Machine Learning I
80-629A

Apprentissage Automatique I
80-629

Sequential Decision Making II
— Week #12

Introduction to Reinforcement Learning

Brief recap

- Markov Decision Processes (MDP)
  - Offer a framework for sequential decision making

    $\langle \mathsf{A}, \mathsf{S}, \mathsf{P}, \mathsf{R}, \gamma \rangle$

  - Goal: find the optimal policy
    - Dynamic programming and several algorithms (e.g., VI,PI)

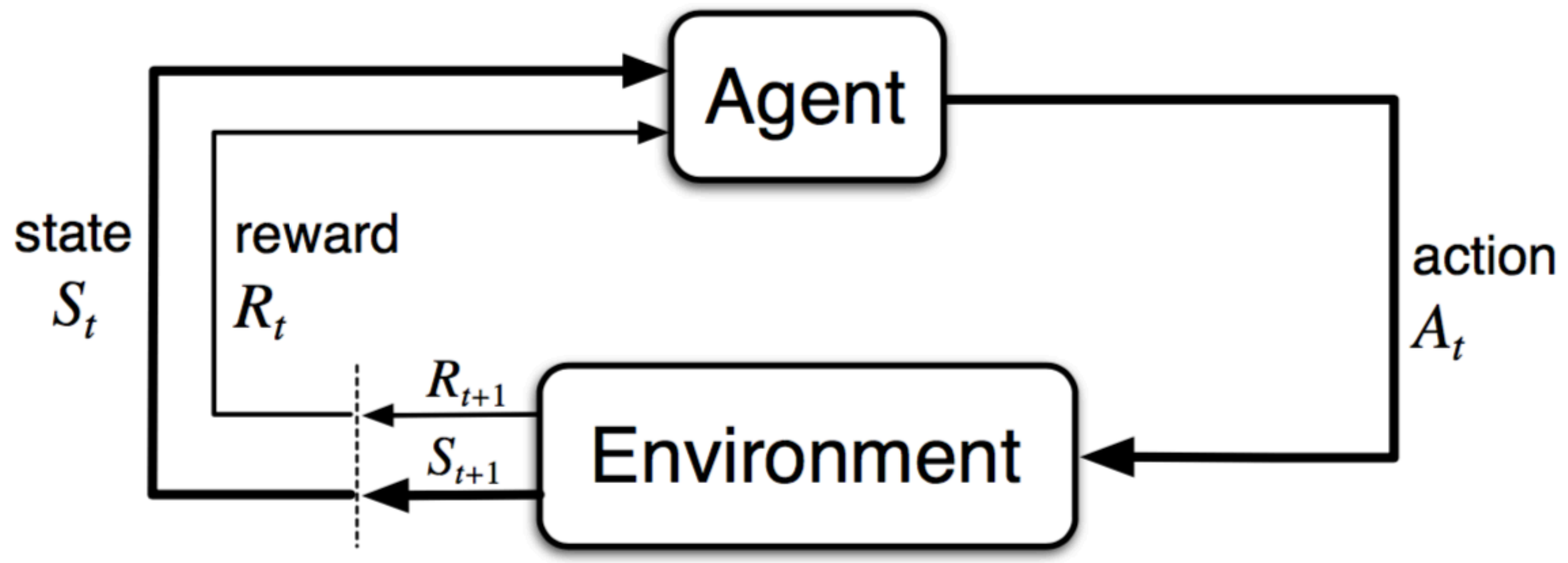

Figure 3.1, RL: An introduction

From MDPs to RL

- **In MDPs we assume that we know**

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- In MDPs we assume that we know
  1. Transition probabilities: P(s' | s, a)
  2. Reward function: R(s)
- RL is more general
  - In RL both are typically unknown
  - RL agents navigate the world to gather this information

Experience

A. Supervised Learning:

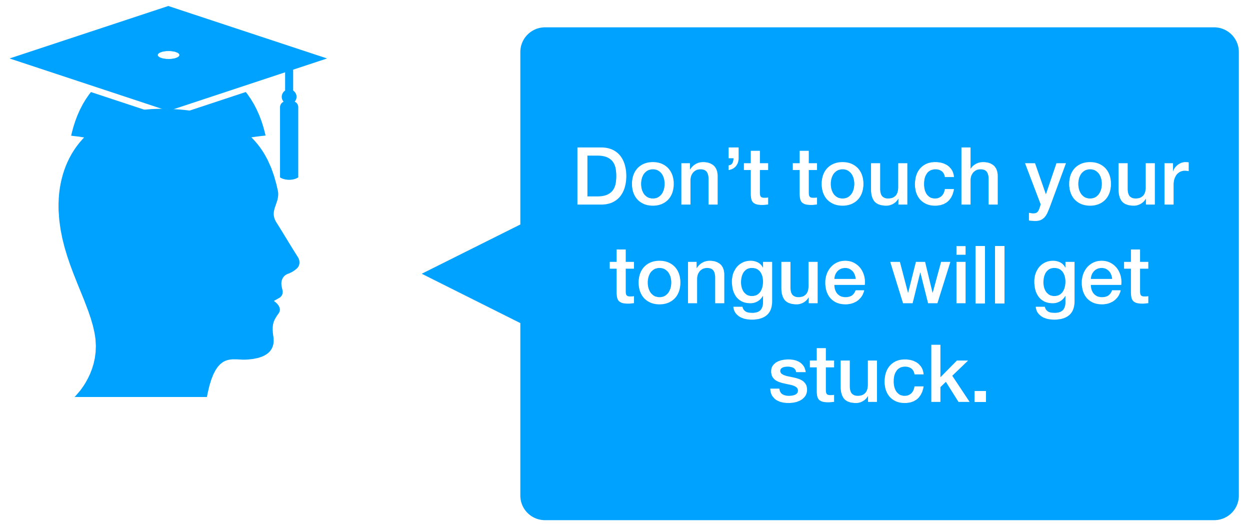
- Given fixed dataset
- Goal: maximize objective on test set (population)

B. Reinforcement Learning

- Collect data as agent interacts with the world
- Goal: maximize sum of rewards

Example

Supervised Learning



Slide adapted from Pascal Poupart

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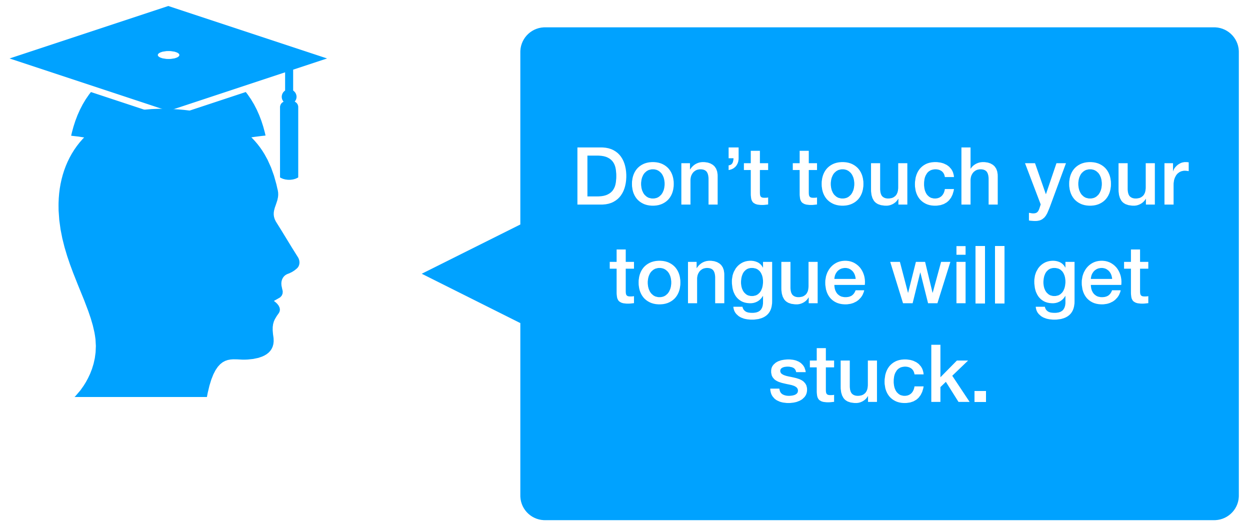

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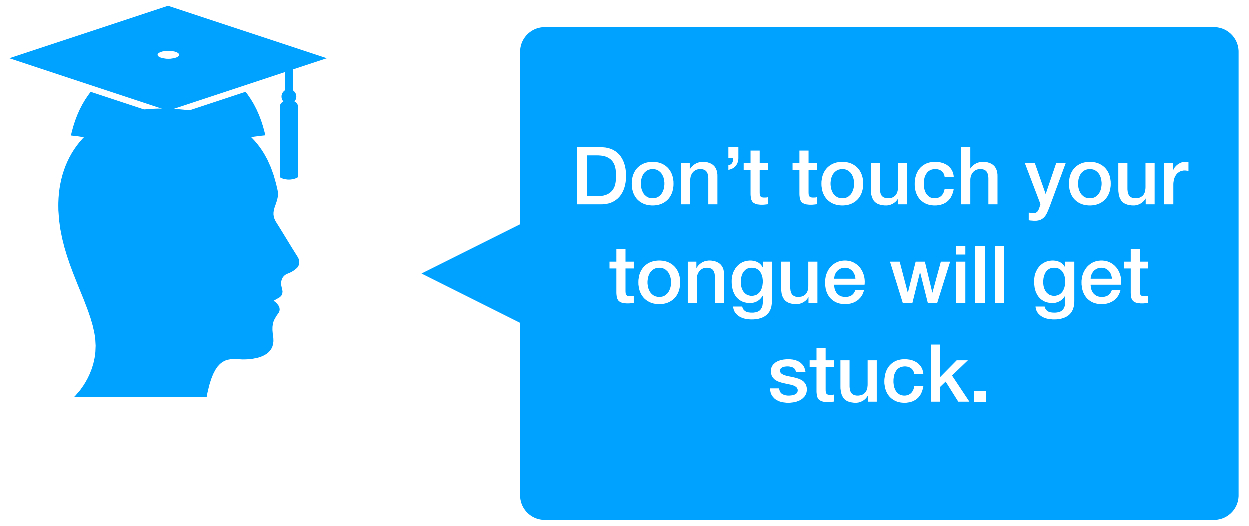

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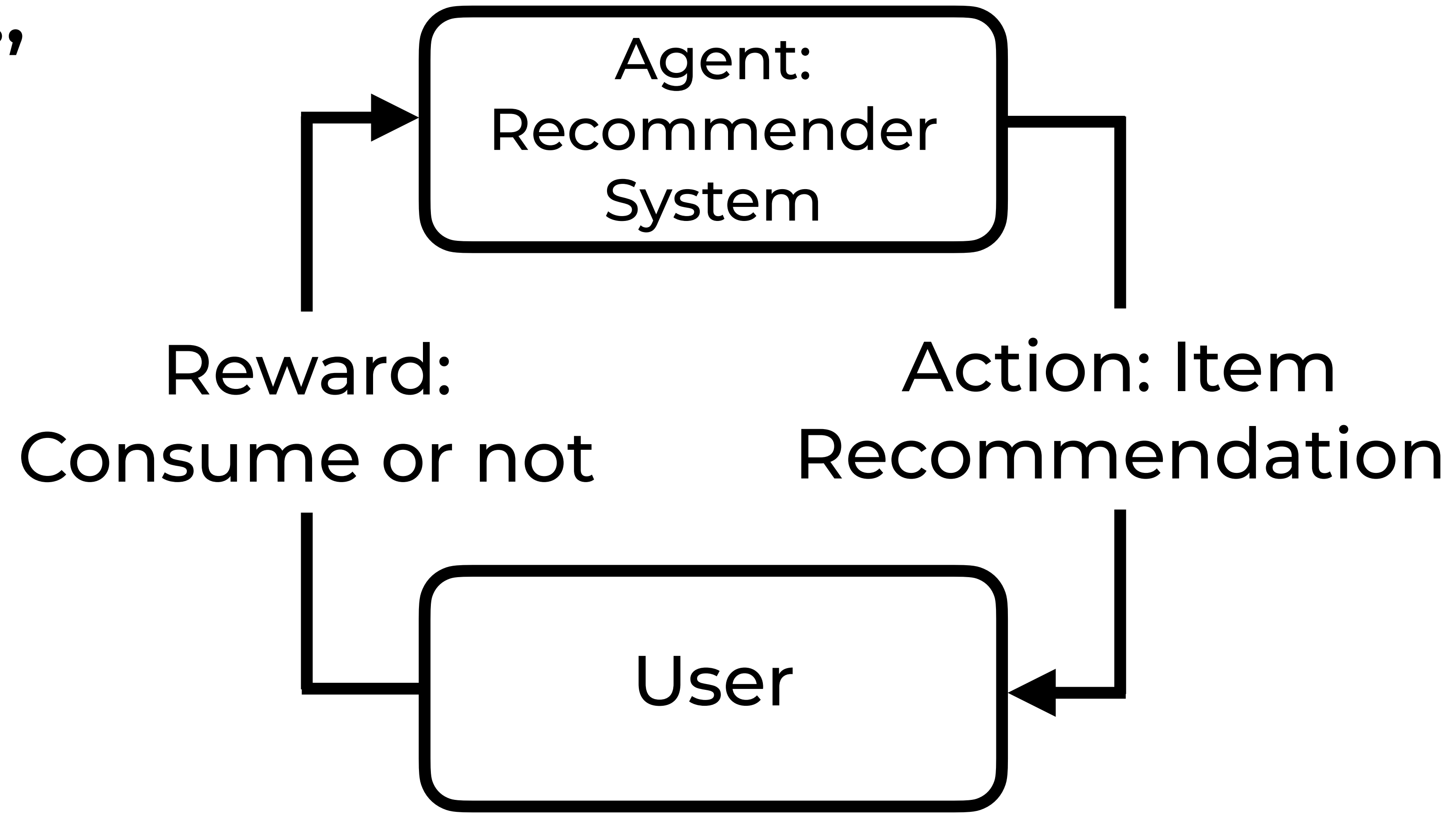
- Interactive systems: recommender systems, chatbots
- Game playing: Backgammon, go
- Healthcare: monitoring systems

