

Machine Learning I

MATH60629A

Modern Generative Models
— Week #11

Rapid evolution of (image) generation capability



2014



2015



2016



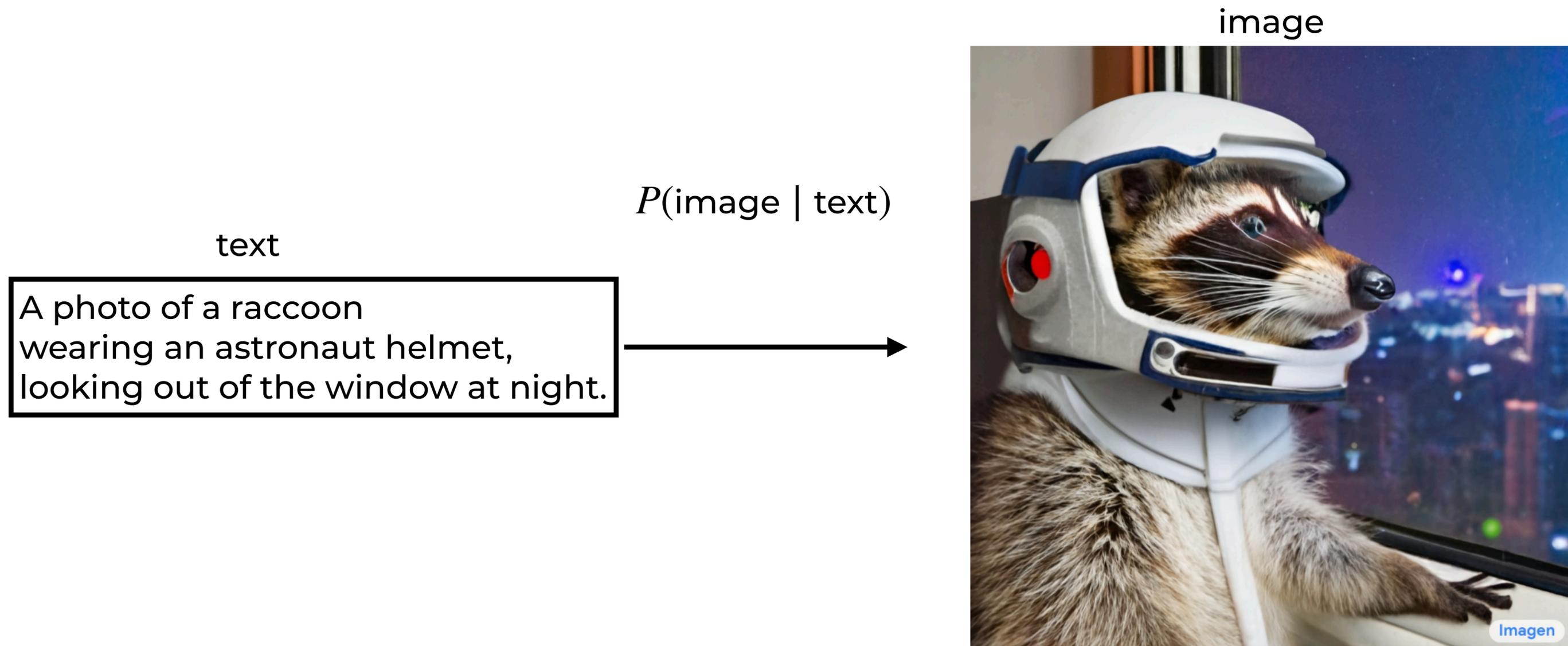
2017



2018

$P(\text{image})$

Conditional Generation



Why generate images?



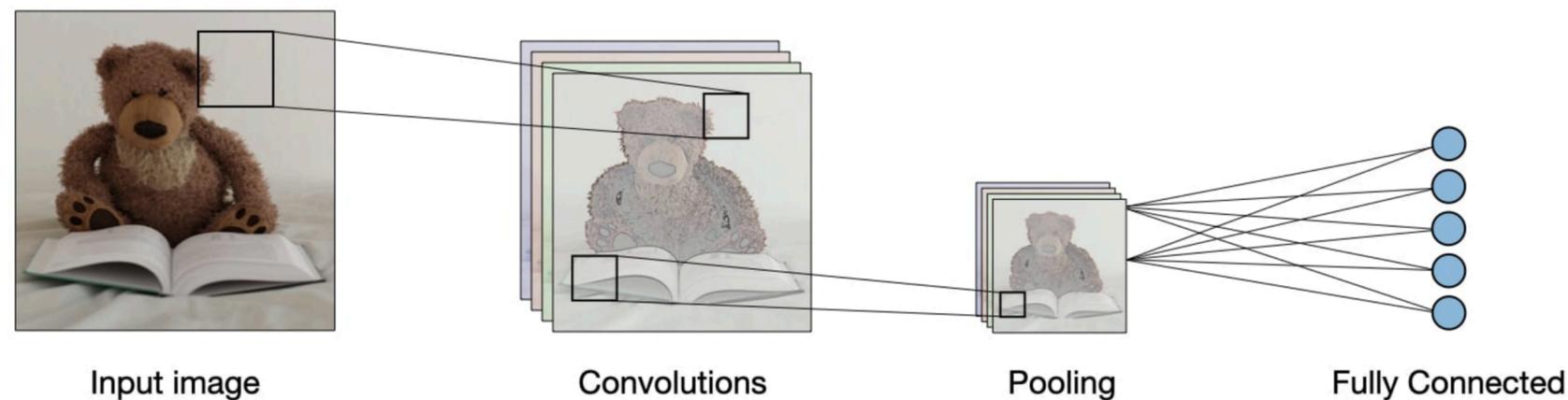
- It used to be a tougher question to answer
- To use wherever images are used (visualizations, video games, presentations, ads, etc...)
- Human-in-the-loop design

Today's Plan*

- “Predict” an image
 - Generative Models
 - Images ($P(x)$):
 - Frameworks: Variational auto-encoders (VAEs), Generative Adversarial Networks (GANs)
 - Images conditioned on text ($P(x | y)$):
 - Dall-E 2, Imagen
- * Like last week, the slides are mostly from David Berger

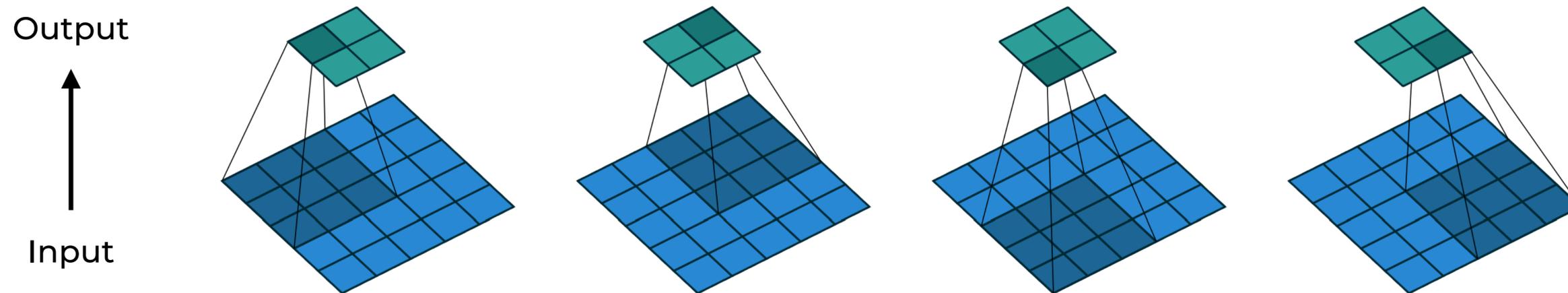
Convolution Neural Network (CNN) — Recall

- Séries of layers (blocs): convolutions + pooling
- Each layer reduces the dimensionality of the representation



