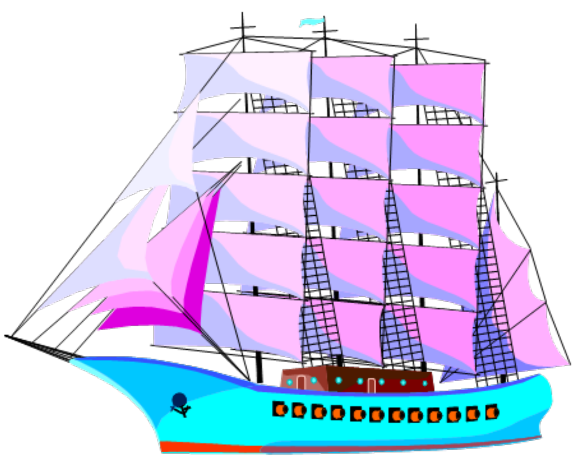
# CSCC43 Introduction to Databases

Relational Normalization Theory



# Redundancy

- Dependencies between attributes cause redundancy
  - Ex. All the people working in the same place have the same zip code

| *SSN* | *Name* | *Department* | *Zip* |
|---|---|---|---|
| 1234 | Joe | DCS | M5S 3H5 |
| 4321 | Mary | DCS | M5S 3H5 |
| 5454 | Tom | DCS | M5S 3H5 |
| …………………. | | | |

Redundancy

| *SSN* | *Name* | *Address* | *Hobby* |
|---|---|---|---|
| 123456789 | Mike | 123 College St. | stamps |
| 123456789 | Mike | 123 College St. | coins |
| ………………. | | | |

Redundancy

# Redundancy and Other Problems

- Example: Person (*SSN, Name, Address, Hobbies*)
  - A person entity with multiple hobbies yields multiple rows in table Person
    - Hence, the association between *Name* and *Address* for the same person is stored redundantly
  - *SSN* is the key of the entity set, but (*SSN, Hobby*) is the key of the corresponding relation
    - The relation Person can't describe people without hobbies

# Anomalies

- Redundancy leads to anomalies:
  - **Update anomaly**: A change in *Address* must be made in several places
  - **Deletion anomaly**: Suppose a person gives up all hobbies. Do we:
    - Set Hobby attribute to null? <u>No</u>, since *Hobby* is part of key
    - Delete the entire row? <u>No</u>, since we lose other information in the row
  - **Insertion anomaly**: *Hobby* value must be supplied for any inserted row since *Hobby* is part of key

# Decomposition

- **Solution**: use two relations to store Person information
  - Person1 (*<u>SSN</u>, Name, Address*)
  - Hobbies (*<u>SSN, Hobby</u>*)
- The decomposition is more general: people with hobbies can now be described
- No update anomalies:
  - Name and address stored once
  - A hobby can be separately supplied or deleted
  - We can represent persons who do not have hobbies

# Normalization Theory

- Result of E-R analysis needs further refinement
- Appropriate decomposition can solve problems
- The underlying theory is referred to as *normalization theory* and is based on *functional dependencies* (and other kinds, like *multivalued dependencies*)

# Functional Dependencies

- **Definition:** A *functional dependency* (FD) on a relation schema **R** is a constraint $\boldsymbol{X} \rightarrow \boldsymbol{Y}$**,** where $X$ and $Y$ are subsets of attributes of **R.**

- **Definition**: An FD $\boldsymbol{X} \rightarrow \boldsymbol{Y}$ is *satisfied* in an instance **r** of **R** if for **every** pair of tuples, *t* and s: if *t* and *s* agree on all attributes in $X$ then they must agree on all attributes in $Y$

# Key Constraint

- Is a special kind of functional dependency:
  - Let *K* be a set of attributes of R, and *U* the set of **all** attributes of R. Then *K* is a *key* if the functional dependency $K \rightarrow U$ is satisfied in R.
    - *SSN* $\rightarrow$ *SSN, Name, Address* (in the Person1 relation)
- A *candidate key* is a minimal superkey
  - *K* is a key in *R*, if for each X $\subset$ K, *X* is not a key
    - *SSN, Hobby* $\rightarrow$ *SSN, Name, Address, Hobby* but
    - *SSN* $\rightarrow$ *SSN, Name, Address, Hobby*
    - *Hobby* $\rightarrow$ *SSN, Name, Address, Hobby*
- A *prime attribute* is an attribute of a key