Reinforcement learning and recommender systems

- Most users have multiple interactions with the system of time
- Making recommendations over time can be advantageous (e.g., you could better explore one's preferences)
- States: Some representation of user preferences (e.g., previous items they consumed)
- Actions: what to recommend (item 1, item 2, item 3, ...)
- Reward:
  - + user consumes the recommendation
  - - user does not consume the recommendation



Challenges of reinforcement learning

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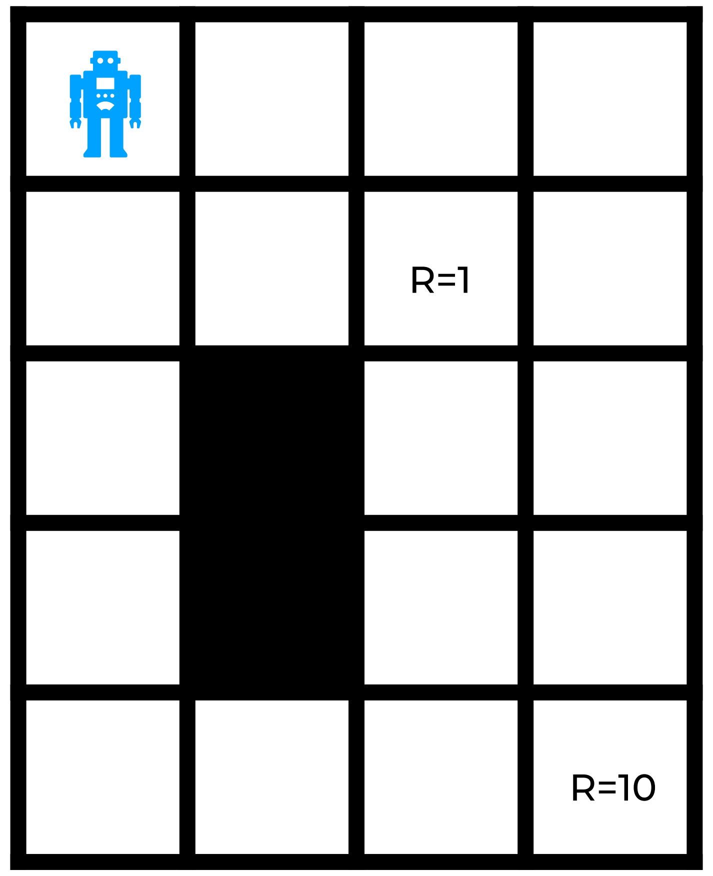
  - A series of actions (getting coffee from cafeteria)
  - A small number of actions several time steps ago may be important (test taking: study before, getting grade long after)

Challenges of reinforcement learning

- Credit assignment problem: which action(s) should be credited for obtaining a reward
    - A series of actions (getting coffee from cafeteria)
    - A small number of actions several time steps ago may be important (test taking: study before, getting grade long after)
- Exploration/Exploitation tradeoff: As agent interacts should it exploit its current knowledge (*exploitation*) or seek out additional information (*exploration*)



Algorithms for Reinforcement Learning

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  - actions, states, discount factor
  - starting state, method for obtaining next state

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- In practice: need a simulator or a real environment for your agent to interact

Algorithms for RL

- **Two main classes of approach**

Algorithms for RL

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1. Model-based

    - Learns a model of the transition and uses it to optimize a policy given the model

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Algorithms for RL

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1. Model-based
   - Learns a model of the transition and uses it to optimize a policy given the model — P(s' | s, a)

2. Model-free
   - Learns an optimal policy without explicitly learning transitions — π

Monte Carlo Methods

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- Assume the environment is episodic
  - Think of playing a card game (like poker). An episode is a hand.
  - Updates the policy after each episode

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- Model-free
- Assume the environment is episodic
  - Think of playing a card game (like poker). An episode is a hand.
  - Updates the policy after each episode
- Intuition
  - Experience many episodes
    - Play many hands (of poker)
  - Average the rewards received at each state
    - What is the proportion of wins given your curent cards

Prediction vs. control

1. Prediction: evaluate a given policy
2. Control: Learn a policy

- Sometimes also called
  - passive (prediction)
  - active (control)

First-visit Monte Carlo

- Given a fixed policy (prediction)
- Calculate the value function V(s) for each state

First-visit MC prediction, for estimating $V \approx v_\pi$

Initialize:
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[Sutton & Barto, RL Book, Ch 5]

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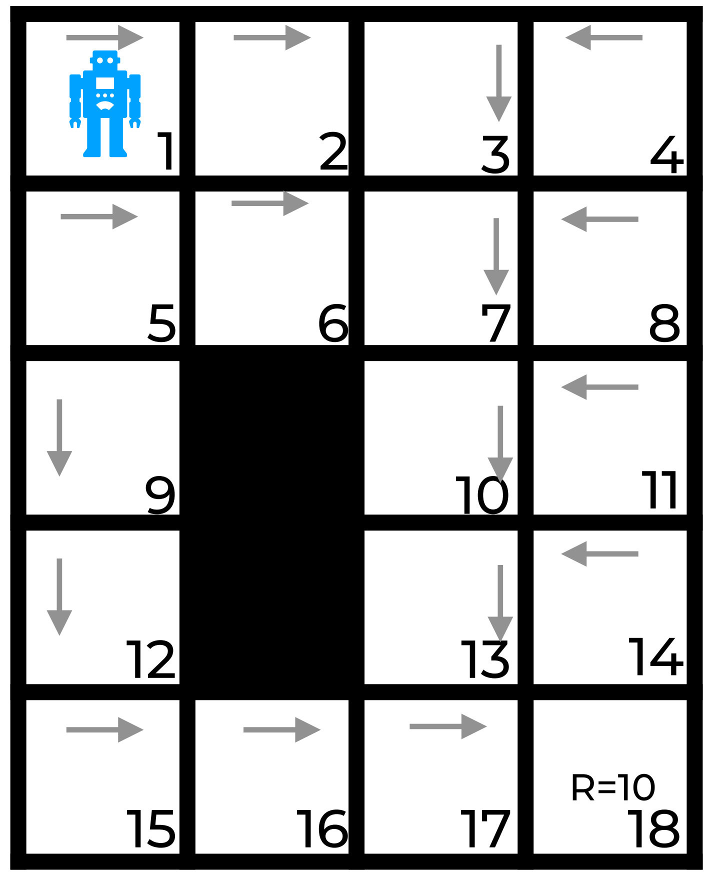
$$V(s_t) = \max_{a_t} \left\{ R(s_t) + \gamma \sum_{s_{t+1}} P(s_{t+1} \mid s_t, a_t) V(s_{t+1}) \right\}$$

Example: grid world



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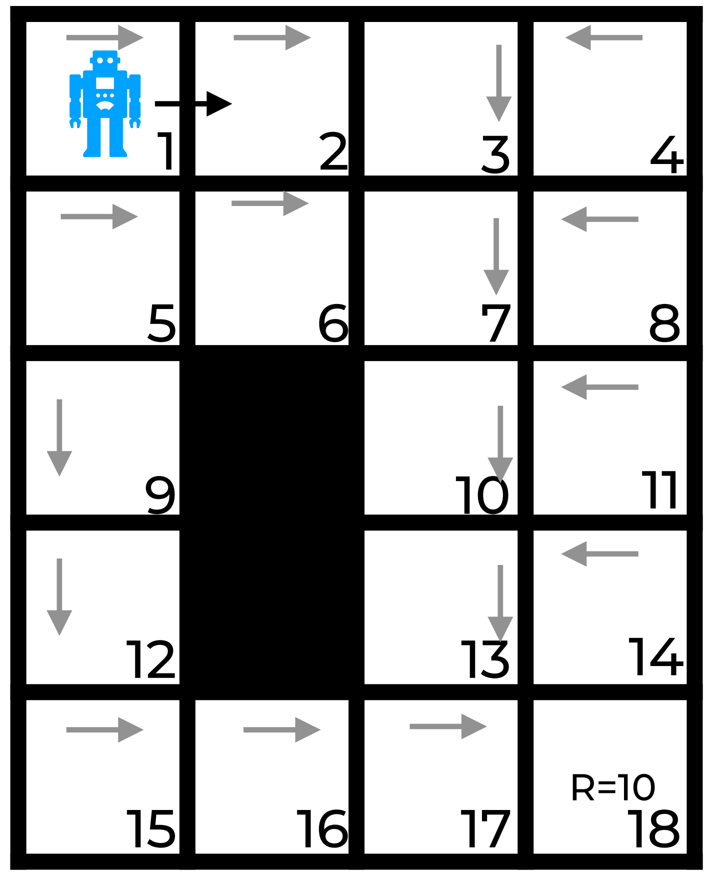

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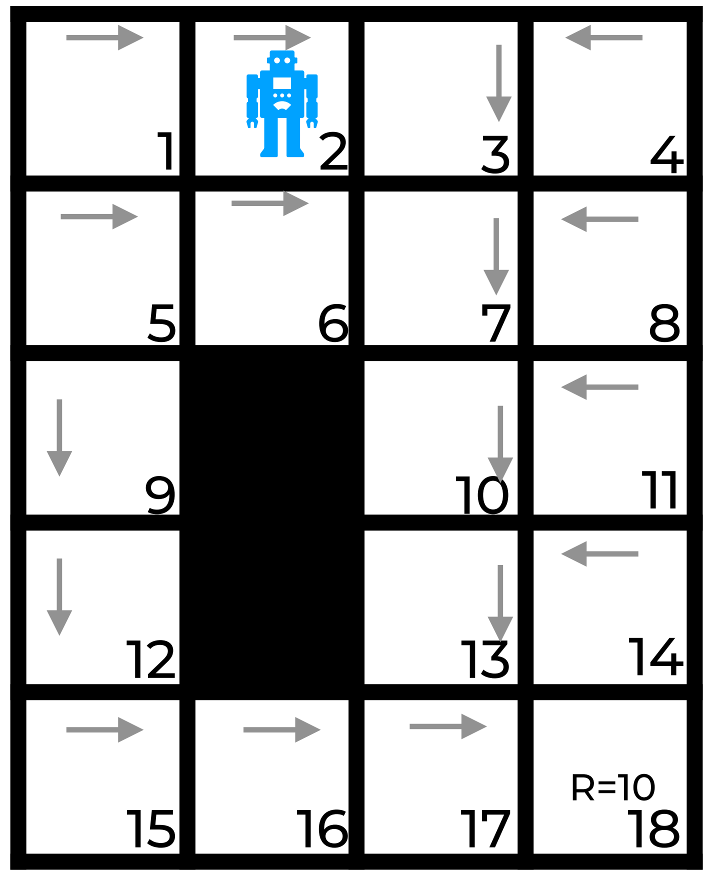

