A Characterization of Combined Traces using Labeled Stratified Order Structures

Dai Tri Man Lê

Department of Computer Science
University of Toronto
Canada

Petri Nets 2010

Outline

1. **Background**
   - Mazurkiewicz Traces
   - Combined Traces (Comtraces)
2. **Comtraces as Labeled Stratified Order Structures**
   - Motivation
   - Construction
   - Representation Theorems
3. **Conclusion**

Mazurkiewicz Traces (Mazurkiewicz 1977)

- (E, ind): an alphabet with an independence relation
- Independent symbols can be commuted.
  - If $(a, b) \in ind$, then $xaby \equiv xbay$.
- A trace is an equivalence class of words.
  - Each equivalence class describes a partial order run of the system
- Simple and elegant algebraic tool providing “true concurrency” semantics for concurrent systems with a static architecture.
  - E.g., elementary net systems, 1-safe Petri nets. . .

Combined Traces (Comtraces)

Limitation of Traces

1. as a special class of labeled partial order, traces cannot model more complicated causality relationships
2. elements of trace alphabet have no visible internal structure

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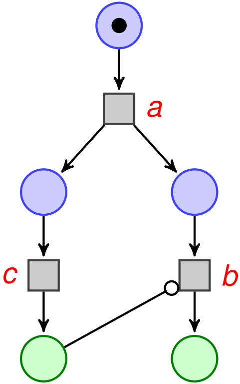
Main Ideas

comtraces (combined traces) [Janicki and Koutny 1995]

1. quotient of step sequence monoid
2. formal-linguistic representation of stratified order structures [Gaifman and Pratt 1987] [Janicki and Koutny 1991]
3. capture so-structure runs of the system

Elementary Net with Inhibitor Arcs [JK ’95]



Equivalent step sequence runs

1.
2.

“a-priori semantics”: event completion takes some time.

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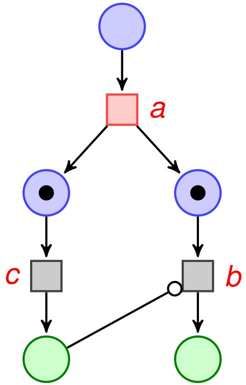

Equivalent step sequence runs

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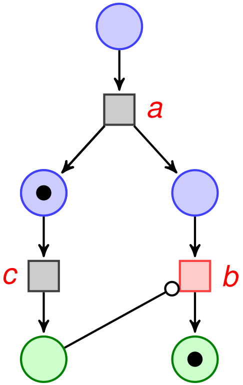

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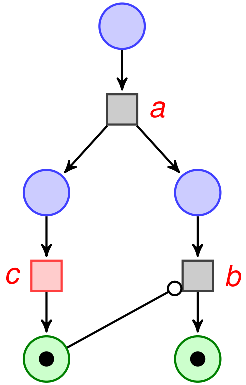

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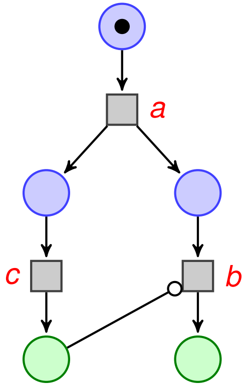

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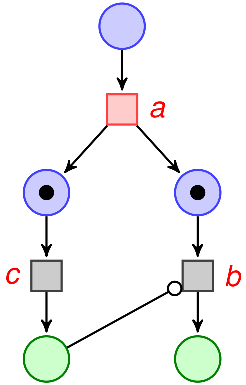

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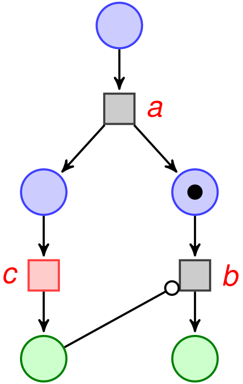