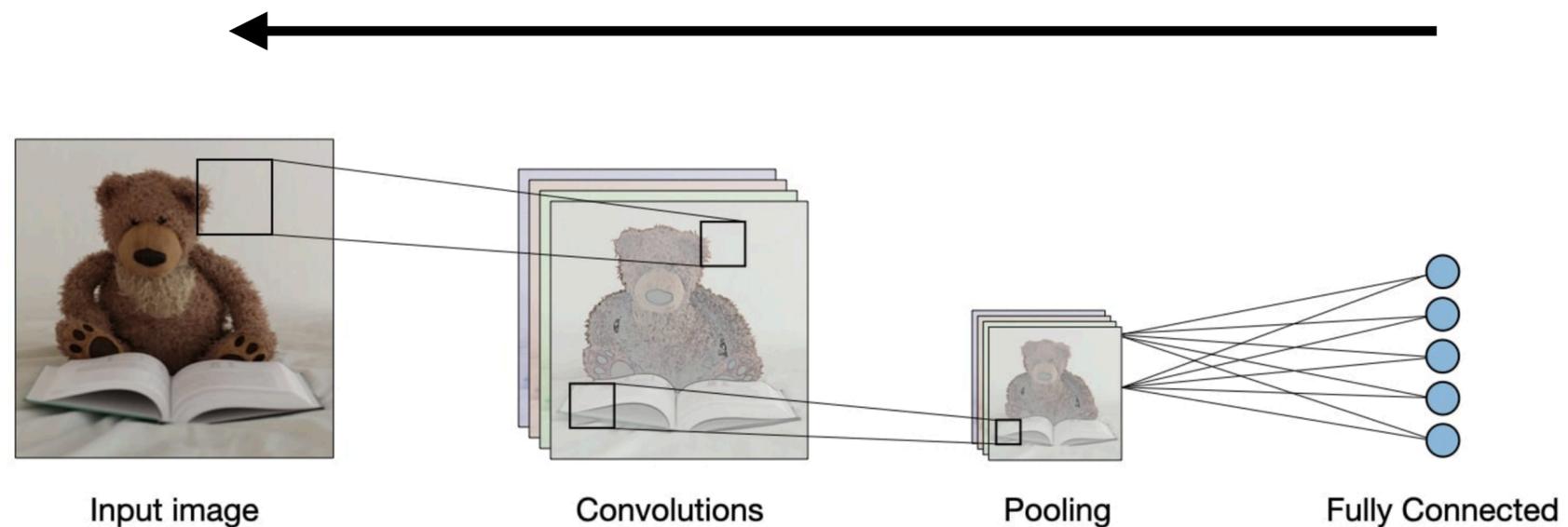
Convolution

- Input: 5 x 5
- Filter (kernel): 3 x 3. Stride 2.
- Output: 2 x 2

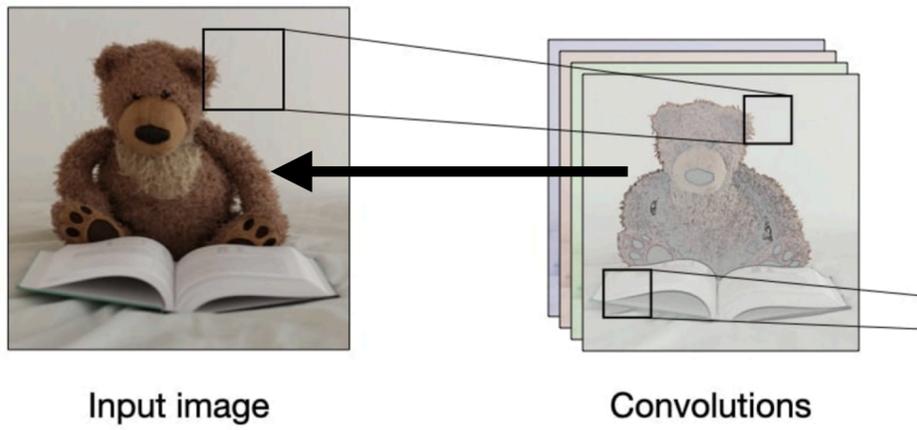


“Reverse” a convolution network

- Each layers increases the dimensionality of the incoming representation

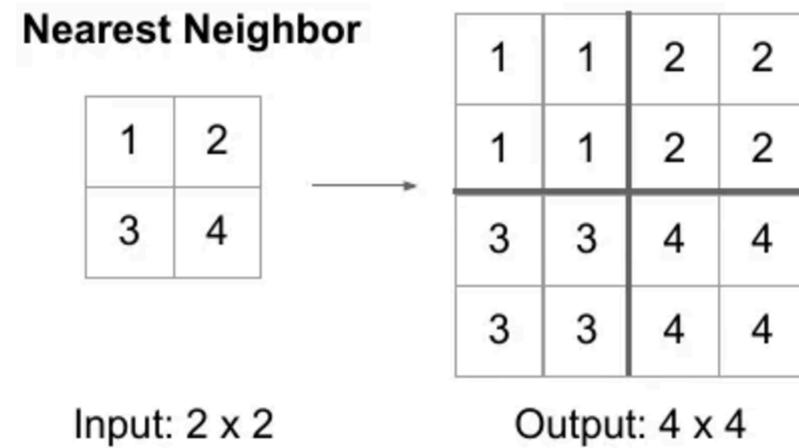


- (Note: this is in fact the same operation as done by backdrop in a CNN.)

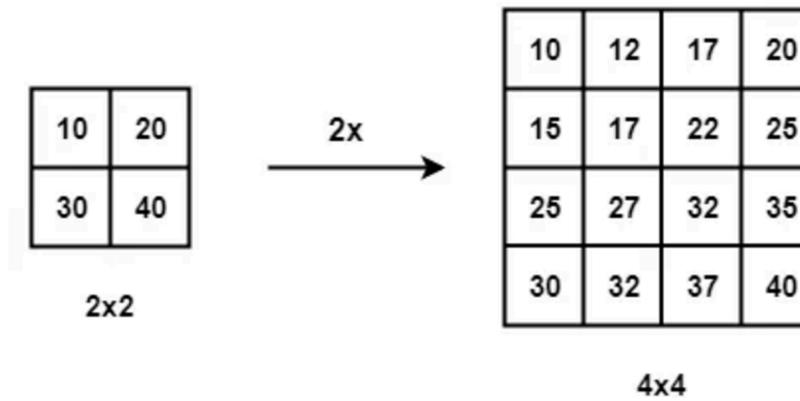


Interpolation

1. Repetition



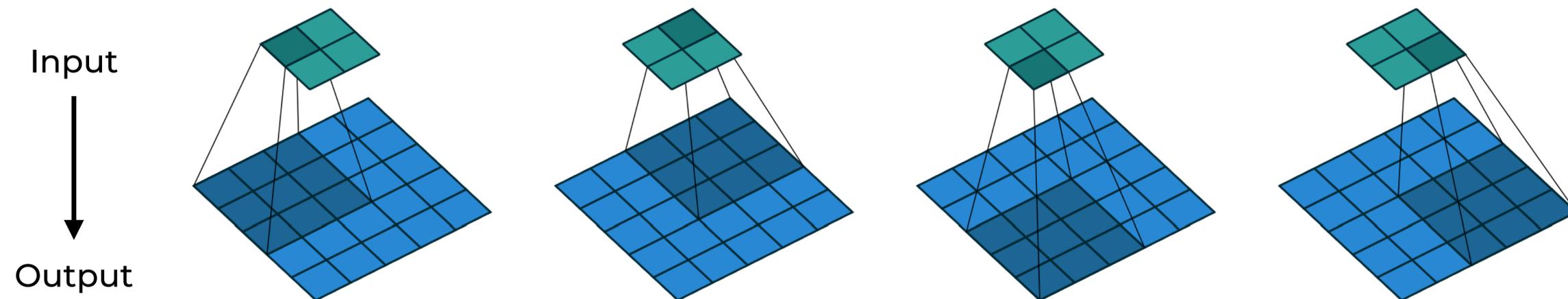
2. (Bi-)linear interpolation



- No parameters to learn

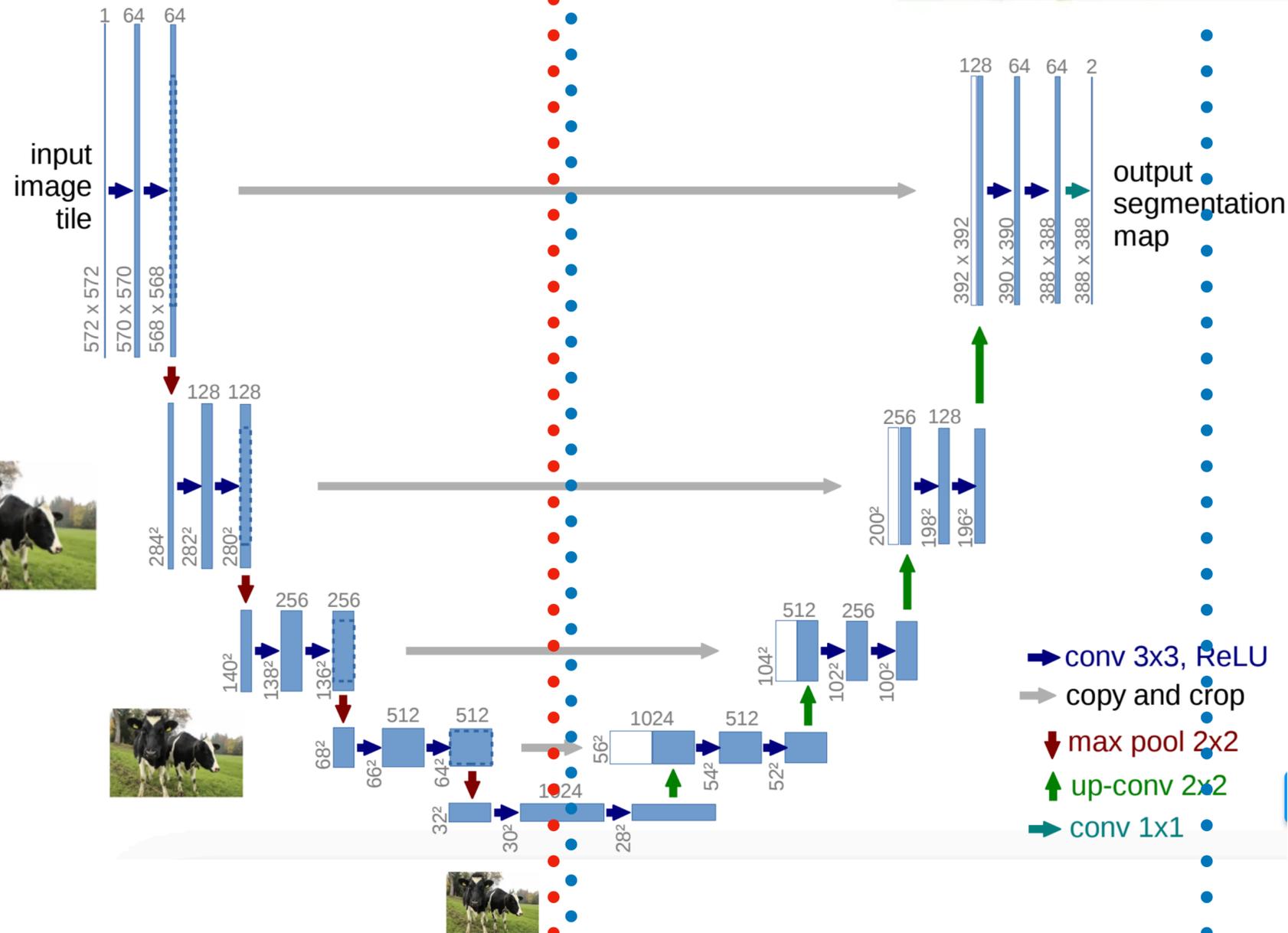
Transposed Convolution

- Useful to generate images or increase the resolution of an image (like in movies)
- Input: 2×2
- Filter: 3×3 . Stride 1.
- Output: 3×3



- If we write down a convolution as a matrix operation (by vectorizing the image), then the reverse operation is multiplying by the transpose (hence the name)

U-Nets



- **Encoder-Decoder Architecture**

- **Encoder:** Obtain a representation of the image (classic CNN without the classification output)
- **Decoder:** From the representation, obtain an image
- Connections between the encoder and decoder allow for precise localization
- **up-conv** -> transposed convolution (for example)
- Proposed for segmentation
- Has become standard for image generation

Generative models

- A U-Net can obtain a representation from an image, but it does not have a probabilistic interpretation
- A generative model is a method for parametrizing $P(x)$ — unsupervised

Why are generative models (often) probabilistic?