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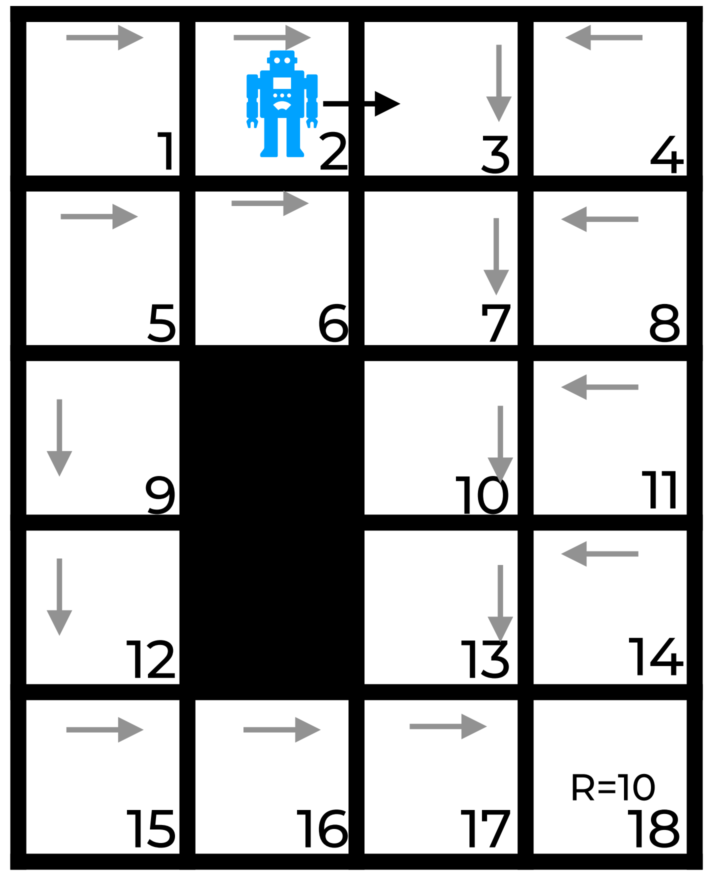

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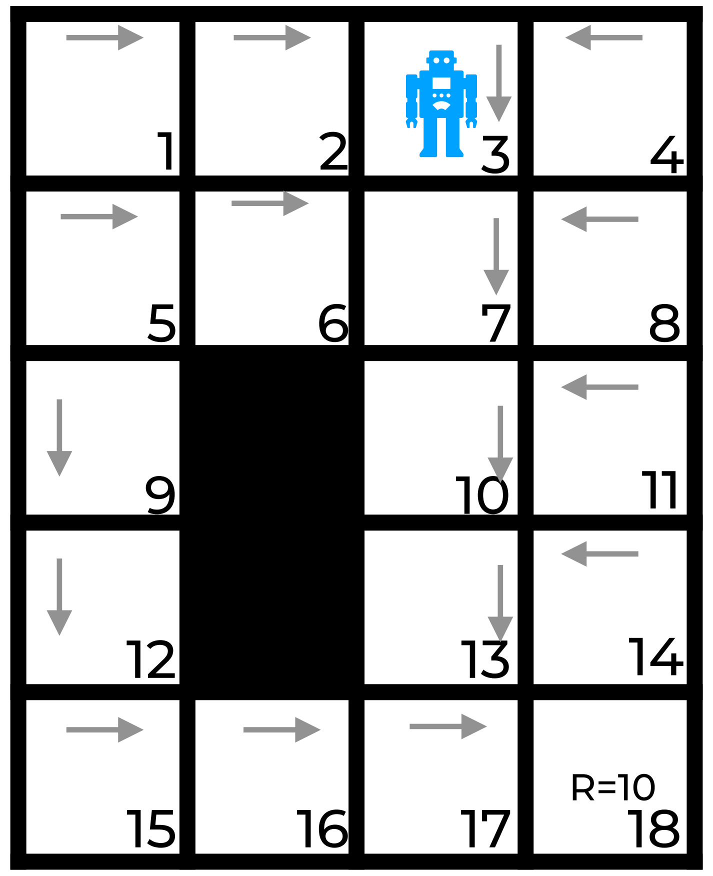

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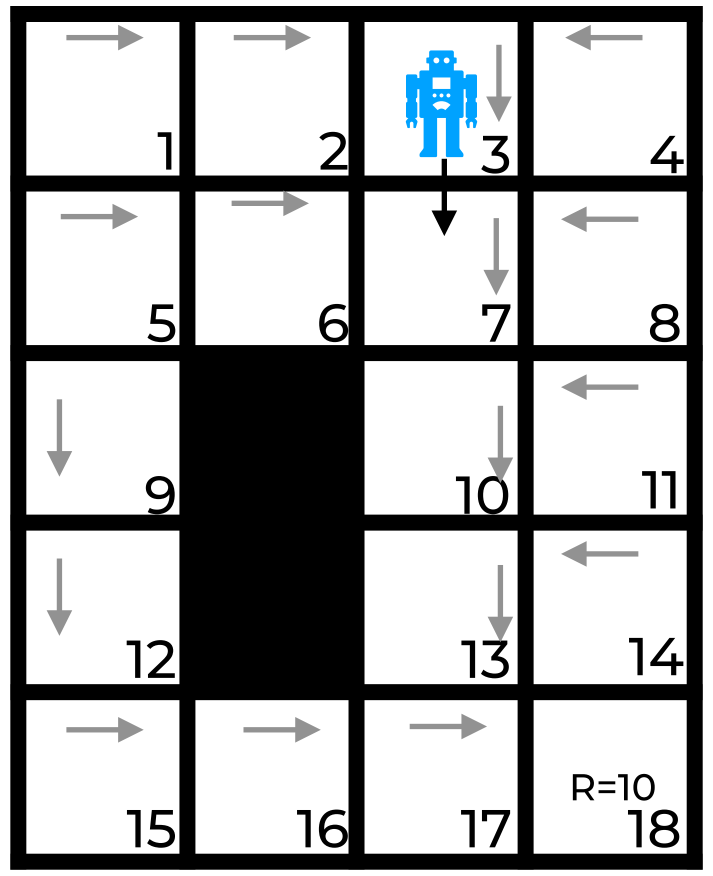

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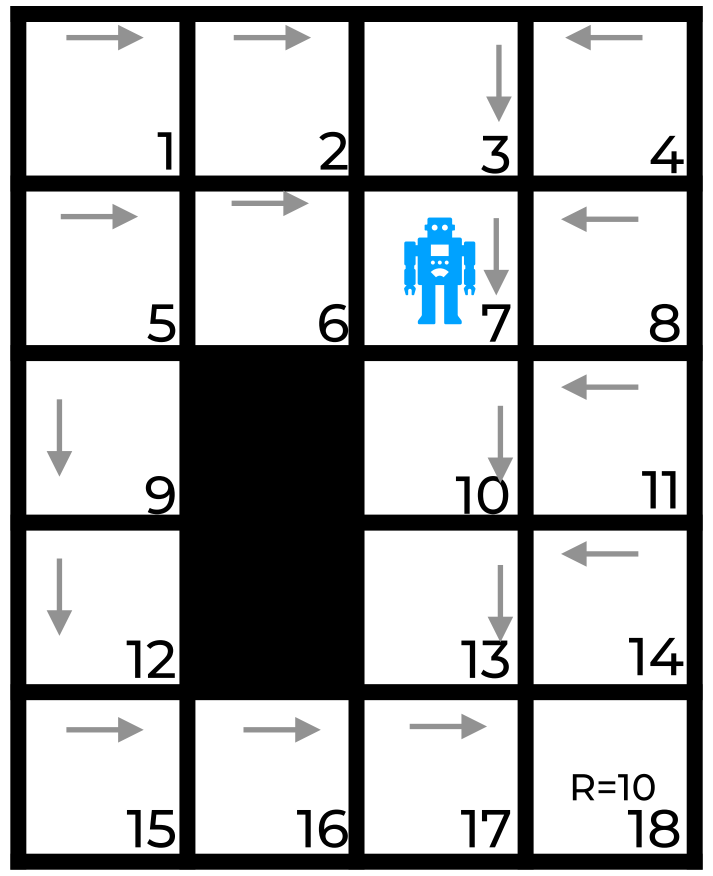

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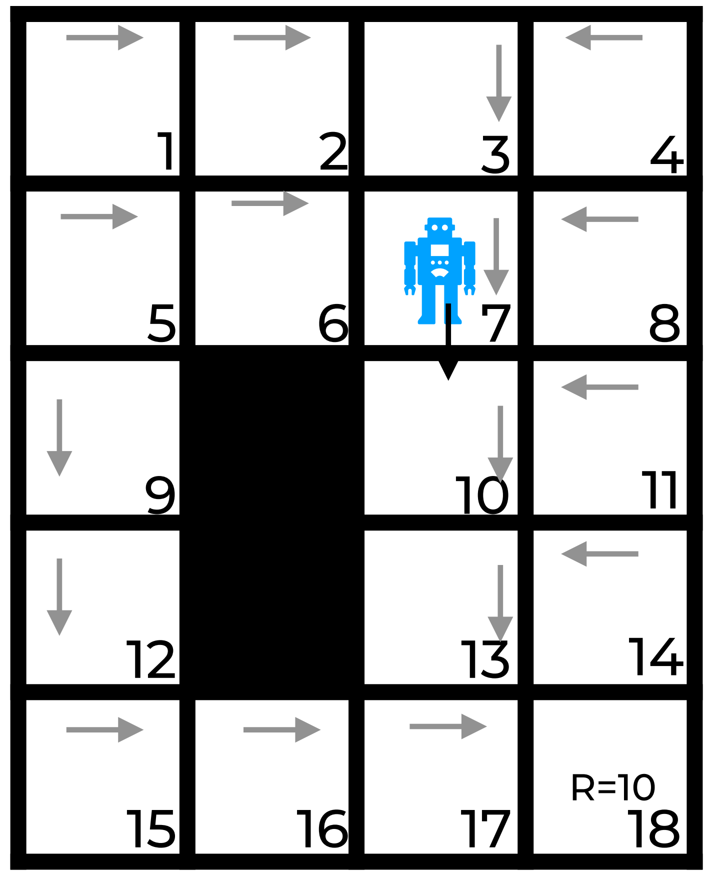

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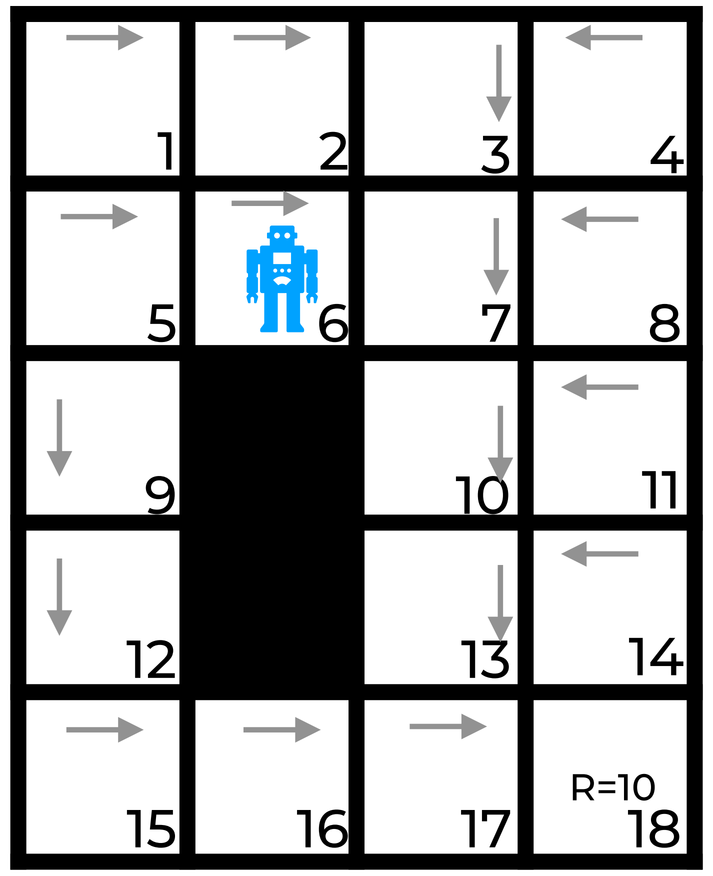

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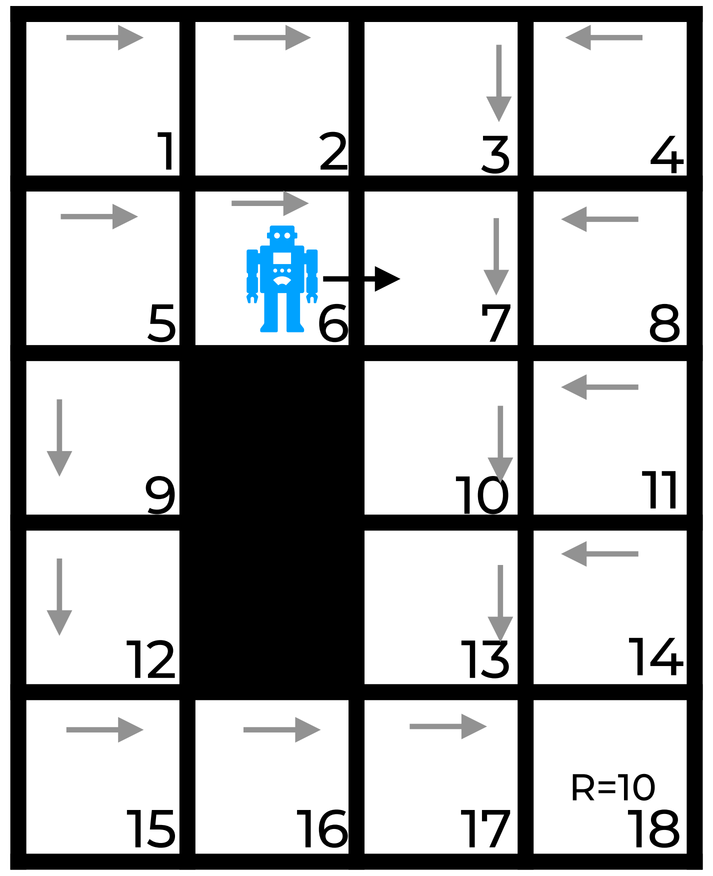