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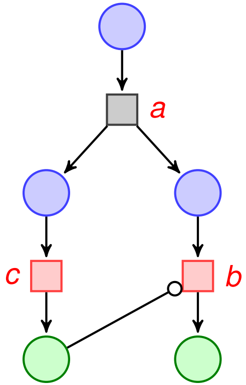

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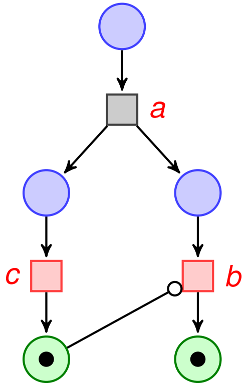

Equivalent step sequence runs

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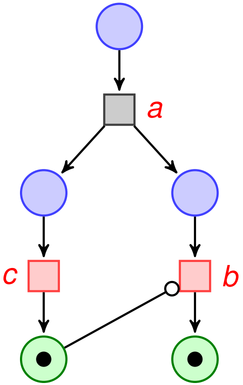

Equivalent step sequence runs

1. $\{a\}\{b\}\{c\}$
2. $\{a\}\{b, c\}$

But NOT equivalent to $\{a\}\{c\}\{b\}$

"a-priori semantics": event completion takes some time.

Comtrace

Comtrace Concurrent Alphabet

a tuple (E, sim, ser), where

- sim and ser model pairwise simutaneity and serializability
- $ser \subseteq sim \subseteq E \times E$
- sim is irreflexive and symmetric
  - defines the valid steps $\Longrightarrow$ valid step sequences.

Comtrace Equivalence

the least congruence $\equiv$ satisfying for all steps A, B and C,

$$\text{if } A \times B \subseteq ser \text{ and } C = A \cup B \text{ then } uCv \equiv uABv$$

Each equivalence class of $\equiv$ is called a *comtrace*.

Comtrace example

From the previous example

- Define $sim = \{(b,c),(c,b)\}$ and $ser = \{(b,c)\}$
- The equivalent runs
  1. $\{a\}\{b\}\{c\}$
  2. $\{a\}\{b,c\}$

  can be grouped together into a comtrace (an equivalence class)

$$[\{a\}\{b\}\{c\}] = \{\{a\}\{b\}\{c\}, \{a\}\{b,c\}\}$$

Motivations

Main research direction

Lift results and techniques from Mazurkiewicz traces to comtraces

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1. Infinite traces and their applications
   - Gastin et al. ('90-'95) provides excellent theoretical foundation
   - Applications:
     i. message sequence charts (Muscholl et al. '98, '99) (Mourin '02) (Kuske '03) (Gazagnaire et al. '09)
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2. Temporal logics for finite and infinite traces
   - (Thiagarajan '94) (Mukund-Thiagarajan '96)
   - (Thiagarajan-Walukiewicz '97) (Walukiewicz '98)
   - (Leucker '02) (Diekert-Gastin '00, '02, '04) (Diekert '02)
   - (Gastin-Mukund '02) (Gastin et al. '03) (Gastin-Kuske '03) . . .

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   - (Gastin-Mukund '02) (Gastin et al. '03) (Gastin-Kuske '03) ...

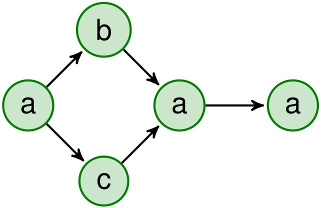
Observation:

Most of the results above utilize labeled-poset definition of traces!

Traces as Labeled Posets

Question:

Given a trace alphabet (E, ind), how to decide if a labeled poset represents a trace?



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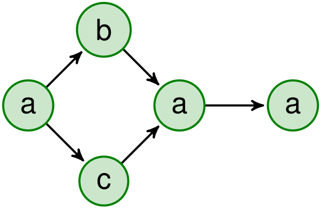

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Two main conditions:

1. Non-connected nodes are labeled with independent events
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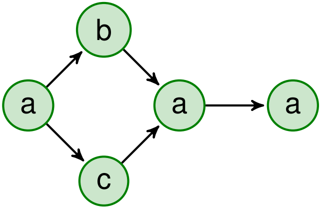

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Our goal

To give a similar order-theoretic characterization for comtraces!