- Allows different types of evaluation
 - For example, the probability of an image according to the model: $P_{\theta}(x_{\text{new}})$
- Can obtain samples $x \sim P_{\theta}(x)$
- Quantifies uncertainty
- It has been popular recently to parametrize distributions with neural networks (think of a softmax layer) — these are not always proper generative models

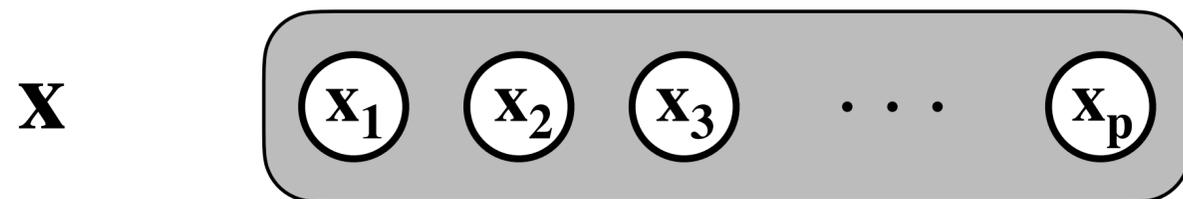
Auto-Encoder (AE)

Auto-encoder - Recall

For an AE with a single hidden layer:

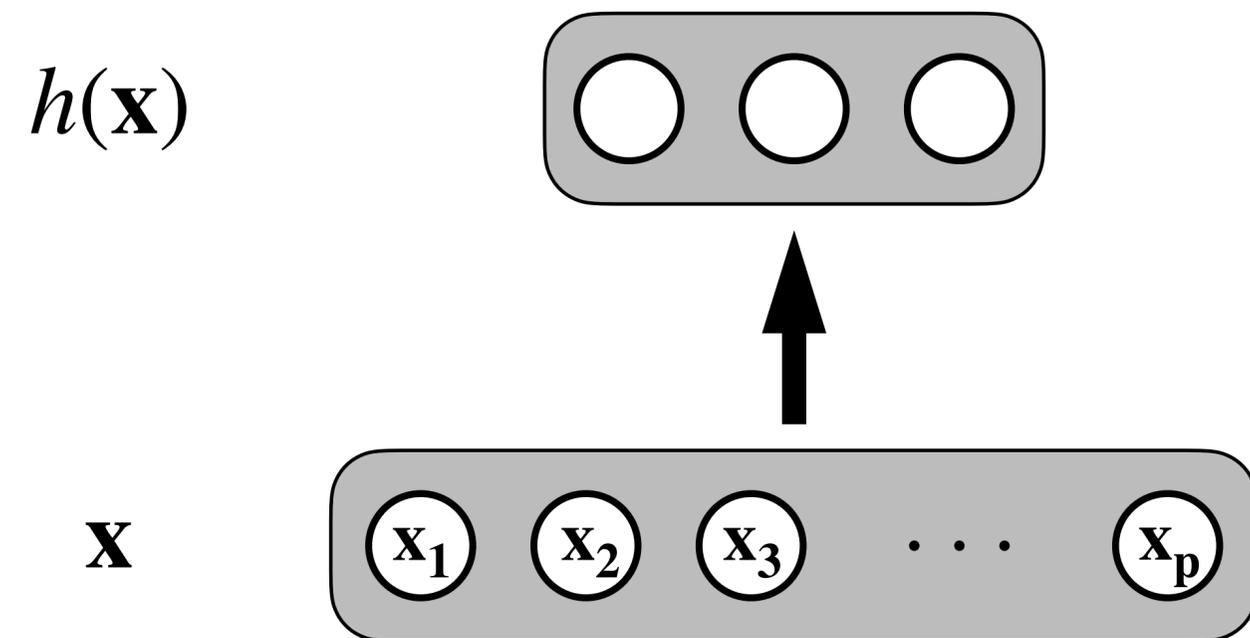
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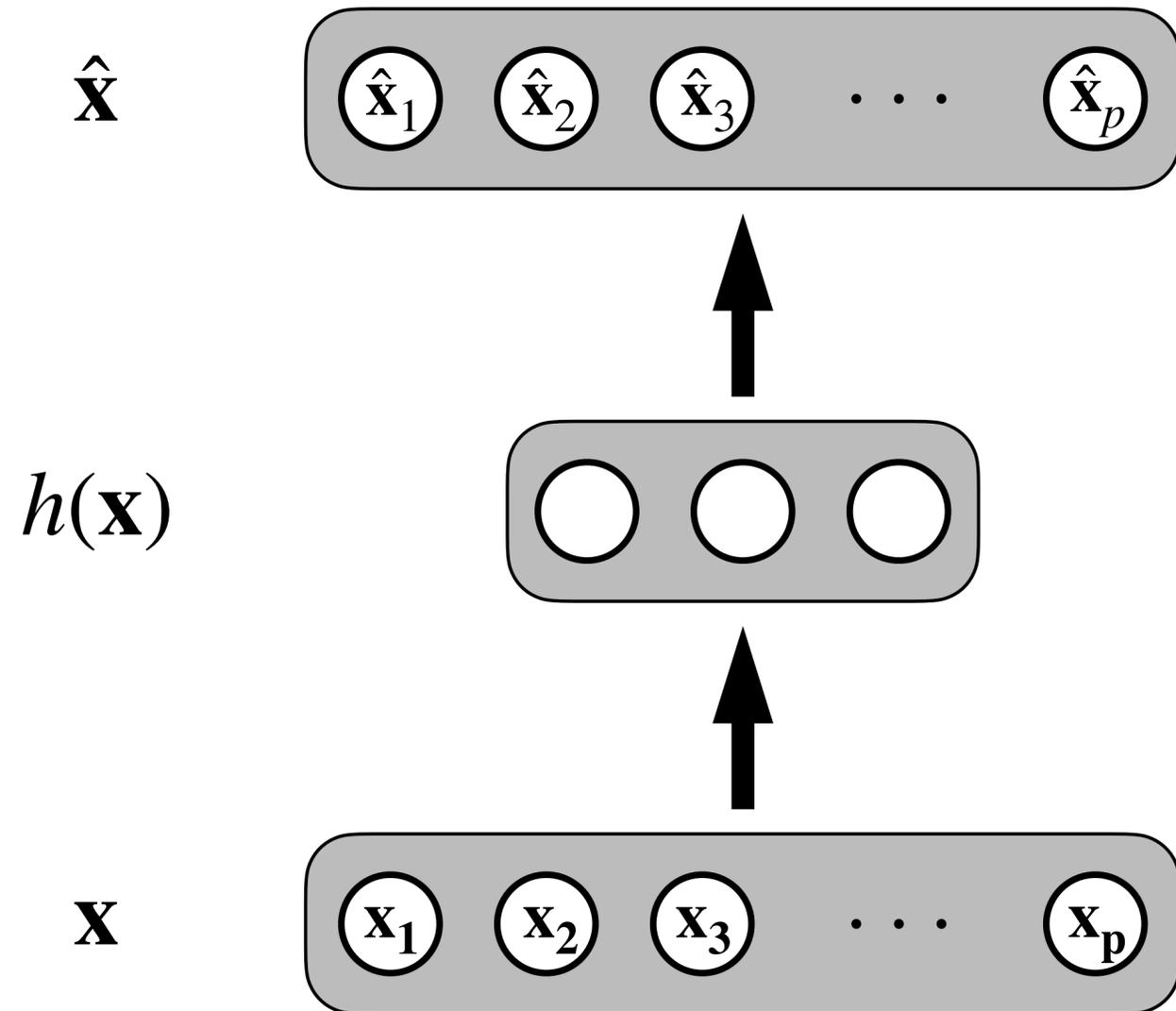
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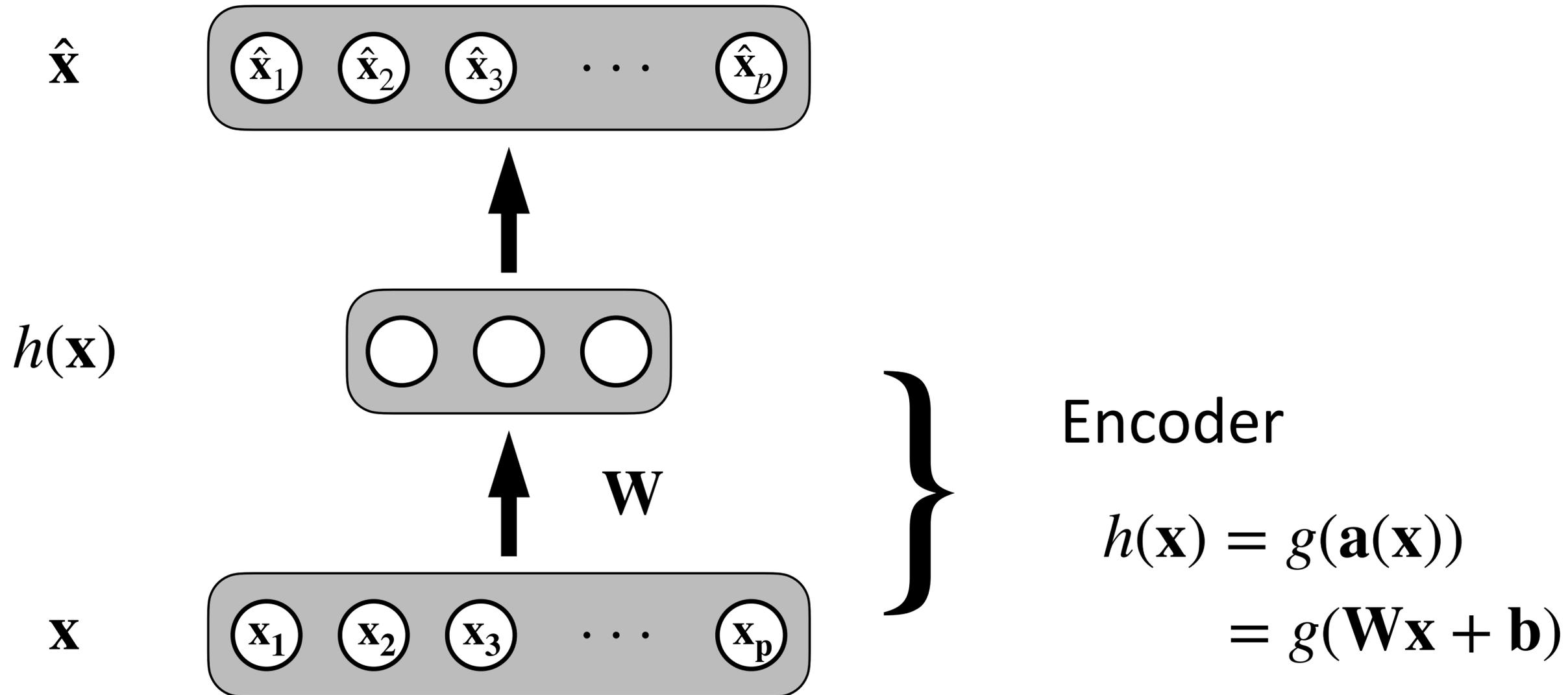
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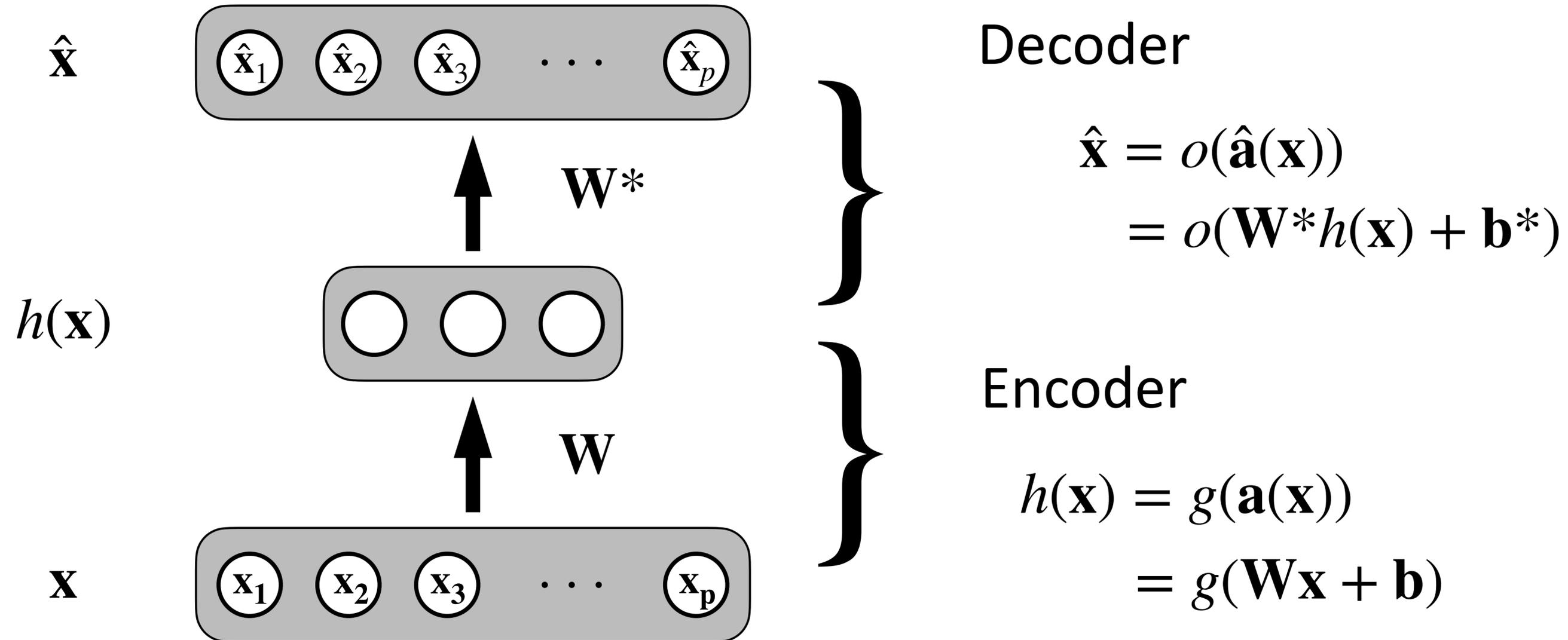
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Auto-encoder - Recall