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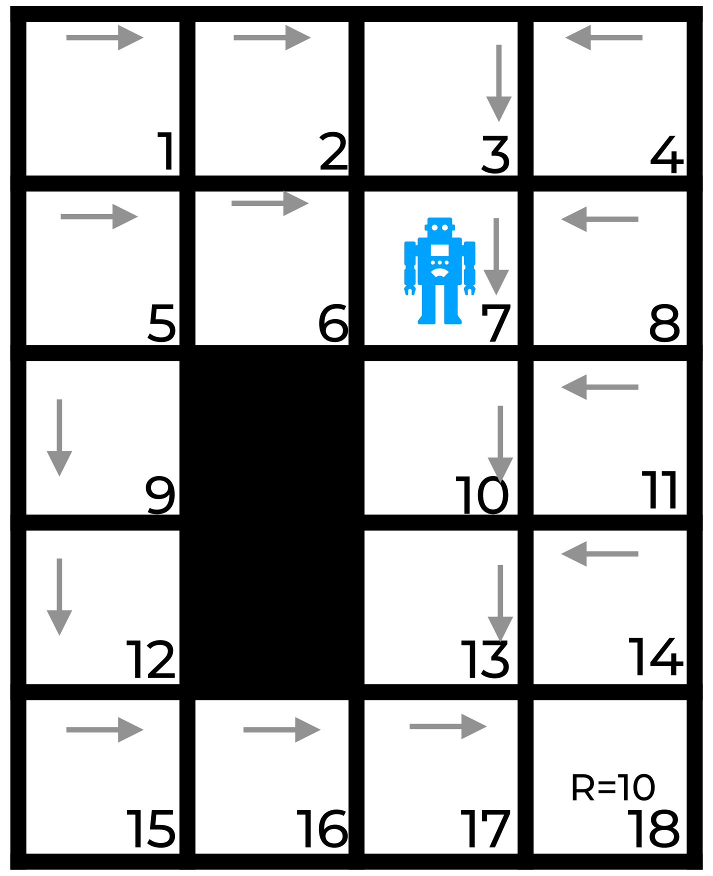

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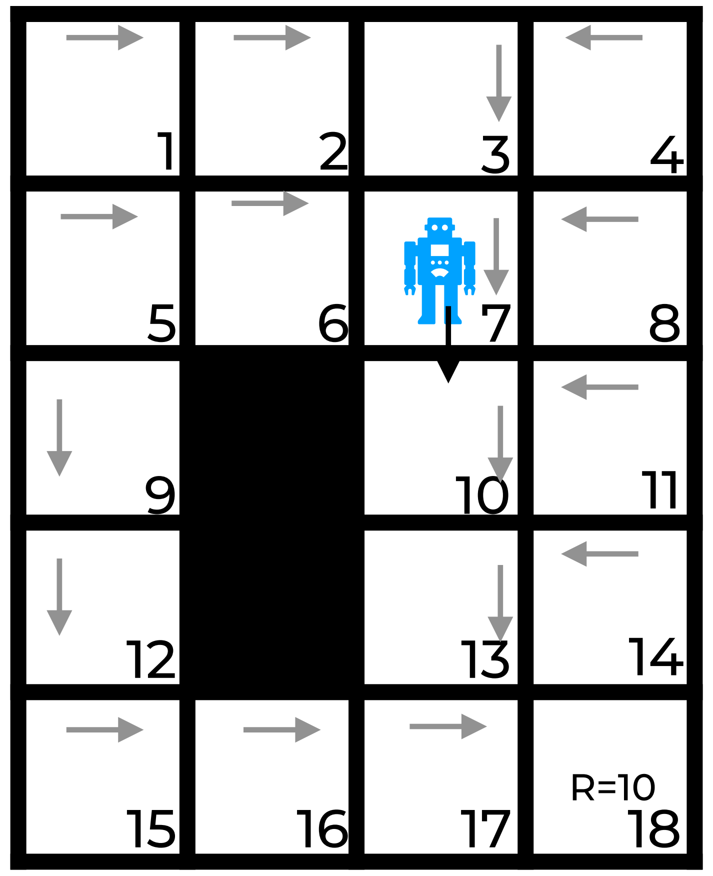

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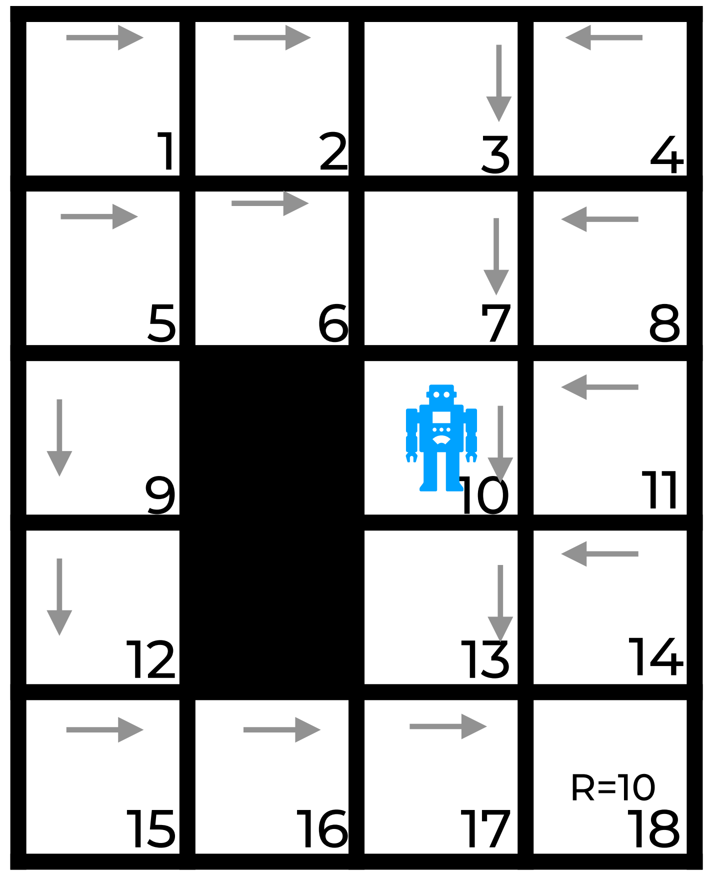

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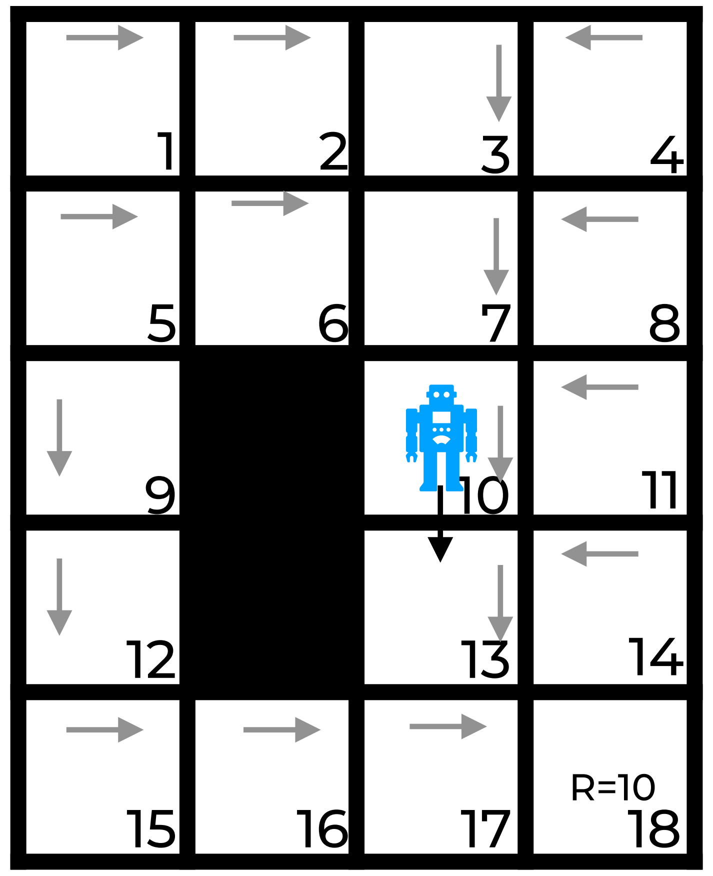

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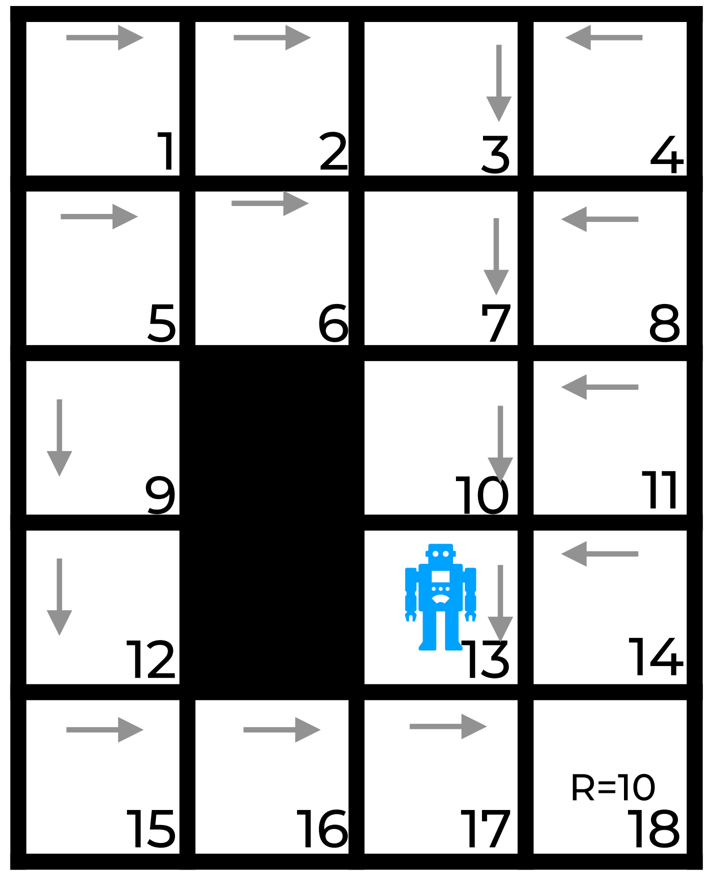

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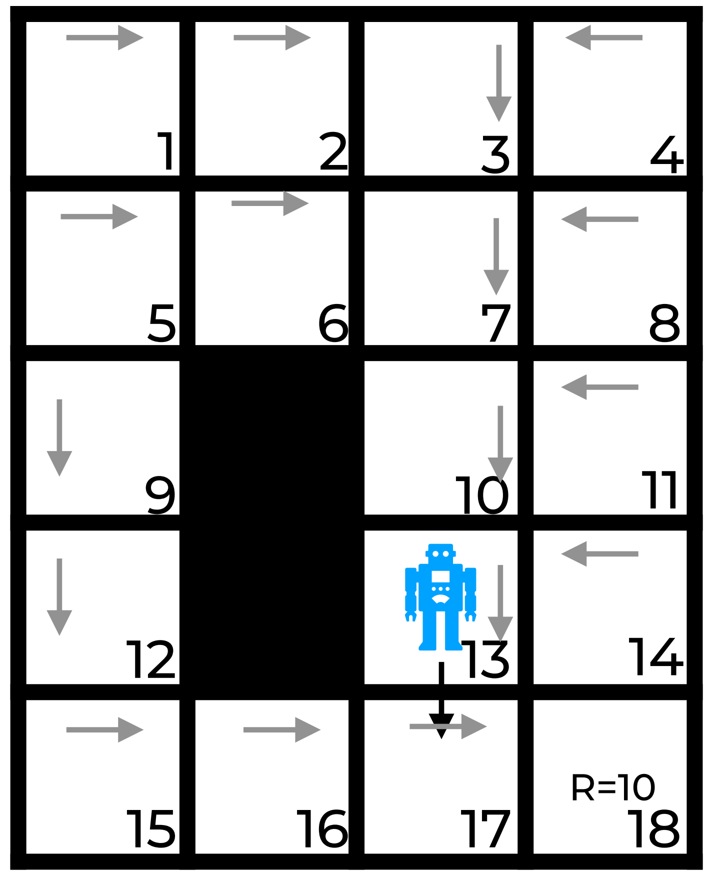