# Functional Dependencies cont'd

- *Address* $\rightarrow$ *ZipCode*
  - DCS's ZIP is M5S 3H5
- *Author, Title, Edition* $\rightarrow$ *PublicationDate*
  - Database Management Systems, by R. Ramakrishnan, and J. Gehrke, McGraw Hill, 2003 (3rd Edition)
- *CourseID* $\rightarrow$ *ExamDate, ExamTime*
  - CSC343's exam date is August 9, starting at 6pm

# Entailment, Closure, Equivalence

- **Definition**: If ***F*** is a set of FDs on schema **R** and *f* is another FD on **R**, then ***F*** *entails f* if every instance **r** of **R** that satisfies every FD in ***F*** also satisfies *f*
  - Ex: $\boldsymbol{F} = \{A \rightarrow B, B \rightarrow C\}$ and *f* is $A \rightarrow C$
    - If *Phone#* $\rightarrow$ *Address*, *Address* $\rightarrow$ *ZipCode* then

      *Phone#* $\rightarrow$ *ZipCode*
- **Definition**: The *closure* of ***F***, denoted $\boldsymbol{F}^{+}$, is the set of all FDs entailed by ***F***
- **Definition**: ***F*** and ***G*** are *equivalent* if ***F*** entails ***G*** and ***G*** entails ***F***

# Armstrong's Axioms for FDs

- This is the *syntactic* way of computing/testing the various properties of FDs
- **Reflexivity**: If $Y \subseteq X$ then $X \rightarrow Y$ (trivial FD)
  - *Name, Address* $\rightarrow$ *Name*
- **Augmentation**: If $X \rightarrow Y$ then $XZ \rightarrow YZ$
  - If *Address* $\rightarrow$ *ZipCode* then *Address, Name* $\rightarrow$ *ZipCode, Name*
- **Transitivity**: If $X \rightarrow Y$ and $Y \rightarrow Z$ then $X \rightarrow Z$
  - If *Phone#* $\rightarrow$ *Address* and *Address* $\rightarrow$ *ZipCode*, then *Phone#* $\rightarrow$ *ZipCode*

# Soundness

- Axioms are *sound*: If an FD $f: X \rightarrow Y$ can be derived from a set of FDs $\boldsymbol{F}$ using the axioms, then $f$ holds in every relation that satisfies every FD in $\boldsymbol{F}$.
- Example: Given $X \rightarrow Y$ and $X \rightarrow Z$ then

$$
\begin{array}{ll}
X \rightarrow XY & \textit{Augmentation by } X \\
YX \rightarrow YZ & \textit{Augmentation by } Y \\
X \rightarrow YZ & \textit{Transitivity}
\end{array}
$$

  - Thus, $X \rightarrow Y\,Z$ is satisfied in every relation where both $X \rightarrow Y$ and $X \rightarrow Z$ are satisfied
    - Therefore, we have derived the *union rule* for FDs

# Completeness

- Axioms are *complete:* If $\boldsymbol{F}$ entails $f$ , then $f$ can be derived from $\boldsymbol{F}$ using the axioms
- A consequence of completeness is the following (naïve) algorithm to determining if $\boldsymbol{F}$ entails $f$:
  - Algorithm: Use the axioms in all possible ways to generate $\boldsymbol{F}^+$ (the set of possible FD's is finite so this can be done) and see if $f$ is in $\boldsymbol{F}^+$

# Correctness

- The notions of *soundness* and *completeness* link the syntax (Armstrong's axioms) with semantics (the definitions in terms of relational instances)
- This is a precise way of saying that the algorithm for entailment based on the axioms is ``correct'' with respect to the definitions

# Generating $F^+$

***F***

$AB \rightarrow C$

union $AB \rightarrow BCD$ decomp

$A \rightarrow D$ aug $AB \rightarrow BD$ trans $AB \rightarrow BCDE$ $AB \rightarrow E$