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Variational Auto-encoder (VAE)

* Understanding these models precisely requires concepts that are beyond our class. Here, we aim for familiarization with the terms and an intuitive understanding.

Variational Auto-Encoder - Motivation

Idea:

- The data are generated conditioned on a random variable (Z):

$$P_{\theta}(\mathbf{x} | \mathbf{z})$$

You can think of Z has an embedding. VAEs will learn a prior $P(z)$ and a posterior $P(z|x)$ over it.

Variational Auto-Encoder - Motivation

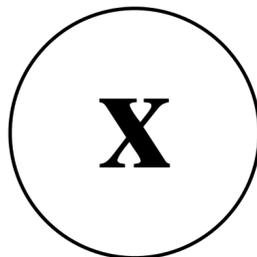
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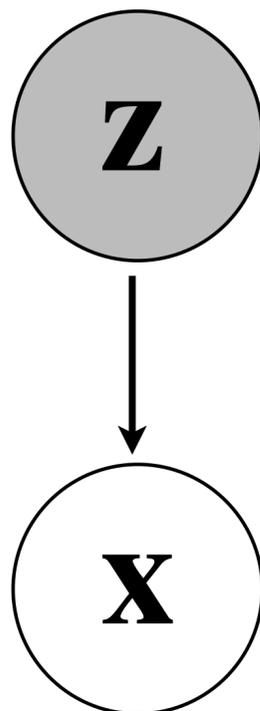
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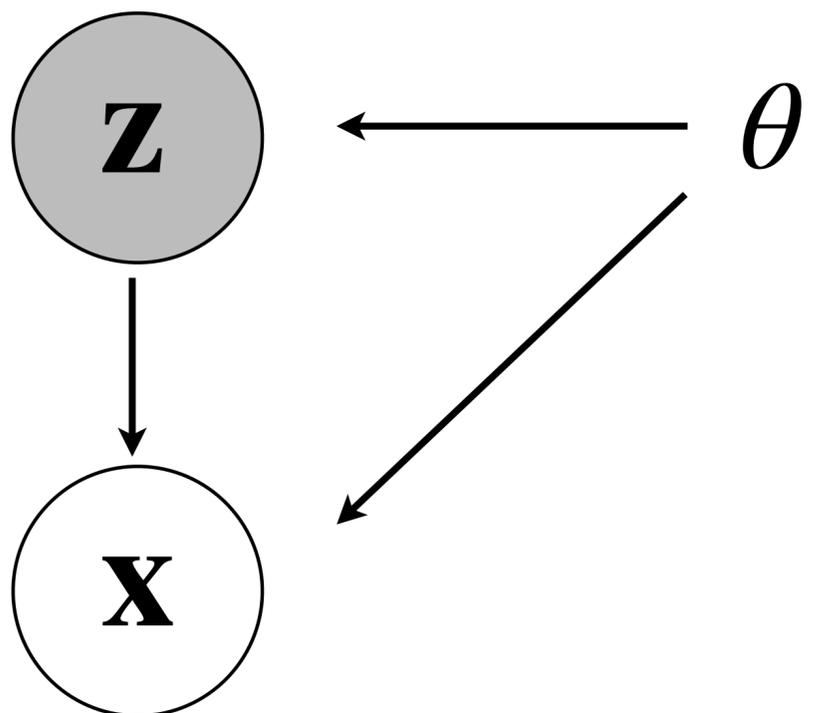
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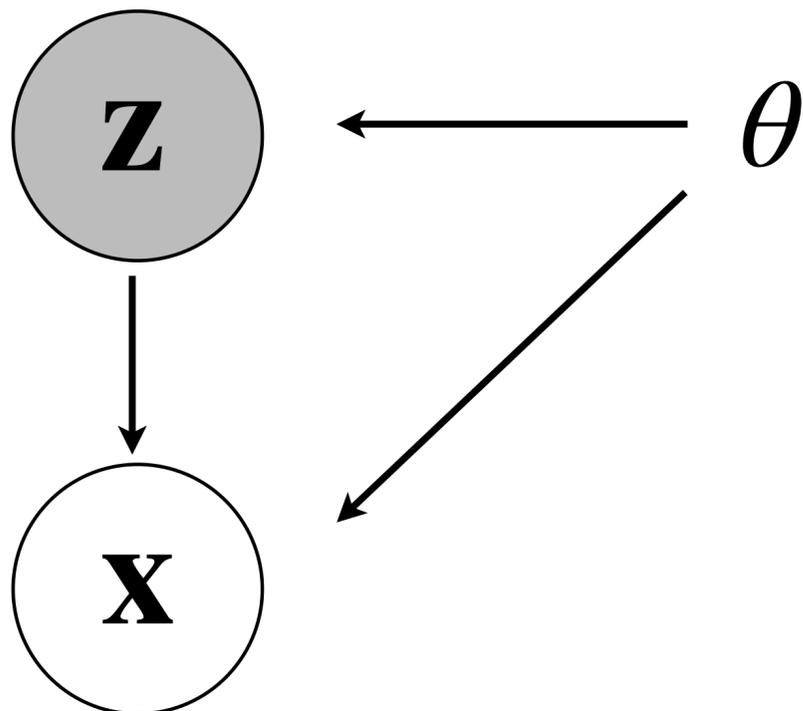
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Question:

- How do we learn such a distribution?

Variational Auto-Encoder - Motivation

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Partial answer:

- A “good” model should maximize the likelihood of the data $P(x)$
- Bayes, provides us with two possibilities:

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Idea:

