$\exists$ence of dom. str. for x depends on P_x and $\mathcal{H}$.

-> strategy proof mech : $\mathcal{H}$, such that $\forall x \; \forall P_x$, P_x is a dominant strategy. => pref elicita° pb solved if it exists.

- $\mathcal{H}$ is pareto efficient iff $\mathcal{H}(q)$ is p.e. $\forall q \in Q$ ~~wrt~~ (2)

  e.g. random serial dictatorship is p.e. and s.p.
  → but can be unstable, because:
  stability $\Leftarrow\Rightarrow$ p.e. $\wedge$ no unacceptable matchings
  [$\Rightarrow$ stability $\Rightarrow$ p.e. but p.e. $\not\Rightarrow$ stability]

- Thm $\nexists$ s.p. $\wedge$ stable matching mech.
  proof counter-example w/ truncated prefs

- Thm ♂ prop DA (/♀) is s.p. for ♂ (/♀)
  $\Rightarrow$ When there prefs P are such that $\exists!$ stable matching, (i.e. DA♂ $\Leftarrow\Rightarrow$ DA♀), then DA is s.p.

- Thm Provided everyone else is truthful, a ♀ can achieve her best possible match by truncating prefs until best achievable ♂ in any stable matching

- Thm Rural Hospital Theorem:
  $\forall$ stable matchings, set of matched (unmatched) is the same across all → proof: A1
  $\Rightarrow$ set of unmatched agents cannot gain by pref manipula°

- DA in practice (e.g. NRMP)
  → ~~[illegible]~~ for large markets:

  when agents ↗:
  - s.p.-ness ↗ if P ~ Unif(Q) popula°
  - given fixed # couples m, prob($\exists\mu$, $|\mu(X)| \geq m$) ↗
  - utility loss between DA♀ and DA♂ ↘

→ Many-to-one matchings

- Extend previous setting. ~~…~~ firms have quota of vacancy
  - simplest extension: responsive prefs
    : ∀ firm, firm strictly ranks workers

  ⇒ stable matching: . each firm does not exceed quota
  . pareto efficient

- Can use DA ~~…~~ by 'duplicating' firm by quota.
  → Rural Hospital thm still holds.
  - Historically, Roth 86 showed that the 'rural hospitals' in NRMP were always the same across ~~…~~ stable matchings.
  → ∃nce of stable match. holds
  → stable mech for hospital (firm) cannot be sp. in general.

- More general extensions:
  - arbitrary prefs:
    → stable matching might not ∃. cf similar to non∃ for match. w/ externalities
  - substitutable prefs:
    → ~~…~~ call W set of workers, $R_f(W)$ set of workers rejected by f if it could choose indep.

rules out IIA. 'complementarities' between worker but not entirely ~grp-IIA

→ Firm f has subst. prefs iff $\forall W, W'$ $W' \subsetneq W \Rightarrow R_f(W') \subseteq R_f(W)$

→ Monotonicity: $W \nearrow \Rightarrow R_f \nearrow$ or $\rightarrow$

<u>Thm</u> If firms have substitutes prefs, then ∃ stable match.

→ In DA, a firm never 'regrets' a rejection.

Δ prefs: resp $\subsetneq$ subs $\subsetneq$ arbit

→ Housing Market

- Two-sided matching market w/ prefs on one side only.
- Initial endowment: each agent a owns a house h_a.

→ Find matching that cannot be pareto-improved.

→ CORE

- Def: allocation is matching s.t. $\begin{cases} \forall a, \text{ a has been assigned a house.} \\ \forall h, \text{ h has been assigned exactly an agent.} \end{cases}$

  ⇒ is a bijection $A \to H$

- Def: coalition $S \subseteq A$ own $H_S \subseteq H$.