$D \rightarrow E$ aug $BCD \rightarrow BCDE$

Thus, $AB \rightarrow BD$, $AB \rightarrow BCD$, $AB \rightarrow BCDE$, and $AB \rightarrow E$ are all elements of $F^+$

# Attribute Closure

- Calculating *attribute closure* leads to a more efficient way of checking entailment
- The *attribute closure* of a set of attributes, $X$, with respect to a set of functional dependencies, $\boldsymbol{F}$, (denoted $X^+_{\boldsymbol{F}}$) is the set of all attributes, $A$, such that $X \rightarrow A$
  - $X^+_{\boldsymbol{F}}$ is not necessarily the same as $X^+_{\boldsymbol{G}}$ if $\boldsymbol{F} \neq \boldsymbol{G}$
- *Attribute closure and entailment*:
  - Algorithm: Given a set of FDs, $\boldsymbol{F}$, then $X \rightarrow Y$ if and only if $Y \subseteq X^+_{\boldsymbol{F}}$

# Example - Computing Attribute Closure

$\boldsymbol{F}$: $AB \rightarrow C$
$A \rightarrow D$
$D \rightarrow E$
$AC \rightarrow B$

| $X$ | $X_F^+$ |
|---|---|
| $A$ | $\{A, D, E\}$ |
| $AB$ | $\{A, B, C, D, E\}$ |
| $AC$ | $\{A, C, B, D, E\}$ |
| $B$ | $\{B\}$ |
| $D$ | $\{D, E\}$ |

Is $AB \rightarrow E$ entailed by $\boldsymbol{F}$? *Yes*
Is $D \rightarrow C$ entailed by $\boldsymbol{F}$? *No*

*Result*: $X_F^+$ allows us to determine FDs of the form
$X \rightarrow Y$ entailed by $\boldsymbol{F}$

# Computation of Attribute Closure $X^{+}_{F}$

*closure* := $X$; // *since* $X \subseteq X^{+}_{F}$

**repeat**

  *old* := *closure*;

  **if** there is an FD $Z \rightarrow V$ in ***F*** such that

    $Z \subseteq$ *closure* **and** $V \subseteq$ *closure*

  **then** *closure* := *closure* $\cup$ $V$

**until** *old* = *closure*

– If $T \subseteq$ *closure* then $X \rightarrow T$ is entailed by ***F***

# Normal Forms

- Each normal form is a set of conditions on a schema that guarantees certain properties (relating to redundancy and update anomalies)

- The two commonly used normal forms are *third normal form* (3NF) and *Boyce-Codd normal form* (BCNF)

# BCNF

- **Definition**: A relation schema **R** is in BCNF if for every FD $X \rightarrow Y$ associated with **R** either
  - $Y \subseteq X$ (i.e., the FD is trivial) or
  - $X$ is a key of **R**
- **Example**: Person1(*SSN, Name, Address*)
  - The only FD is *SSN* $\rightarrow$ *Name, Address*
  - Since *SSN* is a key, Person1 is in BCNF

# (non) BCNF Examples

- Person (*SSN, Name, Address, Hobby*)
  - The FD *SSN* $\rightarrow$ *Name, Address* does <u>not</u> satisfy requirements of BCNF
    - since the key is (*SSN, Hobby*)
- HasAccount (*AcctNum, ClientId, OfficeId*)
  - The FD *AcctNum* $\rightarrow$ *OfficeId* does <u>not</u> satisfy BCNF requirements
    - since keys are (*ClientId, OfficeId*) and (*AcctNum, ClientId*); not *AcctNum*.