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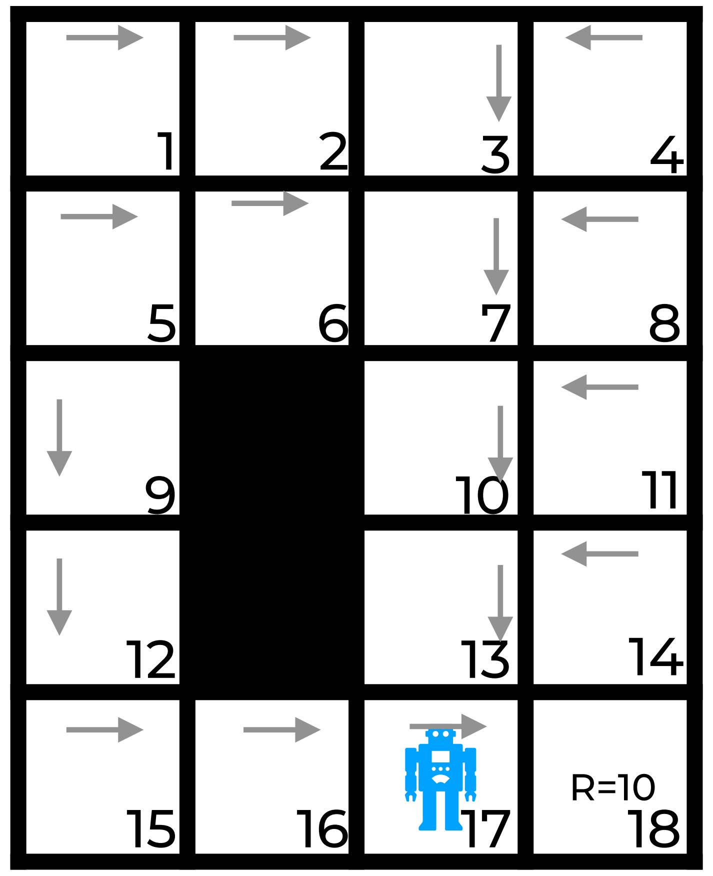
Example: grid world



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First-visit MC prediction, for estimating $V \approx v_\pi$

Initialize:
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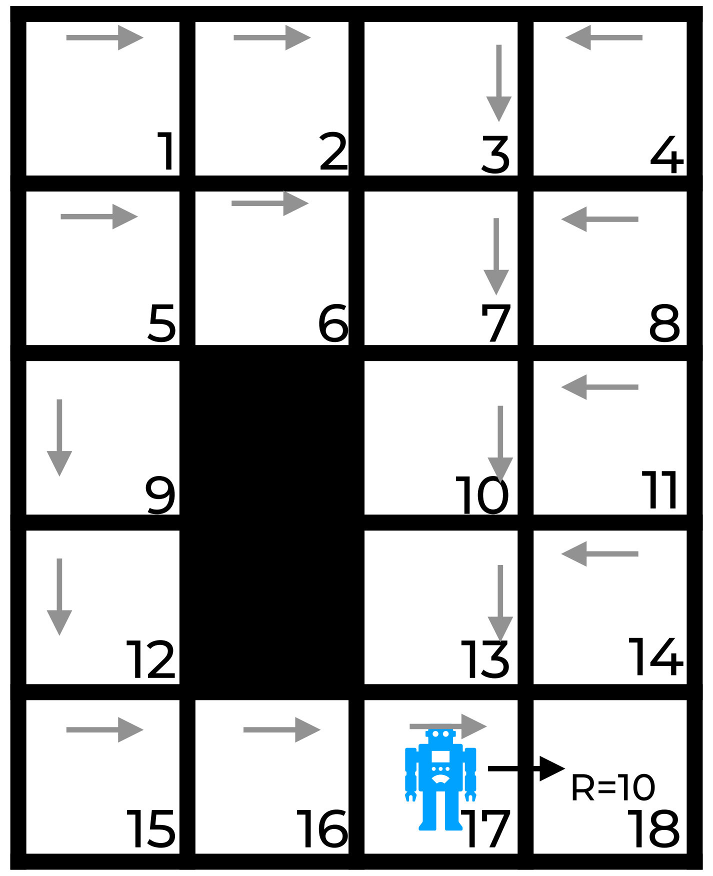

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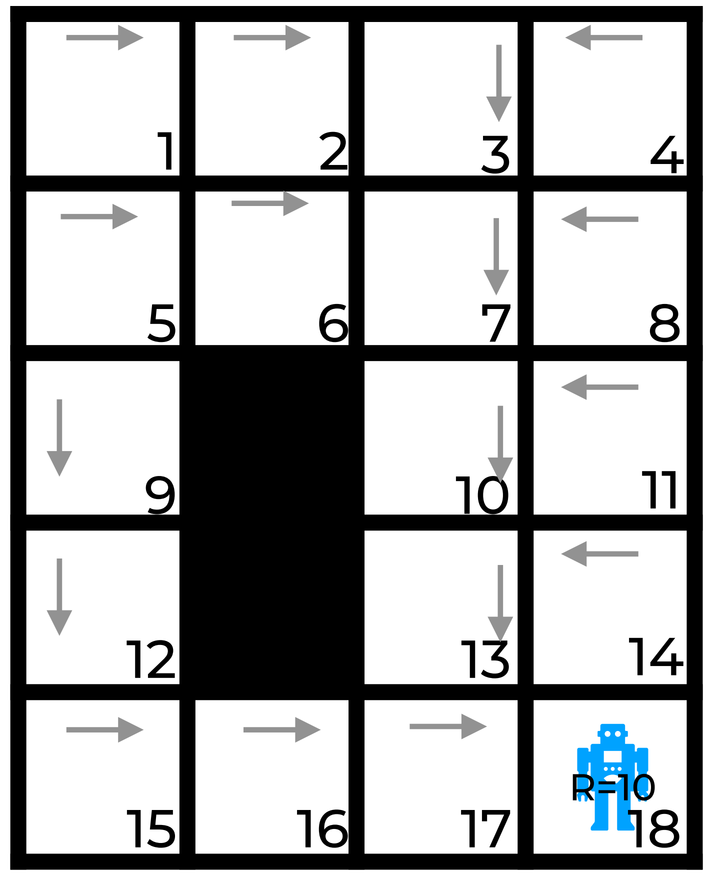

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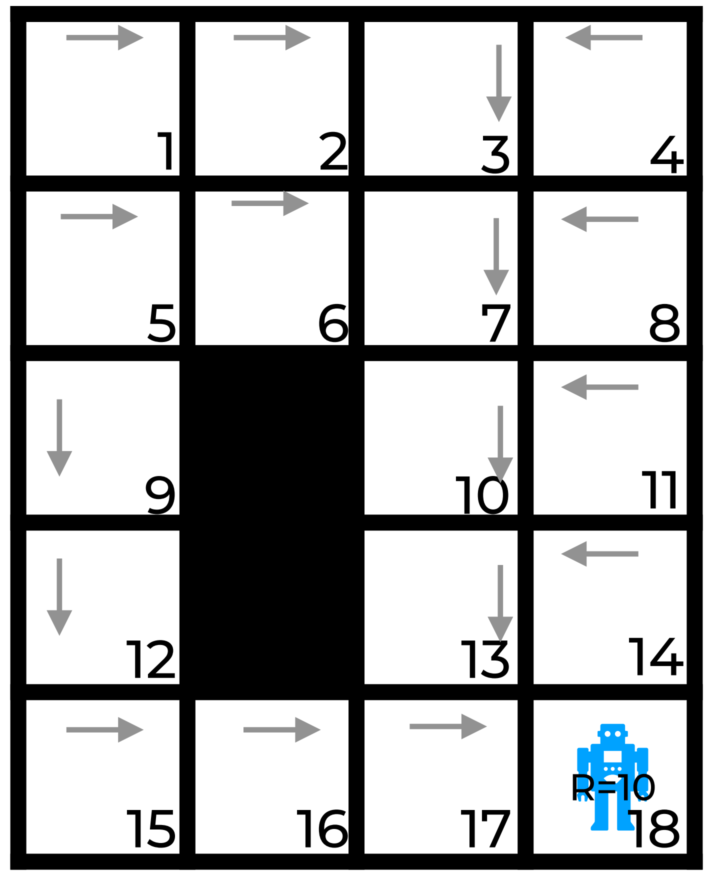

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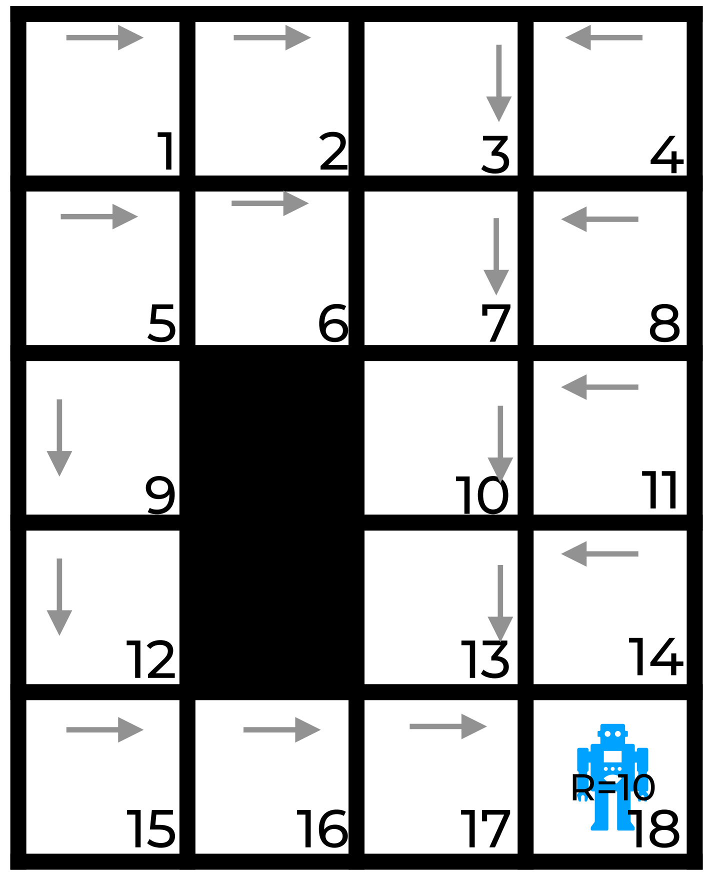

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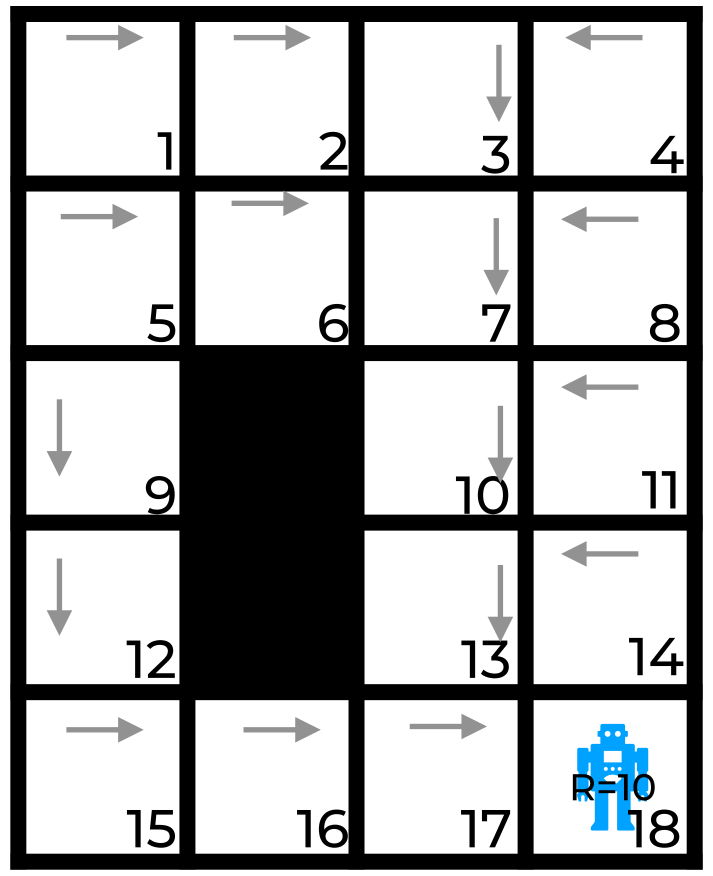