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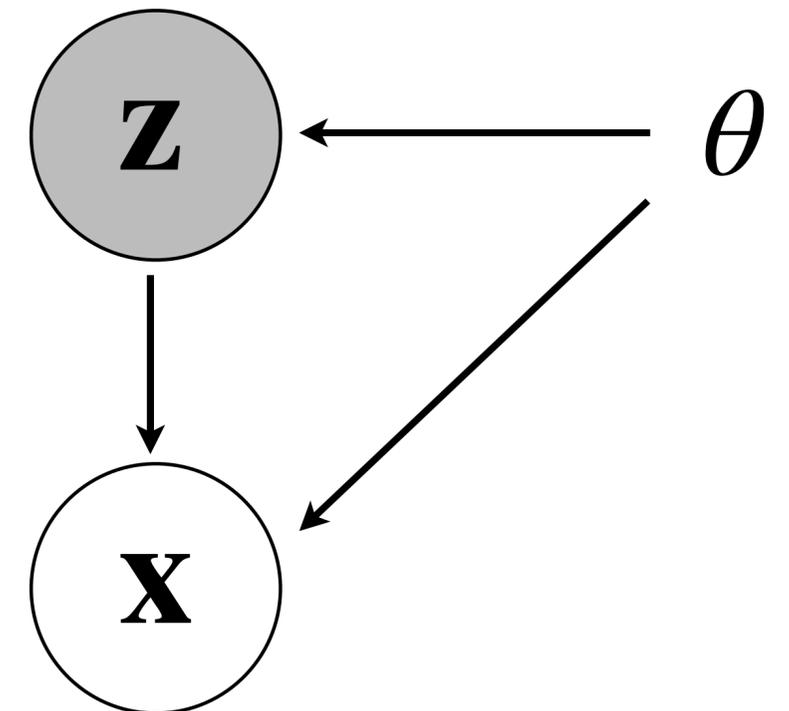
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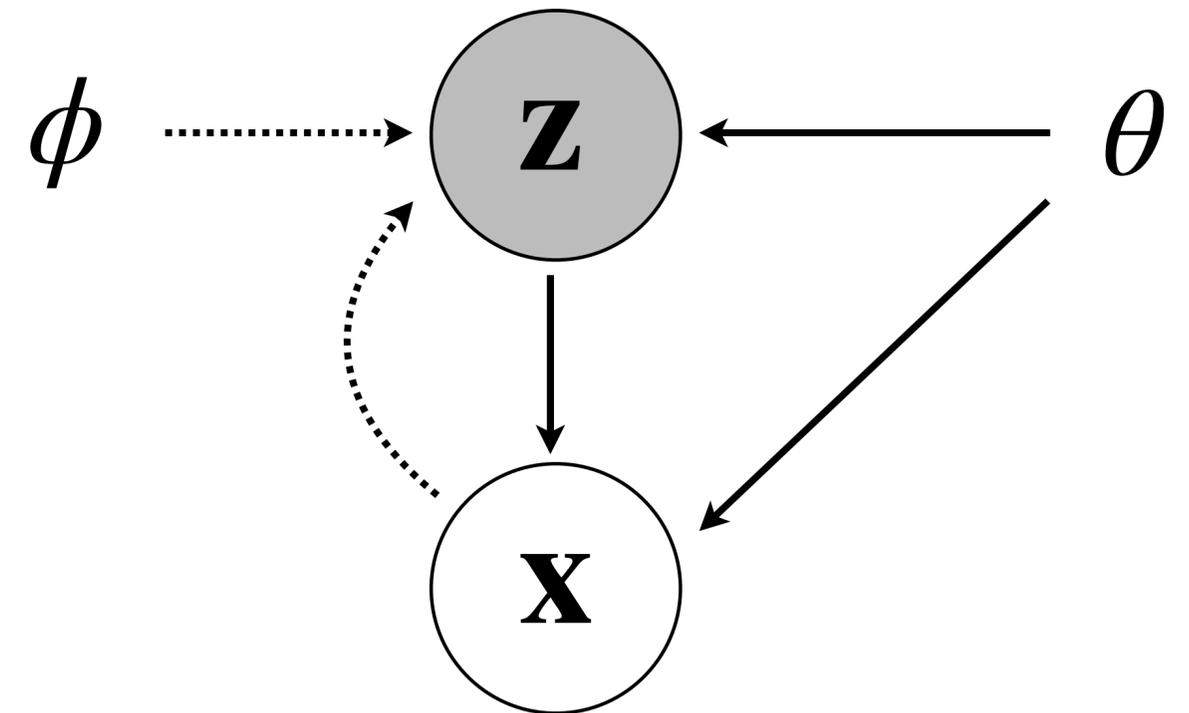
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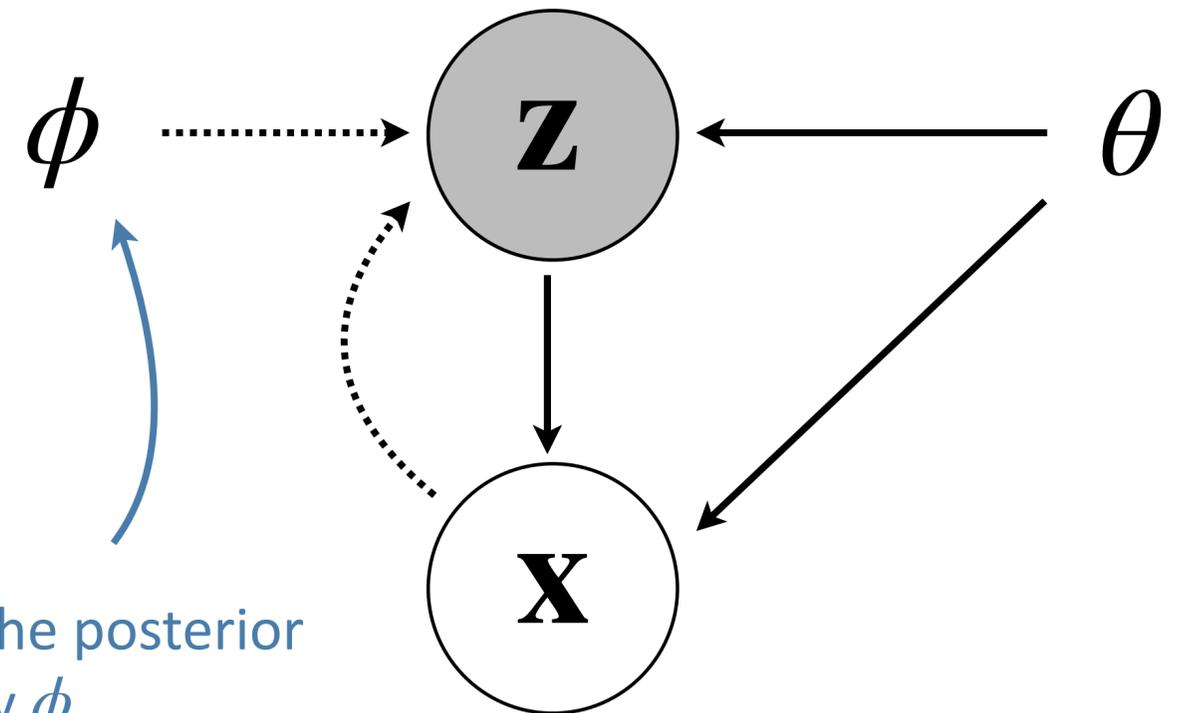
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The approximation of the posterior would be parametric by ϕ

Variational Auto-Encoder - Formalism

We can obtain a bound on the log-likelihood of \mathbf{x} :

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- How do we do this in practice?

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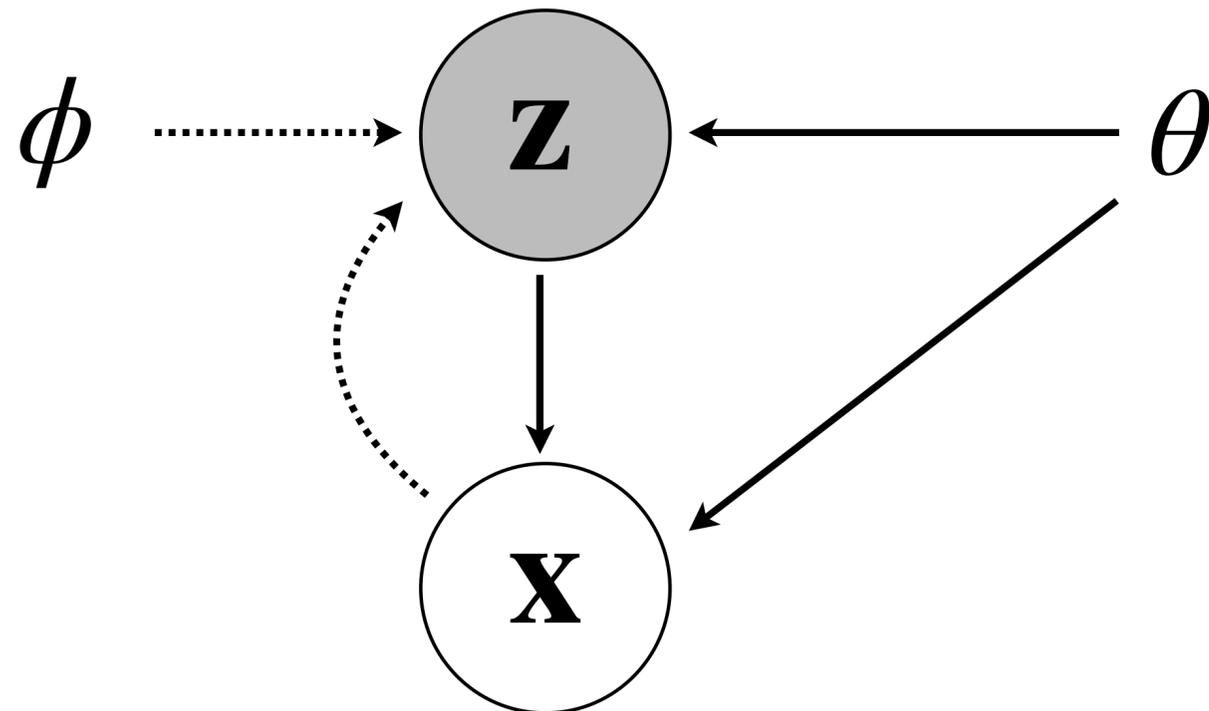
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Graphical view:

Variational Auto-Encoder - Formalism

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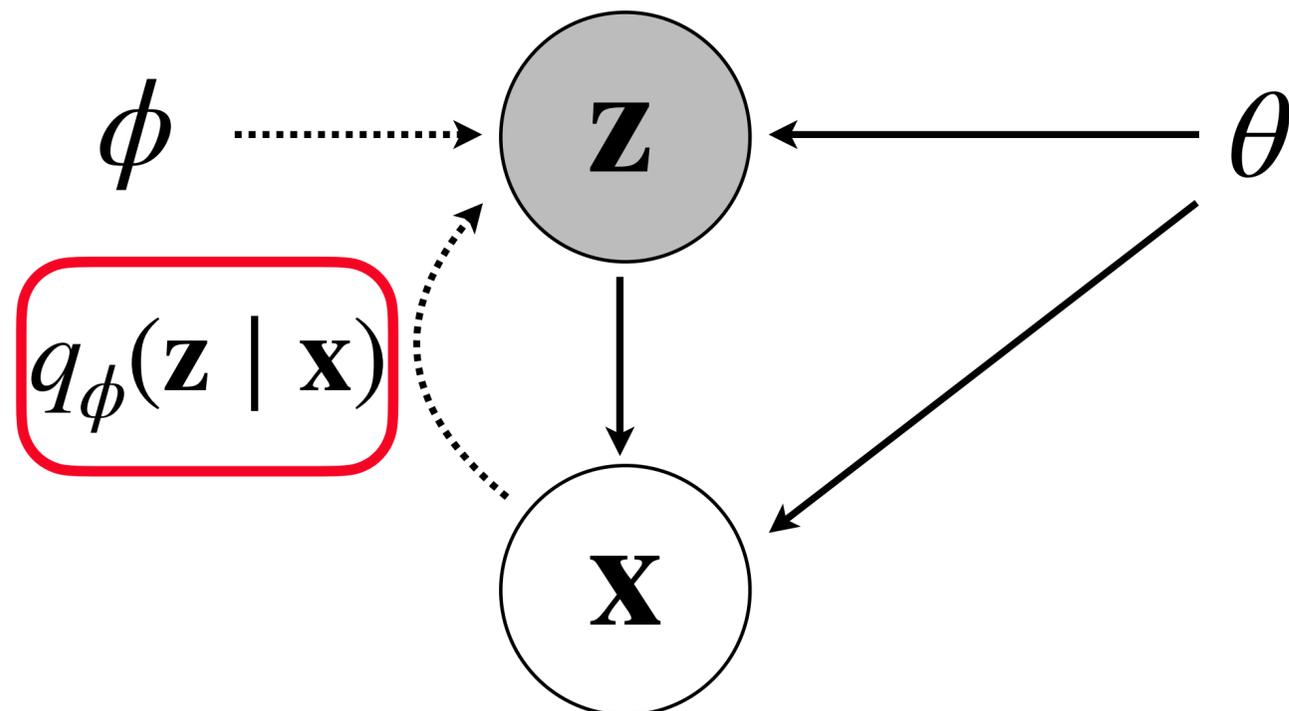
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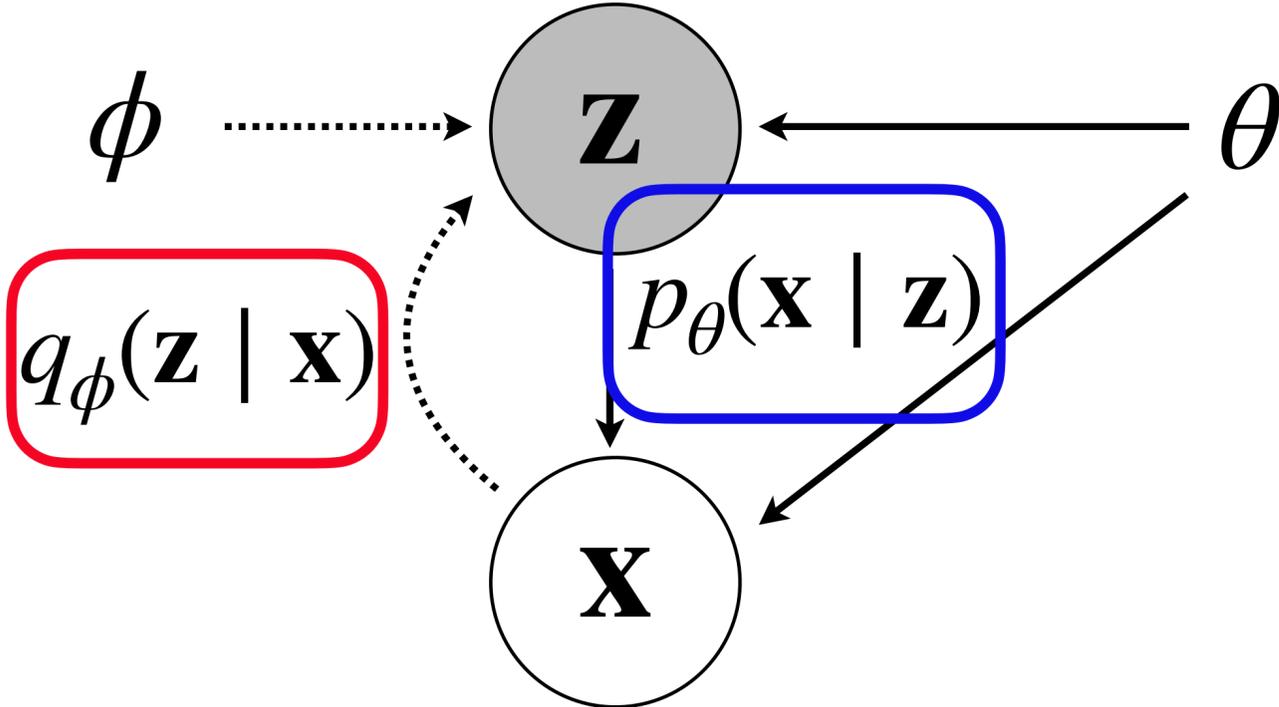
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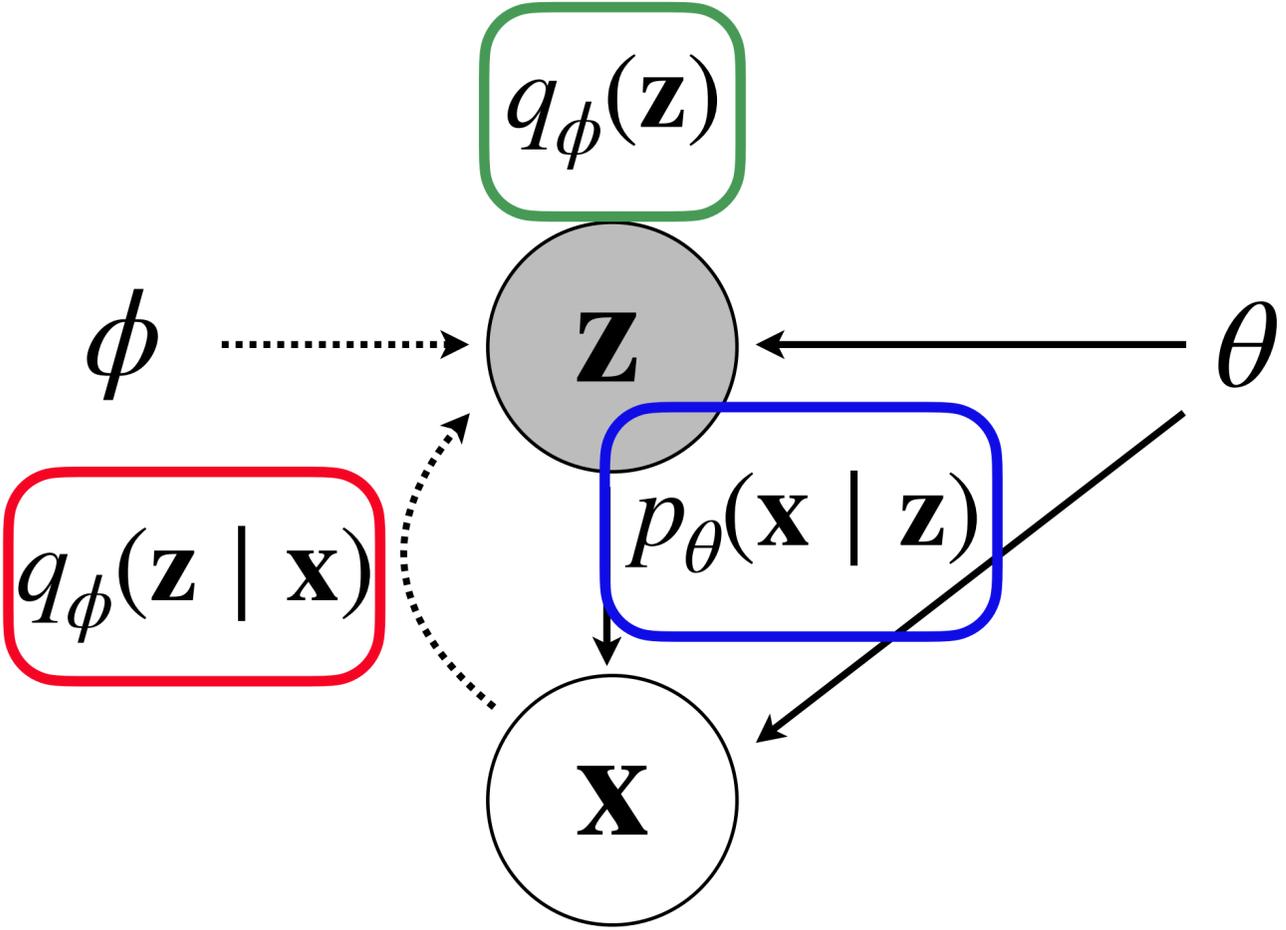
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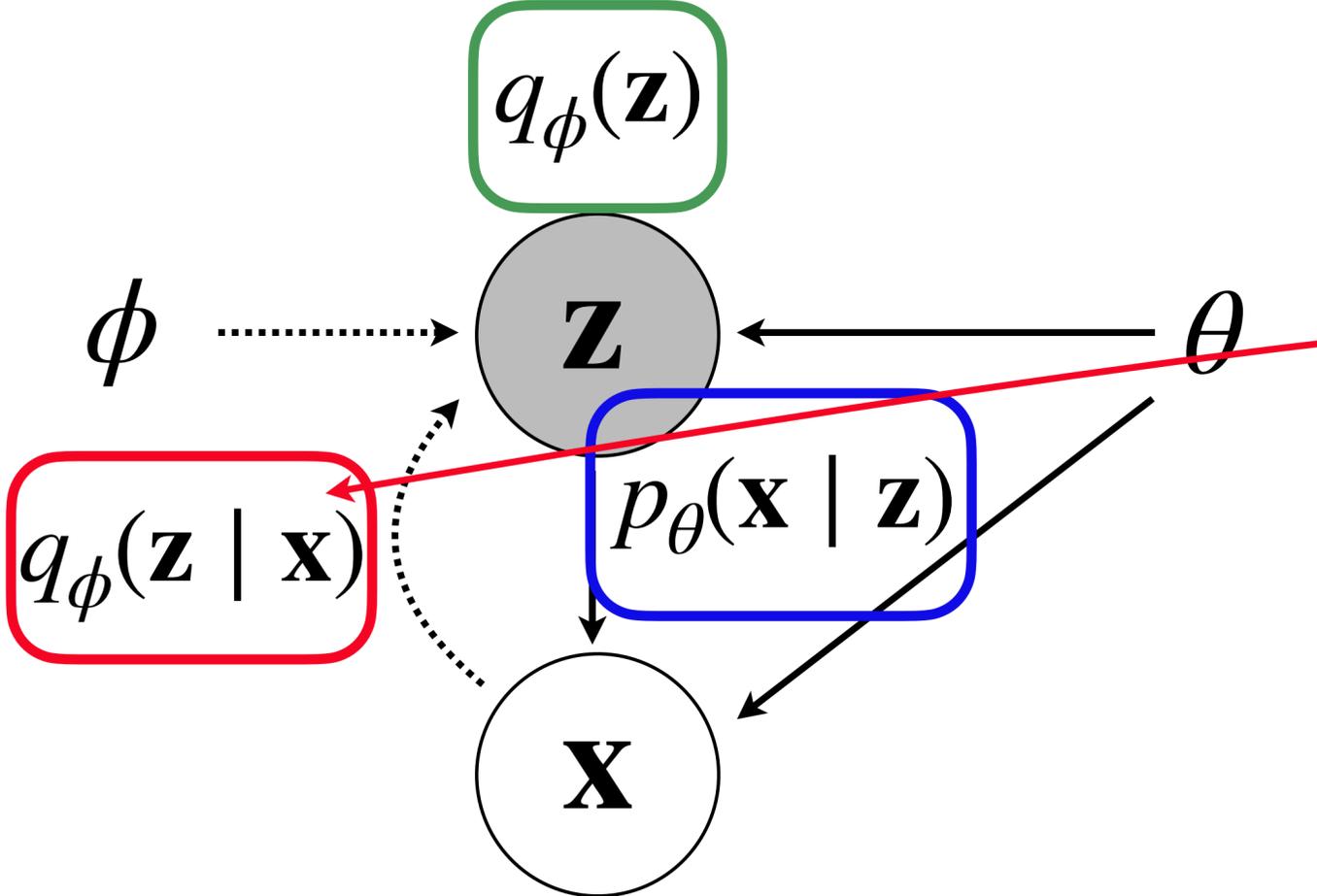
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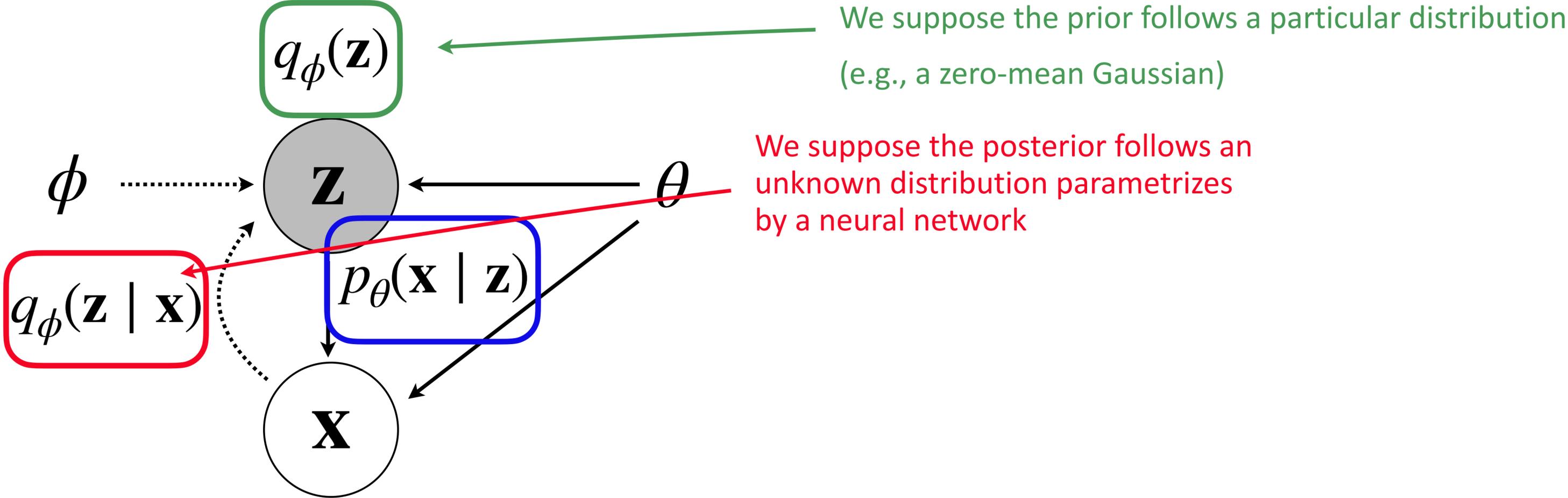