# Redundancy

- Suppose **R** has an FD *A* → *B, and A is not a key.* If an instance has 2 rows with same value in *A*, they *must* also have same value in *B* (=> redundancy, if the A-*value* repeats twice)

| *SSN* | *Name* | *Address* | *Hobby* |
|---|---|---|---|
| 123456789 | Mike | 123 College St. | stamps |
| 123456789 | Mike | 123 College St. | coins |
| ................. | | | |

*Redundancy*

- If *A* is a key, there cannot be two rows with same value of *A*
  - Hence, BCNF eliminates redundancy

# Decomposition

- Schema **R** = ($R$, $\boldsymbol{F}$)
  - $R$ is set a of attributes
  - $\boldsymbol{F}$ is a set of functional dependencies over $R$
    - Each key is described by a FD
- The *decomposition of schema* **R** is a collection of schemas

$\mathbf{R}_i = (R_i, \boldsymbol{F}_i)$ where

  - $R = \cup_i R_i$ for all $i$ (*no new attributes*)
  - $\boldsymbol{F}_i$ is a set of functional dependences involving only attributes of $R_i$
  - $\boldsymbol{F}$ entails $\boldsymbol{F}_i$ for all $i$ (*no new FDs*)
- The *decomposition of an instance*, **r**, of **R** is a set of relations

$\mathbf{r}_i = \pi_{R_i}(\mathbf{r})$ for all $i$

# Example Decomposition

Schema $(R, F)$ where

$R = \{SSN, Name, Address, Hobby\}$

$\boldsymbol{F} = \{SSN \rightarrow Name, Address\}$

can be decomposed into

$R_1 = \{SSN, Name, Address\}$

$\boldsymbol{F}_1 = \{SSN \rightarrow Name, Address\}$

and

$R_2 = \{SSN, Hobby\}$

$\boldsymbol{F}_2 = \{ \}$

# BCNF Decomposition Algorithm

***Input*:** **R** = (*R; **F***)

*Decomp* := **R**
**while** there is **S** = (*S; **F'***) $\in$ *Decomp* and **S** not in BCNF **do**
Find $X \rightarrow Y \in$ ***F'*** that violates BCNF // *X isn't a key in* **S**
Replace **S** in *Decomp* with
$\mathbf{S_1} = (XY; \boldsymbol{F}_1)$, $\mathbf{S_2} = ((S - Y) \cup X); \boldsymbol{F}_2)$
// $\boldsymbol{F}_1$ *= all FDs of **F'** involving only attributes of XY*
// $\boldsymbol{F}_2$ *= all FDs of **F'** involving only attributes of* $(S - Y) \cup X$
**end**
**return** *Decomp*

# Simple Example

- HasAccount :

*(ClientId, OfficeId, AcctNum)*

$ClientId, OfficeId \rightarrow AcctNum$
$AcctNum \rightarrow OfficeId$

- Decompose using $AcctNum \rightarrow OfficeId$ :

*(OfficeId, AcctNum)*

BCNF: *AcctNum* is key
FD: $AcctNum \rightarrow OfficeId$

*(ClientId , AcctNum)*

BCNF (only trivial FDs)

# A Larger Example

**Given:** **R** = ($R$; ***F***) where $R = ABCDEGHK$ and
***F*** $= \{ABH \to C,\ A \to DE,\ BGH \to K,\ K \to ADH,\ BH \to GE\}$

**step 1:** Find a FD that violates BCNF
Not $ABH \to C$ since $(ABH)^+$ includes all attributes
($BH$ is a key)
$A \to DE$ violates BCNF since $A$ is not a key ($A^+ = ADE$)

**step 2:** Split **R** into:
$\mathbf{R_1} = (ADE, \boldsymbol{F_1} = \{A \to DE\})$
$\mathbf{R_2} = (ABCGHK; \boldsymbol{F_1} = \{ABH \to C,\ BGH \to K,\ K \to AH,\ BH \to G\})$
Note 1: $\mathbf{R_1}$ is in BCNF
Note 2: Decomposition <u>is</u> *lossless* since $A$ is a key of $\mathbf{R_1}$.
Note 3: FDs $K \to D$ and $BH \to E$ are not in $\boldsymbol{F_1}$ or $\boldsymbol{F_2}$. But
both can be derived from $\boldsymbol{F_1} \cup \boldsymbol{F_2}$
(*E.g.*, $K \to A$ and $A \to D$ implies $K \to D$)
Hence, decomposition <u>*is*</u> *dependency preserving*.