Stratified Order Structures

Stratified order structure (so-structure) [Janicki-Koutny '91]

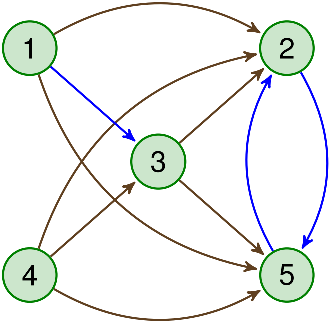
a triple $S = (X, \prec, \sqsubset)$ and binary relations $\prec, \sqsubset \subseteq X \times X$ satisfying

1. $a \not\sqsubset a$
2. $a \prec b \implies a \sqsubset b$
3. $a \sqsubset b \sqsubset c \ \wedge \ a \neq c \implies a \sqsubset c$
4. $a \sqsubset b \prec c \ \vee \ a \prec b \sqsubset c \implies a \prec c$

Intuitively, $\prec$ means *"earlier than"*, and $\sqsubset$ means *"not later than"*

"not later than" = "earlier than" or "simultaneous"

Example of so-structure



"earlier than" ∩ "not later than": ⟶

"not later than": ⟶

Theorem (JK '95)

Every comtrace uniquely defines a labeled so-structure.

Comtraces as labeled so-structures

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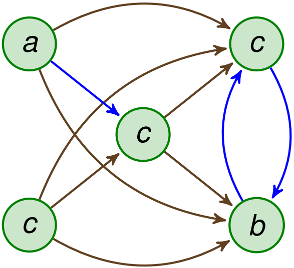

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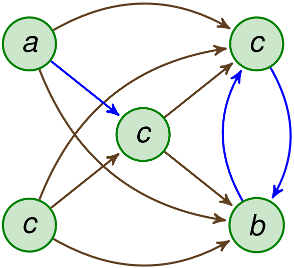

"earlier than" ∩ "not later than": $\longrightarrow$

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Obstacles

1. complication of having both $\longrightarrow$ and $\longrightarrow$
2. complication of having both *sim* and *ser*
3. cycles make things less intuitive

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Observation

The "not later than" relation $\sqsubset$ is a strict pre-order!

Quotient construction

Definition ($\sqsubset$-cycle equivalence relation)

Vertices α and β are $\sqsubset$-cycle equivalent if and only if $\alpha \sqsubset \beta$ and $\beta \sqsubset \alpha$.

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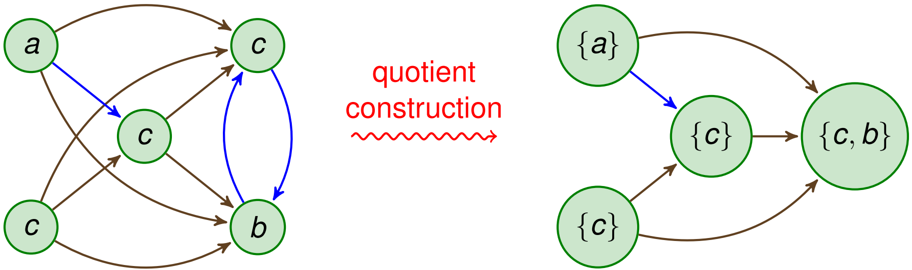

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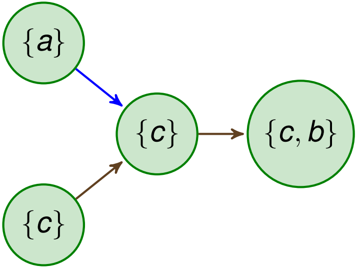

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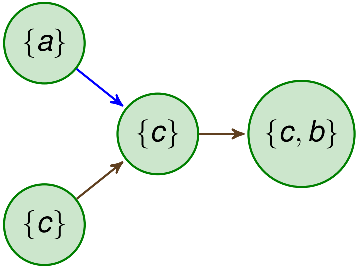

Hasse diagram

Conditions (Definition 10)

λ denotes the labeling function

1. adjacent nodes $[\alpha] \longrightarrow [\beta]$ satisfies $\lambda([\alpha]) \times \lambda([\beta]) \not\subseteq ser$
2. adjacent nodes $[\alpha] \longrightarrow [\beta]$ satisfies $\lambda([\beta]) \times \lambda([\alpha]) \not\subseteq ser$
3. label set of a node $[\alpha]$ can't be serializable w.r.t. ser
4. ...
5. ...