We suppose the posterior follows an unknown distribution parametrized by a neural network

Variational Auto-Encoder - Formalism

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Variational Auto-encoders

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We can estimate these parameters using an auto encoder:

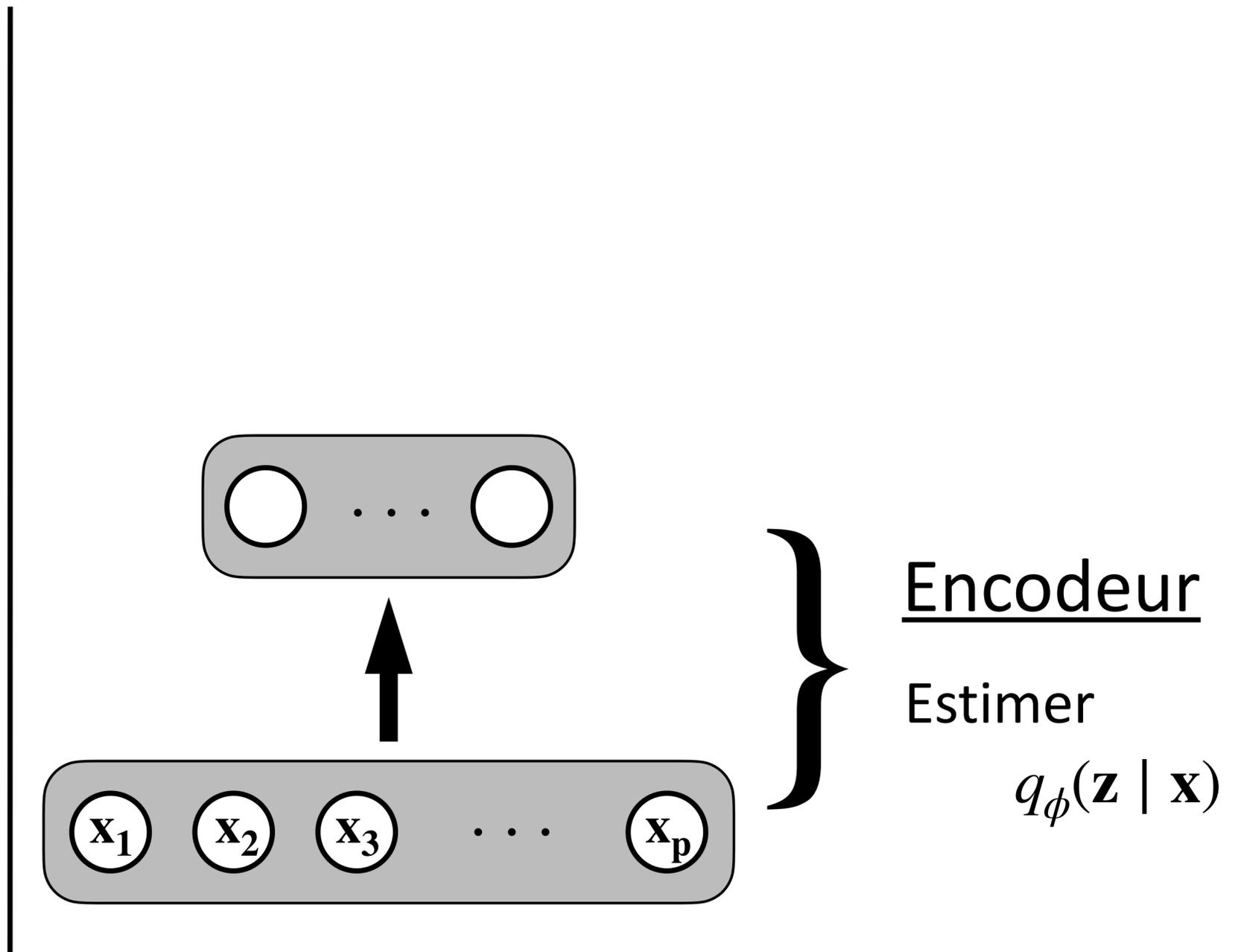