# Example cont'd

**Given: $\mathbf{R_2}$ =** ($ABCGHK$; $\{ABH \rightarrow C,\ BGH \rightarrow K,\ K \rightarrow AH,\ BH \rightarrow G\}$)

**step 1:** Find a FD that violates BCNF.

Not $ABH \rightarrow C$ or $BGH \rightarrow K$, since $BH$ is a key of $\mathbf{R_2}$

$K \rightarrow AH$ violates BCNF since $K$ is not a superkey ($K^+ = AH$)

**step 2:** Split $\mathbf{R_2}$ into:

$\mathbf{R_{21}}$ **=** ($KAH$, $\boldsymbol{F}_{21} = \{K \rightarrow AH\}$)

$\mathbf{R_{22}}$ **=** ($BCGK$; $\boldsymbol{F}_{22} = \{\}$)

Note 1: Both $\mathbf{R_{21}}$ and $\mathbf{R_{22}}$ are in BCNF.

Note 2: The decomposition is *lossless* (since $K$ is a key of $\mathbf{R_{21}}$)

Note 3: FDs $ABH \rightarrow C$, $BGH \rightarrow K$, $BH \rightarrow G$ are not in $\boldsymbol{F}_{21}$ or $\boldsymbol{F}_{22}$, and they can't be derived from $\boldsymbol{F}_1 \cup \boldsymbol{F}_{21} \cup \boldsymbol{F}_{22}$.

Hence the decomposition is *not* dependency-preserving

# Lossless Schema Decomposition

- A decomposition should not lose information
- A decomposition (**R**$_1$,…,**R**$_n$) of a schema, **R**, is *lossless* if every valid instance, **r**, of **R** can be reconstructed from its components:

where each $\mathbf{r}_i = \pi_{\mathbf{R}i}(\mathbf{r})$

$$\mathbf{r} = \mathbf{r}_1 \bowtie \mathbf{r}_2 \bowtie \ldots\ldots \bowtie \mathbf{r}_n$$

# Lossy Decomposition

*The following is always the case* (Think why?):

$$\mathbf{r} \subseteq \mathbf{r}_1 \bowtie \mathbf{r}_2 \bowtie \dots \bowtie \mathbf{r}_n$$

*But the following is not always true*:

$$\mathbf{r} \supseteq \mathbf{r}_1 \bowtie \mathbf{r}_2 \bowtie \dots \bowtie \mathbf{r}_n$$

*Example*: $\mathbf{r} \not\supseteq \mathbf{r}_1 \bowtie \mathbf{r}_2$

| *SSN* | *Name* | *Address* |
|---|---|---|
| 1111 | Joe | 1 Pine |
| 2222 | Alice | 2 Oak |
| 3333 | Alice | 3 Pine |

| *SSN* | *Name* |
|---|---|
| 1111 | Joe |
| 2222 | Alice |
| 3333 | Alice |

| *Name* | *Address* |
|---|---|
| Joe | 1 Pine |
| Alice | 2 Oak |
| Alice | 3 Pine |

*The tuples* (2222, Alice, 3 Pine) and (3333, Alice, 2 Oak) *are in the join, but not in the original*

## **Lossy Decompositions:**
## ***What is Actually Lost?***

- In the previous example, the tuples

*(2222, Alice, 3 Pine)* and *(3333, Alice, 2 Oak)*

were *gained*, not lost!

  - Why do we say that the decomposition was lossy?

- What was lost is *information*:
  - That 2222 lives at 2 Oak: *In the decomposition, 2222 can live at either 2 Oak or 3 Pine*
  - That 3333 lives at 3 Pine: *In the decomposition, 3333 can live at either 2 Oak or 3 Pine*

# Testing for Losslessness

- A (binary) decomposition of **R** = ($R$, $\boldsymbol{F}$) into $\mathbf{R}_1$ = ($R_1$, $\boldsymbol{F}_1$) and $\mathbf{R}_2$ = ($R_2$, $\boldsymbol{F}_2$) is lossless *if and only if* :
  - either the FD
    - $(R_1 \cap R_2) \rightarrow R_1$ is in $\boldsymbol{F}^+$
  - or the FD
    - $(R_1 \cap R_2) \rightarrow R_2$ is in $\boldsymbol{F}^+$