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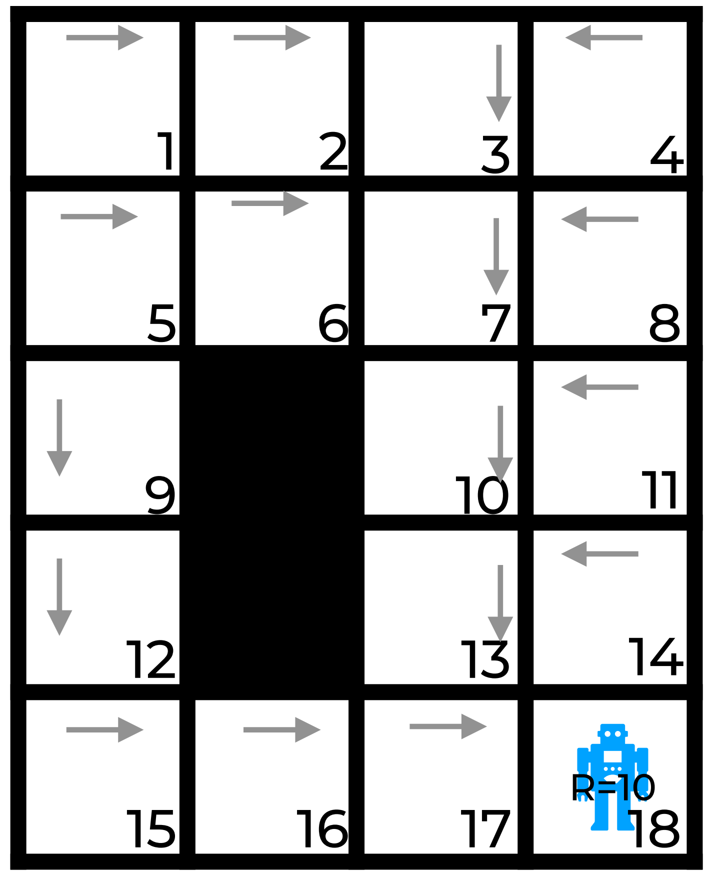
For state 7:

Example: grid world



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$$= \gamma^6 10$$

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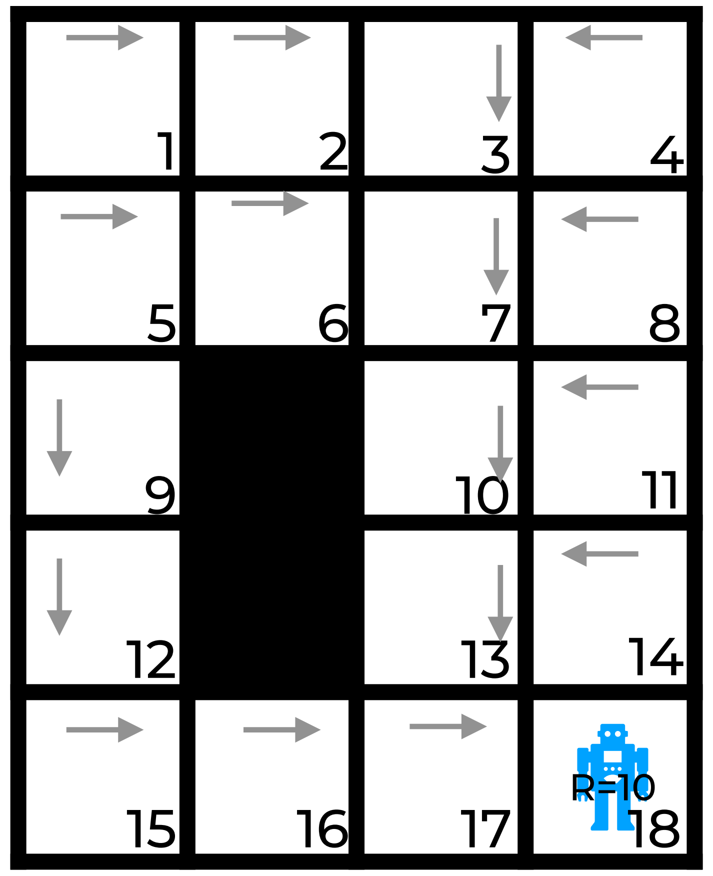

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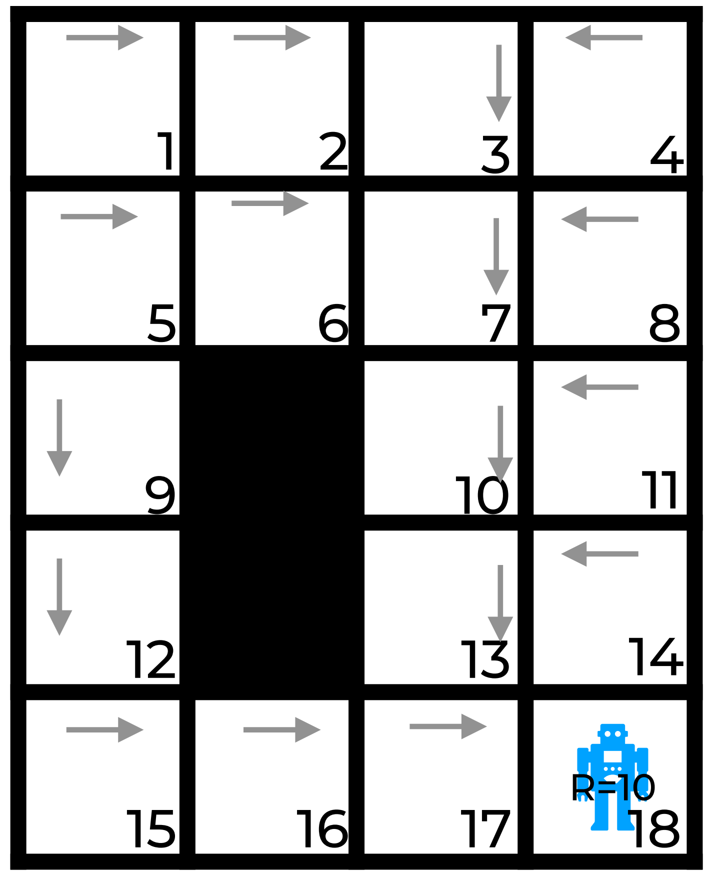

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$$V(7) = \gamma^6 * 10$$

Summary

- Introduced terminology:
  - model based, model-free
- First algorithm for policy evaluation (First-visit MC)
- Compared to MDPs
  - We the agent now has to explore the world to evaluate its value function

Algorithms for RL Control

Q-value function for control

- **We know about state-value functions V(s)**

Q-value function for control

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  - If state transitions are known then they can be used to derive an optimal policy [recall value iteration]:

$$\boldsymbol{\pi}^*(\mathbf{s}) = \arg\max_{\mathbf{a}} \left\{ \mathbf{R}(\mathbf{s}) + \gamma \sum_{\mathbf{s}'} \mathbf{P}(\mathbf{s}' \mid \mathbf{s}, \mathbf{a}) \mathbf{V}^*(\mathbf{s}') \right\} \; \forall \mathbf{s}$$

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  - When state transitions are unknown what can we do?
    - Q(s,a) the value function of a (state,action) pair

$$\pi^*(s) = \arg\max_{a} \{ Q^*(s, a) \} \ \forall s$$

Monte Carlo ES (control)

Monte Carlo ES (Exploring Starts), for estimating $\pi \approx \pi_*$

Initialize, for all $s \in \mathcal{S}$, $a \in \mathcal{A}(s)$:
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Repeat forever:
  Choose $S_0 \in \mathcal{S}$ and $A_0 \in \mathcal{A}(S_0)$ s.t. all pairs have probability > 0
  Generate an episode starting from S_0, A_0, following π
  For each pair s, a appearing in the episode:
    $G \leftarrow$ the return that follows the first occurrence of s, a
    Append G to $Returns(s, a)$
    $Q(s, a) \leftarrow \text{average}(Returns(s, a))$
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[Sutton & Barto, RL Book, Ch.5]

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- Strong reasons to believe that it converges to the optimal policy

- “Exploring starts” requirement may be unrealistic

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- “Exploring starts” insures that all states can be visited regardless of the policy
  - (Specific policy may not visit all states)
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  - (Specific policy may not visit all states)
  - Unrealistic in real-world settings
- Solution: inject some uncertainty in the policy

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On-policy first-visit MC control (for ε-soft policies), estimates $\pi \approx \pi_*$

Initialize, for all $s \in \mathcal{S}$, $a \in \mathcal{A}(s)$:
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  (c) For each s in the episode:
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    For all $a \in \mathcal{A}(s)$:

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- Policy value cannot decrease

$$v_{\pi'}(s) \geq v_\pi(s), \forall s \in S$$

π : policy at current step
π' : policy at next step

Monte-Carlo methods summary

- Allow a policy to be learned through interactions
  - (Does not learn transitions)
- States are effectively treated as being independent
  - Focus on a subset of states (e.g., states for which playing optimally is of particular importance)
- Episodic (with or without exploring starts)

Temporal Difference (TD) Learning

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$$V'(s_t) = V(s_t) + \alpha[G_t - V(s_t)]$$

Step size

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Step size

- TD(0)
  - updates “instantly”

First-visit MC prediction, for estima

Initialize:
  $\pi \leftarrow$ policy to be evaluated
  $V \leftarrow$ an arbitrary state-value function
  $Returns(s) \leftarrow$ an empty list, for all $s \in \mathcal{S}$

Repeat forever:
  Generate an episode using π
  For each state s appearing in the episode
    $G \leftarrow$ the return that follows the first
    Append G to $Returns(s)$
    $V(s) \leftarrow$ average$(Returns(s))$

Temporal Difference (TD) Learning

- One of the "central ideas of RL" [Sutton & Barto, RL book]

- Monte Carlo methods

Observed returned : $G_t = \sum_{t}^{T} \gamma^t R(s_t)$

$$V'(s_t) = V(s_t) + \alpha [G_t - V(s_t)]$$

Step size (α)

First-visit MC prediction, for estima

Initialize:
  $\pi \leftarrow$ policy to be evaluated
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Repeat forever:
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- TD(0)
  - updates "instantly"

$$V'(s_t) = V(s_t) + \alpha[\underbrace{R(s_t) + \gamma V(s_{t+1})}_{\approx G_t} - V(s_t)]$$

TD(0) for prediction

Tabular TD(0) for estimating v_π

Input: the policy π to be evaluated
Initialize $V(s)$ arbitrarily (e.g., $V(s) = 0$, for all $s \in \mathcal{S}^+$)
Repeat (for each episode):
    Initialize S
    Repeat (for each step of episode):
        $A \leftarrow$ action given by π for S
        Take action A, observe R, S'
        $V(S) \leftarrow V(S) + \alpha\big[R + \gamma V(S') - V(S)\big]$
        $S \leftarrow S'$
    until S is terminal

[Sutton & Barto, RL Book, Ch.6]

TD for control

Sarsa (on-policy TD control) for estimating $Q \approx q_*$

Algorithm parameters: step size $\alpha \in (0, 1]$, small $\varepsilon > 0$
Initialize $Q(s, a)$, for all $s \in \mathcal{S}^+, a \in \mathcal{A}(s)$, arbitrarily except that $Q(terminal, \cdot) = 0$

Loop for each episode:
  Initialize S
  Choose A from S using policy derived from Q (e.g., ε-greedy)
  Loop for each step of episode:
    Take action A, observe R, S'
    Choose A' from S' using policy derived from Q (e.g., ε-greedy)
    $Q(S, A) \leftarrow Q(S, A) + \alpha\big[R + \gamma Q(S', A') - Q(S, A)\big]$
    $S \leftarrow S'$; $A \leftarrow A'$;
  until S is terminal

Tabular TD(0) for estimating v_π

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Initialize $V(s)$ arbitrarily (e.g., $V(s) = 0$, for all $s \in \mathcal{S}^+$)
Repeat (for each episode):
  Initialize S
  Repeat (for each step of episode):
    $A \leftarrow$ action given by π for S
    Take action A, observe R, S'
    $V(S) \leftarrow V(S) + \alpha\big[R + \gamma V(S') - V(S)\big]$
    $S \leftarrow S'$
  until S is terminal

Comparing TD and MC

- MC requires going through full episodes before updating the value function. Episodic.

- Converges to the optimal solution

- TD updates each V(s) after each transition. Online.