  → subset of market that can 'freely trade'

- Def: blocking → a coali° S blocks an alloca° μ iff

  $$\exists \mu_S : S \to H_S, \begin{cases} \exists s \in S, \mu_S \succ_s \mu & \text{strict} \\ \forall s \in S, \mu_S \succeq_s \mu & \text{no one else has a reason to change} \end{cases}$$

- Def: core → an alloca° μ is in the core of the housing market if ~~∄~~ there is no coali° that blocks it.

→ TTC (Top Trading Cycle algo Gale)

- build graph:
  - → each agent points to preferred house
  - → each house points to its owner

  $\}\ |V| = |E| \Rightarrow \exists$ cycle
- alg:
  - → remove cycles (assigning houses to agents)
  - → update graph (each agent points to remaining preferred house)
  - → loop until done
- outcome: unique core alloca° of the housing market

. Thm TTC is strategy-proof

<u>proof idea</u>: . agent matched at round n cannot manipulate prefs to break previous cycles (cannot get house assigned to someone else before)

. getting a house after round n does not make one better off.

→ House Allocation Problem

. Same as Housing Market setting but no initial endowment -

→ Mechs:

. (random) serial dictatorship: (randomly) order agent, ~~[illegible]~~ give priority w.r.t. to ordering.

. CORE from (random) assignment: randomly produce initial endowment then use TTC alg.

→ all are s.p. (and grp s.p.!)

→ House Allocation with existing tenants pb

. Some agents have house, others don't : $A - R$; $R \subseteq A$; some houses are empty $E \subseteq H$

→ Mechs:

. extension of serial dictatorship: . $\forall s \in R$, s decides to keep residence or participate in lottery (and give up)

. $\forall s \in A - R$, s participates in lottery

. Outcome of lottery is priority ordering.

→ By participating in lot. $s \in R$ might be worse off ⇒ incentive not to participate ⇒ inefficient outcome -

. YRMH-IGYT : 'you get my house - i get your turn'

TTC where empty houses point to agents with highest priority

→ solves participation pb, is s.p. and p.e.

→ { House alloc setting: YRMH-IGYT → serial dictatorship
Housing market: ———— → TTC

→ Kidney Exchange ④

- Set of patients P, set of kidneys K
- Set of donor-patient pairs $\{(t_1, k_1) \dots\}$ $\begin{cases} t_i \in P \\ k_i \in K \end{cases}$
- For each patient t_i, a strict ordering over set of compatible kidneys $K_i \subseteq K$ and option of exchanging 'own' donor-kidney for priority w on waitlist.

actions → pairwise kidney exchange (multi-way exchanges i.e.)
→ exchange to list

induces selec. pb', since most common blood type is O, it's likely that list exchanges make ~~bias~~ ⊖ O_i blood-O patients

→ pb: maximize supply of live donors (participation pb)

→ TTC & C (and chains) mechanism

- Because of added 'list exchange' there is now the potential formation of chains as well as cycles.
  - → chain selection rule e.g.
    - by priority (choose chain with patient-donor pair with highest priority)
    - by length → minimal chain ⇒ guarantees s.p.
      → maximal chain ⇒ NOT s.p.

$$\underbrace{w \leftarrow t_1 \leftarrow k_1 \leftarrow t_3 \leftarrow k_3}_{\text{small chain}} \qquad \overbrace{\leftarrow t_5 \leftarrow k_5 \leftarrow t_4 \leftarrow k_4}^{\text{long chain}}$$

  - → tail kidney assignment rule
    - assign it to some waitlisted patient ⇒ not p.e.
    - leave it available to remaining ⇒ guarantees p.e.

→ Pairwise Kidney Exchange with binary prefs.

n/a ← Constraints (from KE): only bilateral (pairwise) exchange.
only compatibility (binary) matrix!

R is a $|P| \times |P|$ matrix where $R_{ij} \begin{cases} 1 & \text{if } i\ K_j \text{ are compat.} \\ 0 & \text{otherwise} \end{cases}$

. Priority mechanism

. Use TTC graph

(Ryto) match agents iteratively by priority (~~...~~ keeping unmatched agents until the end).