# Example

Schema ($R$, $F$) where

$R$ = {$SSN$, $Name$, $Address$, $Hobby$}

$\boldsymbol{F}$ = {$SSN \rightarrow Name, Address$}

can be decomposed into

$R_1$ = {$SSN$, $Name$, $Address$}

$\boldsymbol{F}_1$ = {$SSN \rightarrow Name, Address$}

and

$R_2$ = {$SSN$, $Hobby$}

$\boldsymbol{F}_2$ = { }

Since $R_1 \cap R_2 = SSN$ and $SSN \rightarrow R_1$ the decomposition is lossless

# Dependency Preservation

- Consider a decomposition of $\mathbf{R} = (R, \boldsymbol{F})$ into $\mathbf{R}_1 = (R_1, \boldsymbol{F}_1)$ and $\mathbf{R}_2 = (R_2, \boldsymbol{F}_2)$
  - An FD $X \rightarrow Y$ of $\boldsymbol{F}^+$ is in $\boldsymbol{F}_i$ iff $X \cup Y \subseteq R_i$
  - An FD, $f \in \boldsymbol{F}^+$ may be in neither $\boldsymbol{F}_1$, nor $\boldsymbol{F}_2$, nor even

  $(\boldsymbol{F}_1 \cup \boldsymbol{F}_2)^+$
    - Checking that $f$ is true in $\mathbf{r}_1$ or $\mathbf{r}_2$ is (relatively) easy
    - Checking $f$ in $\mathbf{r}_1$ $\mathbf{r}_2$ is harder – requires a join
    - *Ideally*: want to check FDs locally, in $\mathbf{r}_1$ and $\mathbf{r}_2$, and have a guarantee that every $f \in F$ holds in $\mathbf{r}_1 \bowtie \mathbf{r}_2$
- The decomposition is *dependency preserving* iff the sets $\boldsymbol{F}$ and $\boldsymbol{F}_1 \cup \boldsymbol{F}_2$ are equivalent: $\boldsymbol{F}^+ = (\boldsymbol{F}_1 \cup \boldsymbol{F}_2)^+$
  - Then checking all FDs in $\boldsymbol{F}$, as $\mathbf{r}_1$ and $\mathbf{r}_2$ are updated, can be done by checking $\boldsymbol{F}_1$ in $\mathbf{r}_1$ and $F_2$ in $\mathbf{r}_2$

# Dependency Preservation

- If $f$ is an FD in $\boldsymbol{F}$, but $f$ is not in $\boldsymbol{F}_1 \cup \boldsymbol{F}_2$, there are two possibilities:
  - $f \in (\boldsymbol{F}_1 \cup \boldsymbol{F}_2)^+$
    - If the constraints in $\boldsymbol{F}_1$ and $\boldsymbol{F}_2$ are maintained, $f$ will be maintained automatically.
  - $f \notin (\boldsymbol{F}_1 \cup \boldsymbol{F}_2)^+$
    - $f$ can be checked only by first taking the join of $\mathbf{r}_1$ and $\mathbf{r}_2$. This is costly.

# Example

Schema ($R$, $\boldsymbol{F}$) where

$R = \{SSN, Name, Address, Hobby\}$

$\boldsymbol{F} = \{SSN \rightarrow Name, Address\}$

can be decomposed into

$R_1 = \{SSN, Name, Address\}$

$\boldsymbol{F}_1 = \{SSN \rightarrow Name, Address\}$

and

$R_2 = \{SSN, Hobby\}$

$\boldsymbol{F}_2 = \{\ \}$

Since $\boldsymbol{F} = \boldsymbol{F}_1 \cup \boldsymbol{F}_2$ the decomposition is dependency preserving

# Example

- Schema: ($ABC;\ F$) , $\boldsymbol{F} = \{A \rightarrow B, B \rightarrow C, C \rightarrow B\}$
- Decomposition:
  - ($AC$, $\boldsymbol{F}_1$), $\boldsymbol{F}_1 = \{A \rightarrow C\}$
    - Note: $A \rightarrow C \notin \boldsymbol{F}$, but in $\boldsymbol{F}^+$
  - ($BC$, $\boldsymbol{F}_2$), $\boldsymbol{F}_2 = \{B \rightarrow C, C \rightarrow B\}$