- Converges to the optimal solution (some conditions on α)

- Empirically TD methods tend to converge faster

Q-learning for control

Q-learning (off-policy TD control) for estimating $\pi \approx \pi_*$

Initialize $Q(s, a)$, for all $s \in \mathcal{S}, a \in \mathcal{A}(s)$, arbitrarily, and $Q(\textit{terminal-state}, \cdot) = 0$
Repeat (for each episode):
    Initialize S
    Repeat (for each step of episode):
        Choose A from S using policy derived from Q (e.g., ϵ-greedy)
        Take action A, observe R, S'
        $Q(S, A) \leftarrow Q(S, A) + \alpha\big[R + \gamma \max_a Q(S', a) - Q(S, A)\big]$
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[Sutton & Barto, RL Book, Ch.6]

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[Sutton & Barto, RL Book, Ch.6]

ϵ -greedy policy

$$a = \begin{cases} \arg\max_a Q(a,s) & \text{with probability } 1-\epsilon, \\ \text{random } a & \text{with probability } \epsilon. \end{cases}$$

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[Sutton & Barto, RL Book, Ch.6]

- Converges to Q* as long as all (s,a) pairs continue to be updated and with minor constraints on learning rate

Summary

- Introduced algorithms for learning the optimal policy
  - Monte Carlo and TD methods
  - On-policy and off-policy methods

Practical difficulties

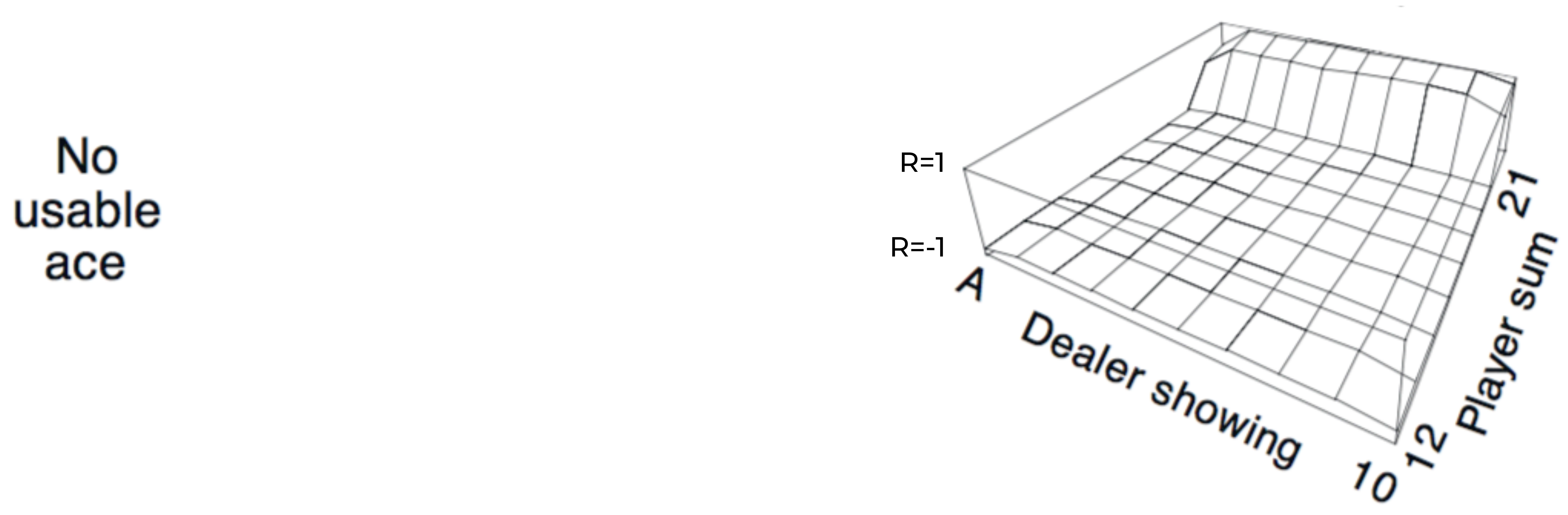
- Compared to supervised learning setting up an RL problem is often harder
  - Need an environment (or at least a simulator)
- Rewards
  - In some domains it's clear (e.g., in games)
  - In others it's much more subtle (e.g., you want to please a human)

Extra material
(Some will be used for this week's exercises)

Example: Black Jack 🂡

- Episode: one hand
- States: Sum of player’s cards, dealer’s card, usable ace
- Actions: {Stay, Hit}
- Rewards: {Win +1, Tie 0, Loose -1}
- A few other assumptions: infinite deck

- **Evaluates the policy that hits except when the sum of the cards is 20 or 21**

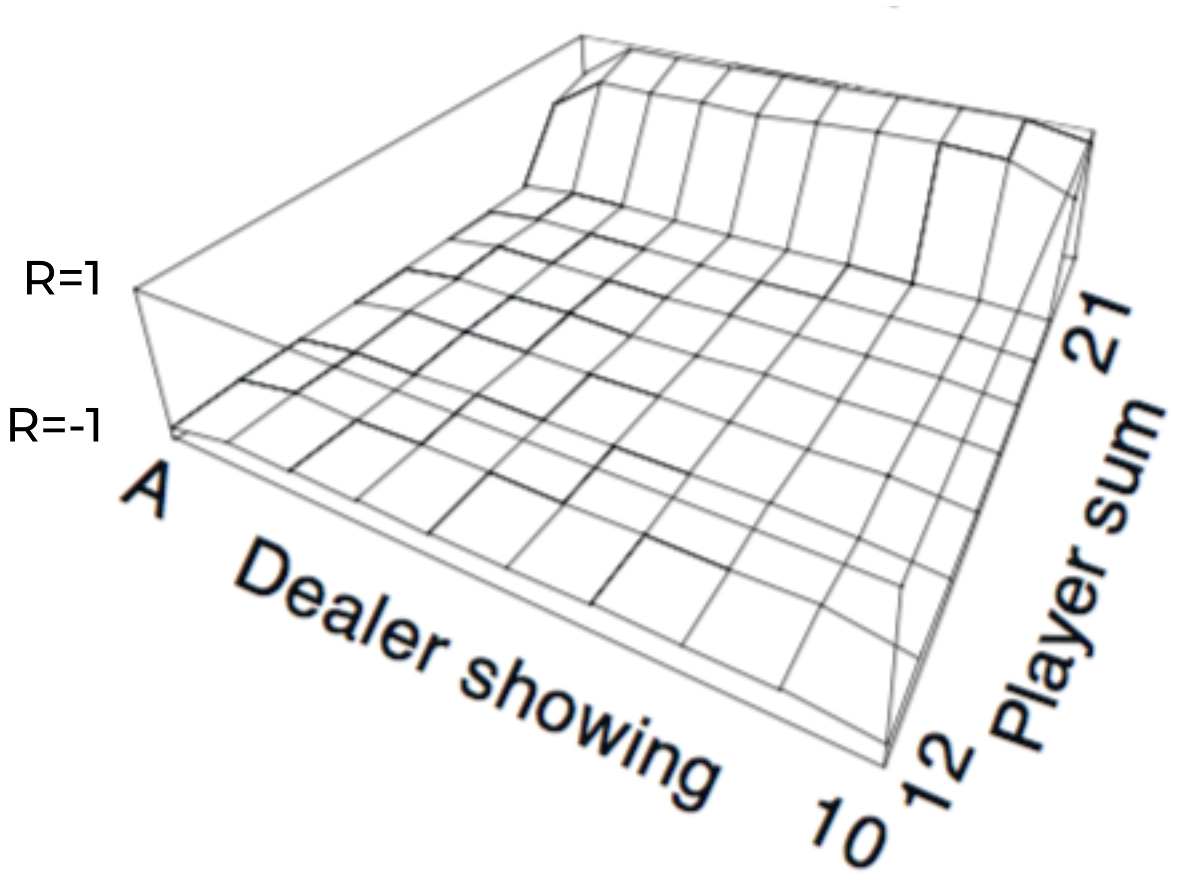


[Figure 5.1, Sutton & Barto]

- **Evaluates the policy that hits except when the sum of the cards is 20 or 21**

Usable ace

No usable ace



[Figure 5.1, Sutton & Barto]

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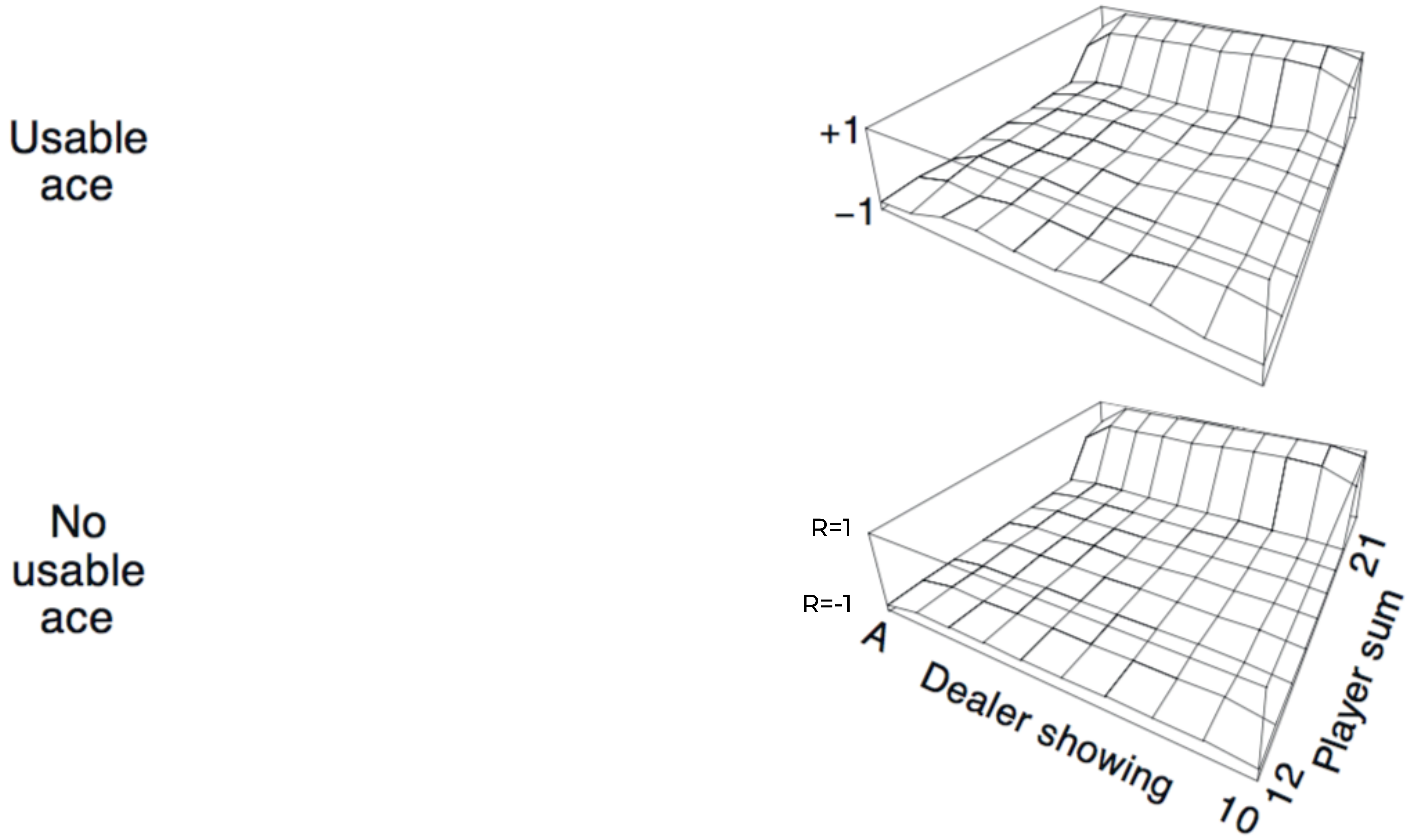

[Figure 5.1, Sutton & Barto]