→ p.e. & s.p.

→ limited bc only bilateral exchanges considered.

→ S

. Student-School Alloca° redux

→ College Admission

. Students & Schools have prefs (welfare of both matters)

. Many-to-one matchings (Gale Shapley 1962)

→ Student Placement

. Students have prefs (only economic agents who matter)

. Schools assign endogenous ranks → e.g. std test scores

→ School Choice

. Students have prefs (same, only their welfare matters)

. Schools assign exogenous priorities → e.g.

. Model (Student Placement)

. Students I with strict prefs over schools.

. Schools S with a quota q_s for each $s \in S$.

. Categories C s.t. $\forall s \in S, \exists c_s \in C$ → e.g. medecine, law...

. Exam score profile $\{e_c^i\}_{i \in I, c \in C}$ (e_c^i denotes score of student i in category c) s.t. $\forall c \in C$, each student is strictly ranked within C.

Auctions

→ Ascending Price Auction

- Each bidder has dominant strategy: stay in auction as long as price is less than valuation
- Outcome: efficient, highest bidder pays 2nd price

→ First Price Auction

- Setting: submit bid in envelope
- No dominant strategy → depends on other bidders

→ Descending Price Auction

- Equivalent to AP, FP, SP

→ Revenue Equivalence Theorem

- model: N bidders, have values v_i i.i.d. from F.
- If in equilibrium: highest bidder wins, ~~lowest~~ bidder w/ lowest valuation doesn't pay any.

$\Rightarrow$ avg revenue & bidder profits are same across any such auctions

→ Envelope Theorem on auctions

- Bidder has objective function: $u(b,v) = v\,Pr(win\,|\,b) - E[Payment\,|\,b]$

$$\Rightarrow u_v(b,v) = Pr(win\,|\,b)$$

→ Let $b^*(v)$ be the equilibrium bidding strategy, then by the envelope thm: $U'(v) = Pr(win\,|\,b^*(v))$

$$\Rightarrow U(v) = U(0) + \int_0^v Pr(win\,|\,\tilde{v})\,d\tilde{v}$$

→ All pay auction
- every bidder pays w/ bid to participate
- $U(v) = Pr(\text{win}|v) \times v - b(v) = v^2 - b(v)$

  in 2^{nd} price auc: $U_{2P}(v) = v^2/2$

  Thus, by RET: $v^2 - b(v) = v^2/2 \Rightarrow \underline{b(v) = v^2/2}$

  equilibrium bidding strategy

→ Auction w/ reserve price
- bidding own value is optimal when $v \geq R$ – (also when $v \leq R$)

→ ~~Auct~~ Common Value Auction
- e.g. wallet / jar of pennies auction

→ loser's ~~&~~ winner's curses → under or over estimating

→ Rev. Equ. doesn't hold!

~~seller~~ bidder revenue → AP > SP > FP

→ Milgrom-Weber: Open auctions better

→ Linkage Principle: ↑ info made public ⇒ ↑ bidder utility

→

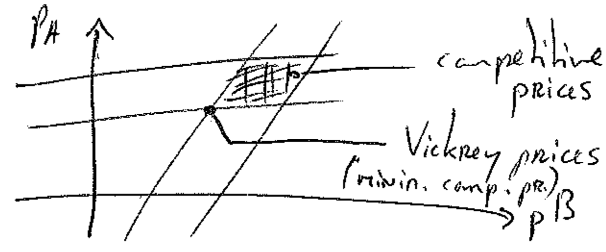
→ Multi-item auctions
- uniform price auction vs discriminatory price auction

→ clock auction:

→ $k+1^{th}$ price auction: K highest bidders pay $K+1^{th}$ highest bid –

→ Vickrey auction: $\begin{cases} \text{dominant strategy to be truthful} \\ \text{efficiency} \end{cases}$

. displaced bidders



→ Competitive Equilibrium –
- such that highest bidder wins –

→ Generalized Second Price Auction
- Not SP