- $A \rightarrow B \notin (\boldsymbol{F}_1 \cup \boldsymbol{F}_2)$, **but** $A \rightarrow B \in (\boldsymbol{F}_1 \cup \boldsymbol{F}_2)^+$.
  - So $\boldsymbol{F}^+ = (\boldsymbol{F}_1 \cup \boldsymbol{F}_2)^+$ and thus the decompositions is still dependency preserving

# Example

- HasAccount (*AcctNum, ClientId, OfficeId*)

  $f_1$: $AcctNum \rightarrow OfficeId$

  $f_2$: $ClientId, OfficeId \rightarrow AcctNum$
- Decomposition:

  $R_1 = (AcctNum, OfficeId;\ \{AcctNum \rightarrow OfficeId\})$

  $R_2 = (AcctNum, ClientId;\ \{\})$
- Decomposition <u>is</u> lossless:

  $R_1 \cap R_2 = \{AcctNum\}$ and $AcctNum \rightarrow OfficeId$
- In BCNF
- <u>Not</u> dependency preserving: $f_2 \notin (\boldsymbol{F_1} \cup \boldsymbol{F_2})^+$
- HasAccount *does not* have BCNF decompositions that are both lossless and dependency preserving! (Check, eg, by enumeration)
- Hence: BCNF+lossless+dependency preserving decompositions are not always achievable!

# Third Normal Form

- Compromise – Not all redundancy removed, but dependency preserving decompositions are <u>always</u> possible (and, of course, lossless)
- 3NF decomposition is based on a *minimal cover*

# Third Normal Form

- A relational schema **R** is in 3NF if for every FD $X \rightarrow Y$ associated with **R** either:
- $Y \subseteq X$ (i.e., the FD is trivial); or
  - $X$ is a superkey of **R;** or
  - Every $A \in Y$ is part of some key of **R**
- 3NF is weaker than BCNF (every schema that is in BCNF is also in 3NF)

*BCNF conditions*

# 3NF Example

- HasAccount (*AcctNum, ClientId, OfficeId*)
  - *ClientId*, *OfficeId* $\rightarrow$ *AcctNum*
    - OK since LHS contains a key
  - *AcctNum* $\rightarrow$ *OfficeId*
    - OK since RHS is part of a key
- HasAccount is in 3NF but it might still contain redundant information due to *AcctNum* → *OfficeId* (which is not allowed by BCNF)

# 3NF (Non) Example

- Person (*SSN, Name, Address, Hobby*)
  - (*SSN, Hobby*) is the only key.
  - $SSN \rightarrow Name$ violates 3NF conditions since *Name* is not part of a key and *SSN* is not a superkey

# Minimal Cover

- A *minimal cover* of a set of dependencies, ***F,*** is a set of dependencies, ***U,*** such that:
  - *U* is equivalent to ***F*** ($F^+ = U^+$)
  - All FDs in ***U*** have the form $X \rightarrow A$ where *A* is a single attribute
  - It is not possible to make ***U*** smaller (while preserving equivalence) by
    - Deleting an FD
    - Deleting an attribute from an FD (either from LHS or RHS)
  - FDs and attributes that can be deleted in this way are called *redundant*

# Computing Minimal Cover

- **Example**: $\boldsymbol{F} = \{ABH \rightarrow CK,\ A \rightarrow D,\ C \rightarrow E,$
  $BGH \rightarrow L,\ L \rightarrow AD,\ E \rightarrow L,\ BH \rightarrow E\}$
- **step 1**: Make RHS of each FD into a single attribute
  - *Algorithm*: Use the decomposition inference rule for FDs
  - Example: $L \rightarrow AD$ replaced by $L \rightarrow A,\ L \rightarrow D$ ;
    $ABH \rightarrow CK$ by $ABH \rightarrow C$, $ABH \rightarrow K$
- **step 2**: Eliminate redundant attributes from LHS.
  - *Algorithm*: If FD $XB \rightarrow A \in \boldsymbol{F}$ (where $B$ is a single attribute) and $X \rightarrow A$ is entailed by $\boldsymbol{F}$, then $B$ was unnecessary
  - Example: Can an attribute be deleted from $ABH \rightarrow C$ ?
    - Compute $AB^+_{\boldsymbol{F}}$, $AH^+_{\boldsymbol{F}}$, $BH^+_{\boldsymbol{F}}$.
    - Since $C \in (BH)^+_{\boldsymbol{F}}$ , $BH \rightarrow C$ is entailed by $\boldsymbol{F}$ and $A$ is redundant in $ABH \rightarrow C$.