Representation Theorems

Theorem 3

Given a comtrace alphabet θ, let

- $\mathbb{S}^*/\equiv_{\boldsymbol{\theta}}$: comtraces over θ,
- $\mathsf{LCT}(\theta)$: lsos-comtraces over θ.

Then the following diagram commutes

$$\begin{tikzcd}
\mathbb{S}^*/\equiv_{\boldsymbol{\theta}} \arrow[loop left, "\mathrm{id}_{\mathbb{S}^*/\equiv_\theta}"] \arrow[rr, bend left, "\mathrm{ct2lct}"] & & \mathsf{LCT}(\theta) \arrow[loop right, "\mathrm{id}_{\mathsf{LCT}(\theta)}"] \arrow[ll, bend left, "\mathrm{lct2ct}"]
\end{tikzcd}$$

- This is the converse of the main theorem in [JK ’95].

Representation Theorems

Theorem 4

Given a comtrace alphabet θ, let

- LCT(θ): lsos-comtraces over θ
- CDG(θ): combined dependency graphs [Kleijn-Koutny '08] over θ
  - analogous to dependency graphs for Mazurkiewicz traces

Then the following diagram commutes

$$\begin{tikzcd}
\mathrm{LCT}(\theta) \arrow[loop left, "\mathrm{id}_{\mathrm{LCT}(\theta)}"] \arrow[rr, bend left, "\mathrm{lct2dep}"] & & \mathrm{CDG}(\theta) \arrow[loop right, "\mathrm{id}_{\mathrm{CDG}(\theta)}"] \arrow[ll, bend left, "\mathrm{dep2lct}"]
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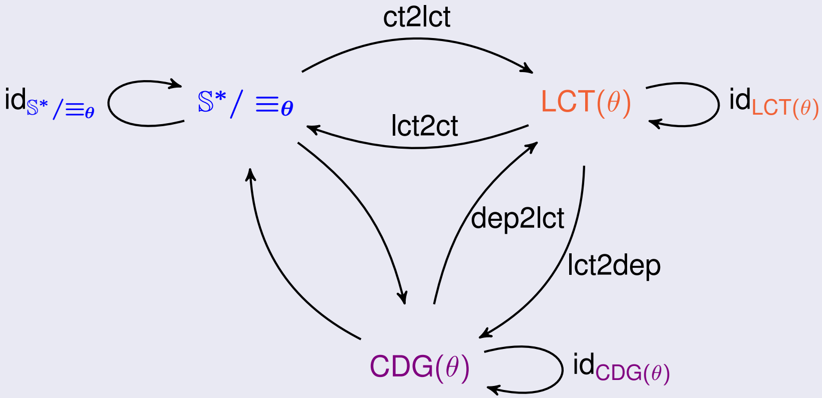
Representation Theorems

- Theorems 3 and 4: the following diagram commutes



- Theorems 5 and 6: these mappings are monoid isomorphisms.

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Summary

1. We formally show that comtraces and lsos-comtraces and combined dependency graphs are equivalent models.
2. Formal-linguistic, order-theoretic and graph-theoretic respectively.

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3. More generalized trace languages using step sequences, where "congruence" is defined from interactions of steps.

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Future Works

1. Similar results for generalized combined traces [JL '08] [JL '09]
2. Infinite comtraces?
3. Linear temporal logics for comtraces?
4. Applications of comtraces?

Thank you very much for your attention!