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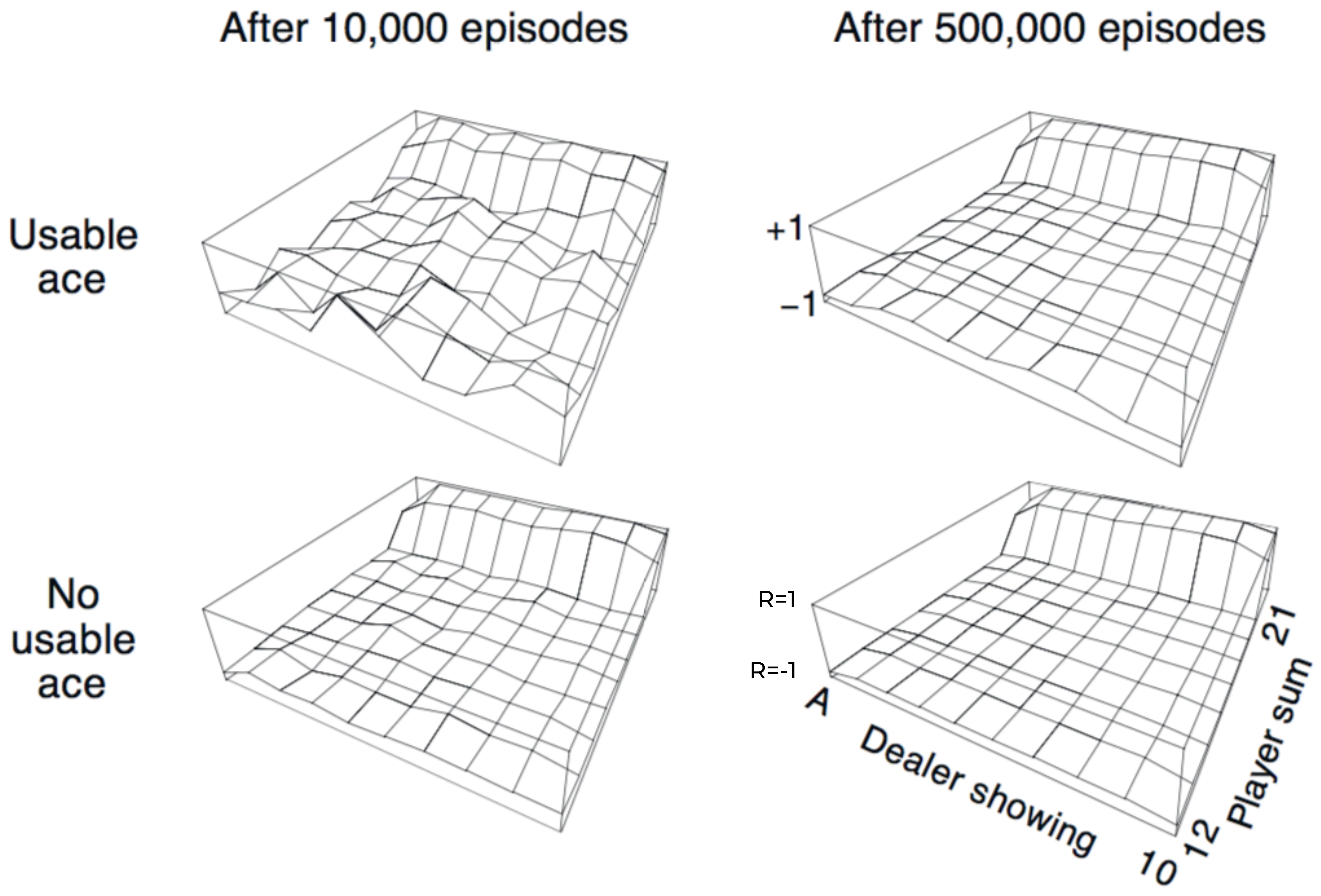

[Figure 5.1, Sutton & Barto]

Approximation techniques

- Methods we studied are "tabular"
- State value functions (and Q) can be approximated
  - Linear approximation: $\mathbf{V}(\mathbf{s}) = \mathbf{w}^\top \mathbf{x}(\mathbf{s})$
    - Coupling between states through x(s)
  - Adapt the algorithms for this case.
    - Updates to the value function now imply updating the weights w using a gradient

Approximation techniques (prediction)

- Linear approximation: $V(s) = w^\top x(s)$

Approximation techniques (prediction)

- **Linear approximation:** $V(s) = w^\top x(s)$
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[Sutton & Barto, RL Book, Ch.9]

Gradient Monte Carlo Algorithm for Estimating $\hat{v} \approx v_\pi$

Input: the policy π to be evaluated
Input: a differentiable function $\hat{v} : \mathcal{S} \times \mathbb{R}^d \to \mathbb{R}$

Initialize value-function weights $\mathbf{w}$ as appropriate (e.g., $\mathbf{w} = \mathbf{0}$)
Repeat forever:
  Generate an episode $S_0, A_0, R_1, S_1, A_1, \ldots, R_T, S_T$ using π
  For $t = 0, 1, \ldots, T-1$:
    $\mathbf{w} \leftarrow \mathbf{w} + \alpha\left[G_t - \hat{v}(S_t,\mathbf{w})\right]\nabla\hat{v}(S_t,\mathbf{w})$

First-visit MC prediction, for estimating $V \approx v_\pi$

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[Sutton & Barto, RL Book, Ch.9]

G_t is an unbiased estimator of $v_\pi(s_t)$

Approximation techniques

- **Works both for prediction and control**

Approximation techniques

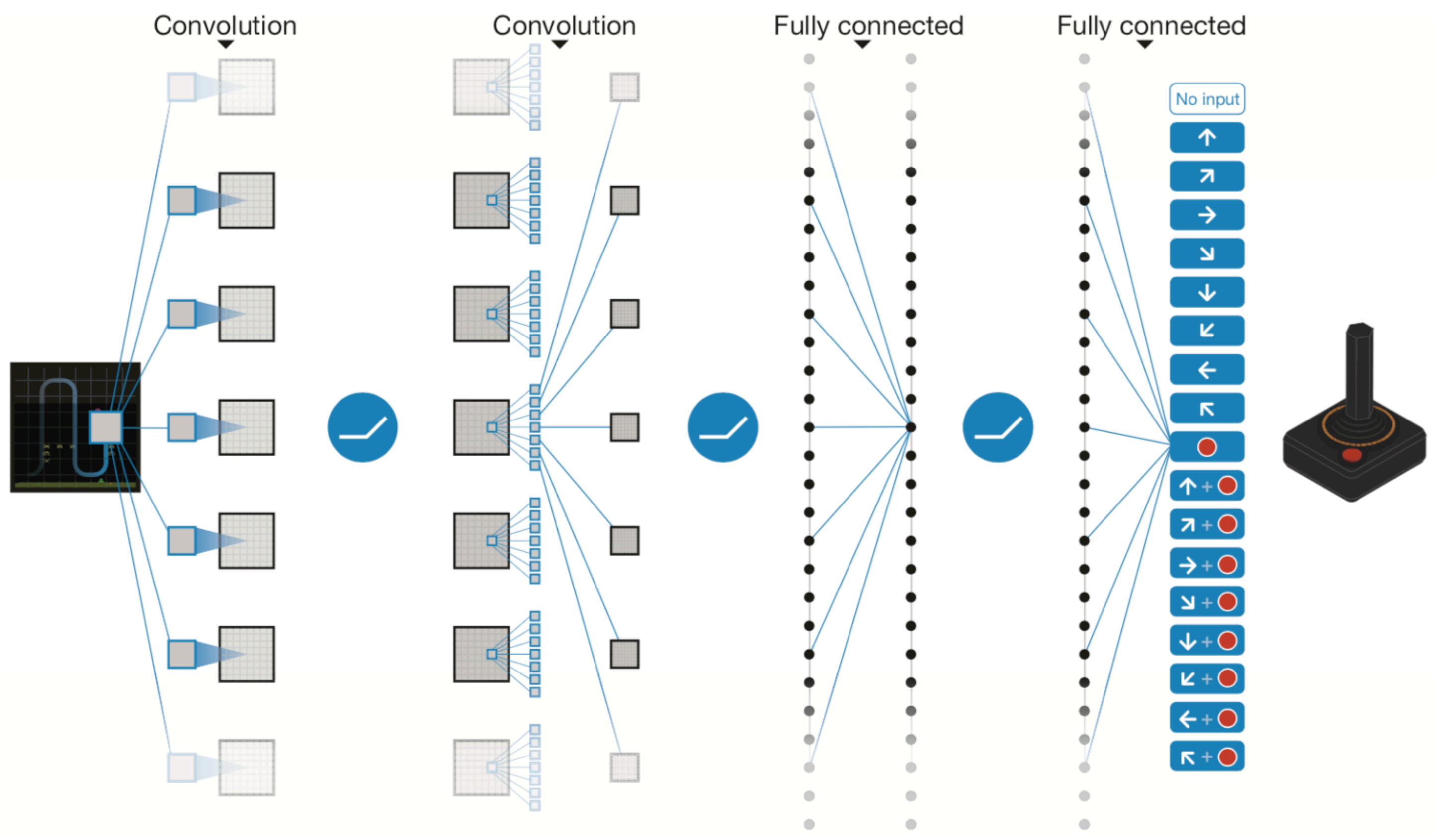
- Works both for prediction and control
- Any model can be used to approximate

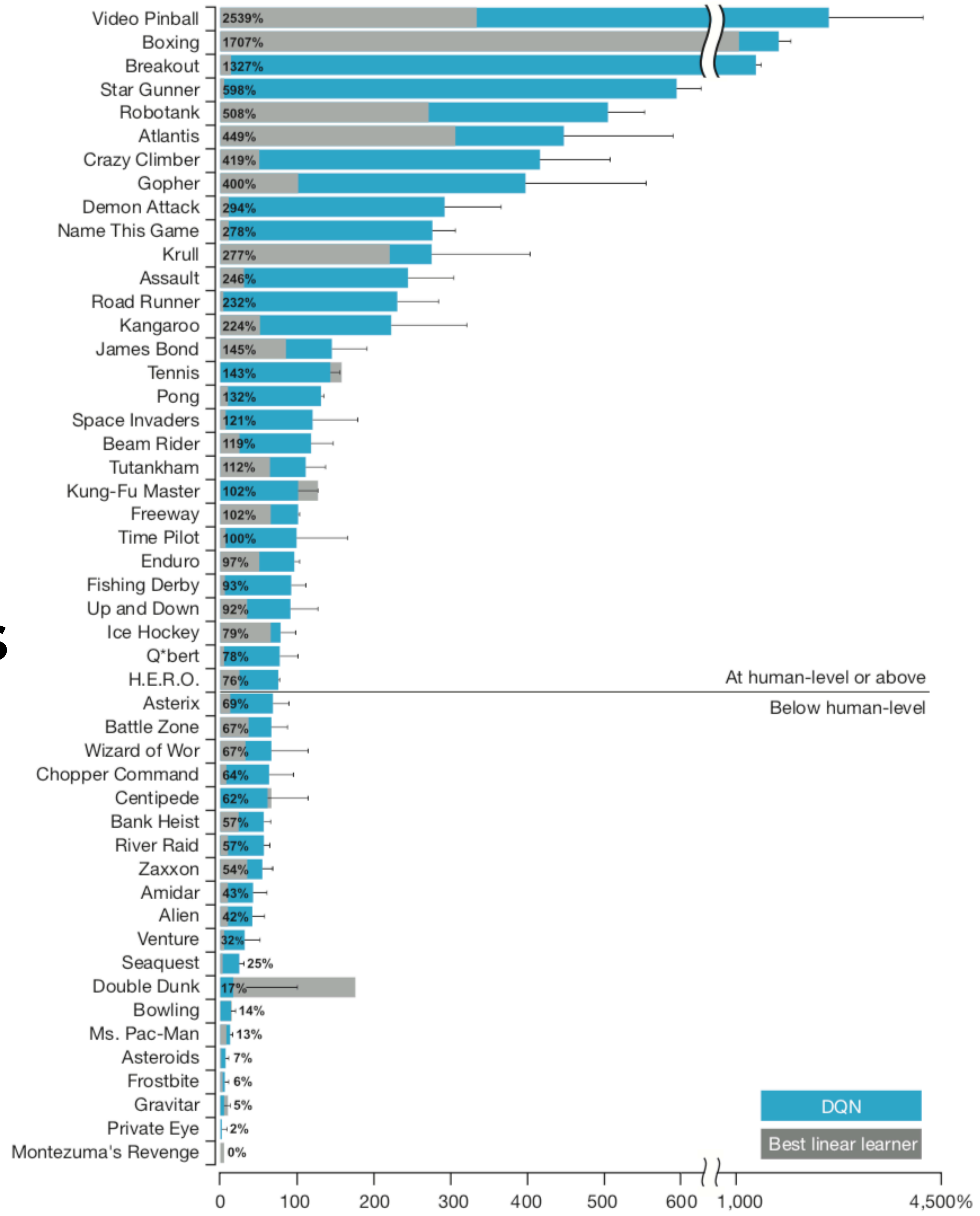
Approximation techniques

- Works both for prediction and control

- Any model can be used to approximate

- Recent work using deep neural networks yield impressive performance on computer (Atari) games





Summary

- Today we have defined RL studied several algorithms for solving RL problems (mostly for for tabular case)
- Main challenges
  - Credit assignment
  - Exploration/Exploitation tradeoff
- Algorithms
  - Prediction
    - Monte Carlo and TD(0)
  - Control
    - Q-learning
- Approximation algorithms can help scale reinforcement learning

Practical difficulties

- Compared to supervised learning setting up an RL problem is often harder
    - Need an environment (or at least a simulator)
- Rewards
    - In some domains it's clear (e.g., in games)
    - In others it's much more subtle (e.g., you want to please a human)

Acknowledgements

- The algorithms are from “Reinforcement Learning: An Introduction” by Richard Sutton and Andrew Barto
  - The definitive RL reference
- Some of these slides were adapted from Pascal Poupart’s slides (CS686 U.Waterloo)
- The TD demo is from Andrej Karpathy