# Computing Minimal Cover cont'd

- **step 3**: Delete redundant FDs from ***G***
  - *Algorithm*: If $\boldsymbol{G} - \{f\}$ entails $f$, then $f$ is redundant
    - If $f$ is $X \to A$ then check if $A \in X^{+}_{\boldsymbol{G}-\{f\}}$
  - Example: $BH \to L$ is entailed by $E \to L$, $BH \to E$, so it is redundant

- *Note*: The order of steps 2 and 3 cannot be interchanged!!

# Synthesizing a 3NF Schema

Starting with a schema **R** = ($R$, $\boldsymbol{F}$)

- **step 1**: Compute a minimal cover, $\boldsymbol{U}$, of $\boldsymbol{F}$. The decomposition is based on $\boldsymbol{U}$, but since $\boldsymbol{U}^+ = \boldsymbol{F}^+$ the same functional dependencies will hold

  A minimal cover for

  $\boldsymbol{F} = \{ABH \rightarrow CK, A \rightarrow D, C \rightarrow E, BGH \rightarrow L, L \rightarrow AD, E \rightarrow L, BH \rightarrow E\}$

  is

  $U = \{BH \rightarrow C, BH \rightarrow K, A \rightarrow D, C \rightarrow E, L \rightarrow A, E \rightarrow L\}$

# Synthesizing a 3NF schema cont'd

- **step 2**: Partition $\boldsymbol{U}$ into sets $\boldsymbol{U}_1, \boldsymbol{U}_2, \ldots \boldsymbol{U}_n$ such that the LHS of all elements of $\boldsymbol{U}_i$ are the same
  - $\boldsymbol{U}_1 = \{BH \rightarrow C, BH \rightarrow K\}$, $U_2 = \{A \rightarrow D\}$,

    $\boldsymbol{U}_3 = \{C \rightarrow E\}$, $U_4 = \{L \rightarrow A\}$, $U_5 = \{E \rightarrow L\}$
- **step 3**: For each $\boldsymbol{U}_i$ form schema $\mathbf{R_i} = (R_i, \boldsymbol{U}_i)$, where $R_i$ is the set of all attributes mentioned in $\boldsymbol{U}_i$
  - Each FD of $\boldsymbol{U}$ will be in some $\mathbf{R_i}$. Hence the decomposition is *dependency preserving*
  - $\mathbf{R_1} = (BHCK;\ BH \rightarrow C, BH \rightarrow K)$, $\mathbf{R_2} = (AD;\ A \rightarrow D)$, $\mathbf{R_3} = (CE;\ C \rightarrow E)$, $\mathbf{R_4} = (AL;\ L \rightarrow A)$, $\mathbf{R_5} = (EL;\ E \rightarrow L)$

# Synthesizing a 3NF schema cont'd

- **step 4**: If no $R_i$ is a superkey of **R,** add schema $\mathbf{R_0}$ = ($R_0$,{}) where $R_0$ is a key of **R**.
  - $\mathbf{R_0}$ = (*BGH,* {})
    - $\mathbf{R_0}$ might be needed when not all attributes are necessarily contained in $R_1 \cup R_2 \ldots \cup R_n$
      - A missing attribute, *A*, must be part of all keys

        (since it's not in any FD of *U*, deriving a key constraint from *U* involves the augmentation axiom)
    - $\mathbf{R_0}$ might be needed even if all attributes are accounted for in $R_1 \cup R_2 \ldots \cup R_n$
      - Example: (*ABCD*; {$A \rightarrow B$, $C \rightarrow D$}).

        Step 3 decomposition:

        $R_1$ = (*AB*; {$A \rightarrow B$}), $R_2$ = (*CD*; {$C \rightarrow D$}).

        Lossy! Need to add (AC; { }), for losslessness
  - Step 4 guarantees lossless decomposition.