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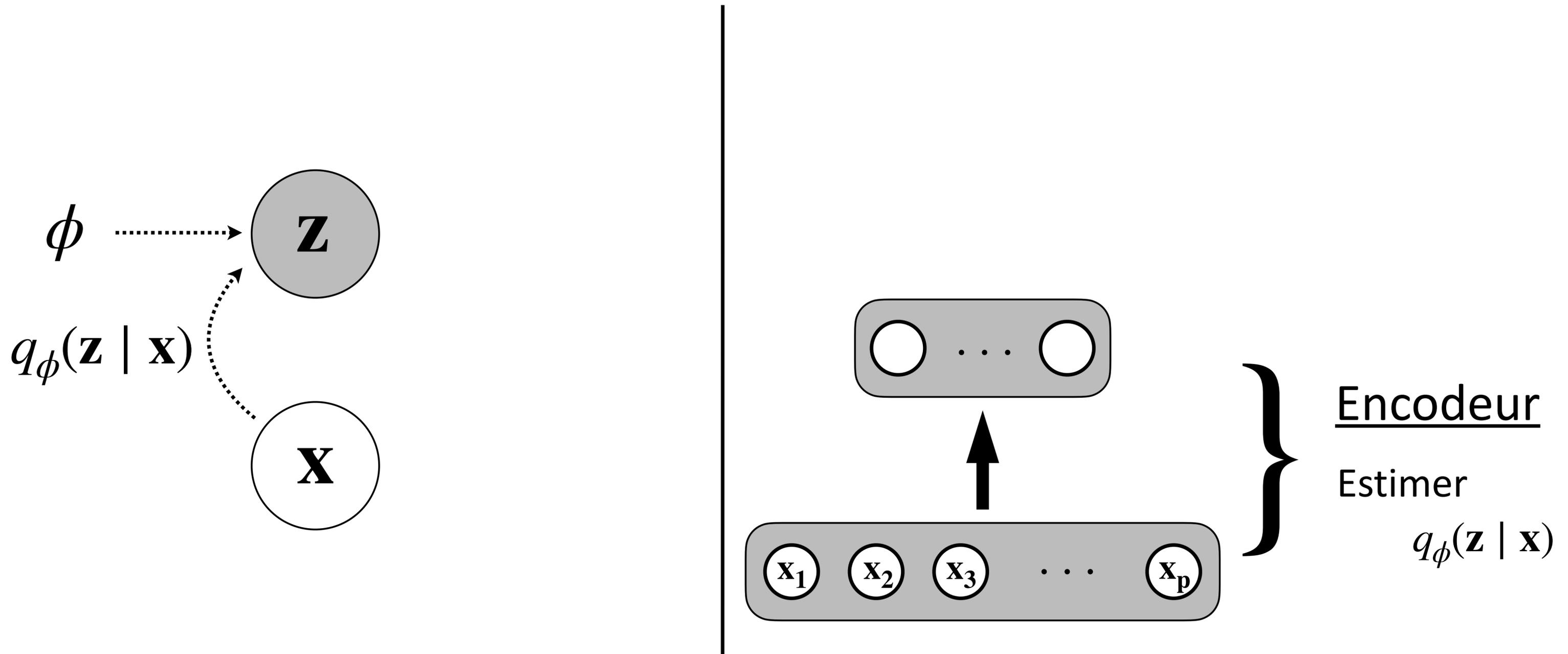
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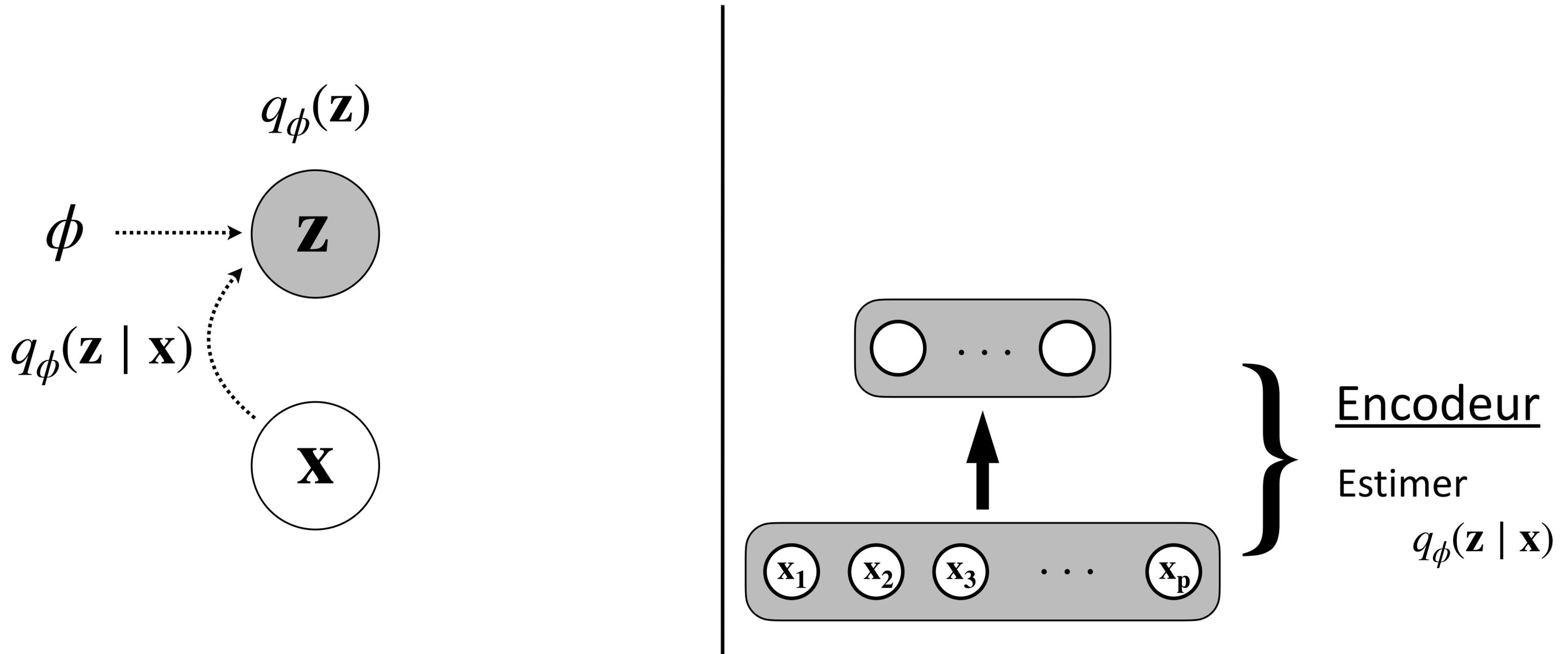
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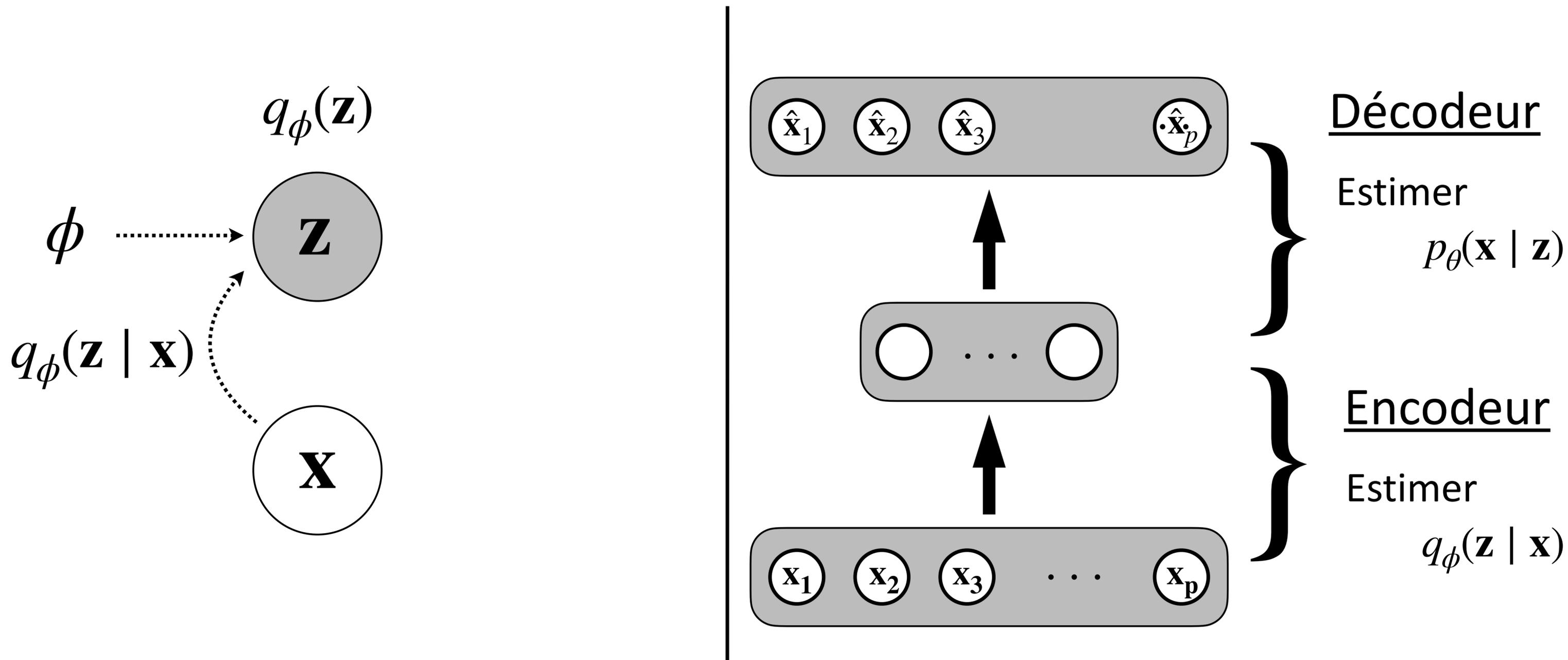
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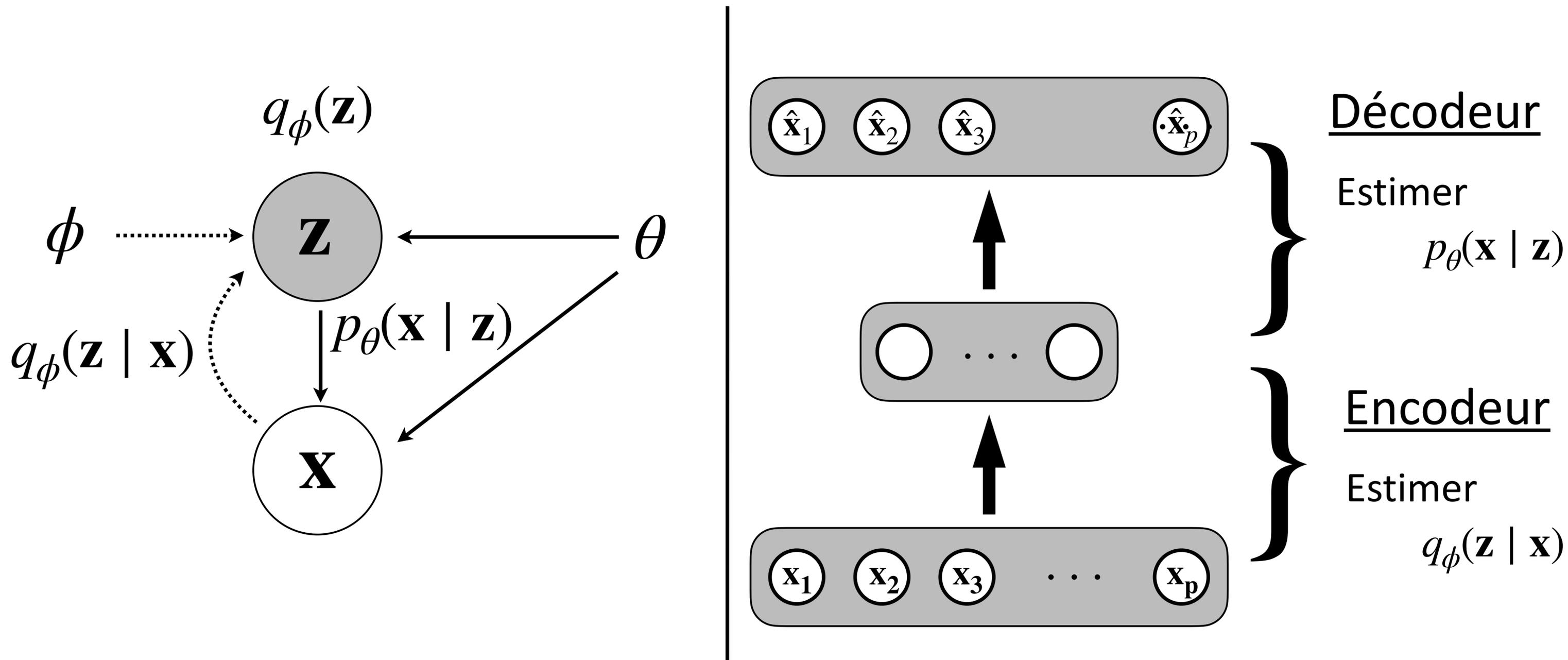
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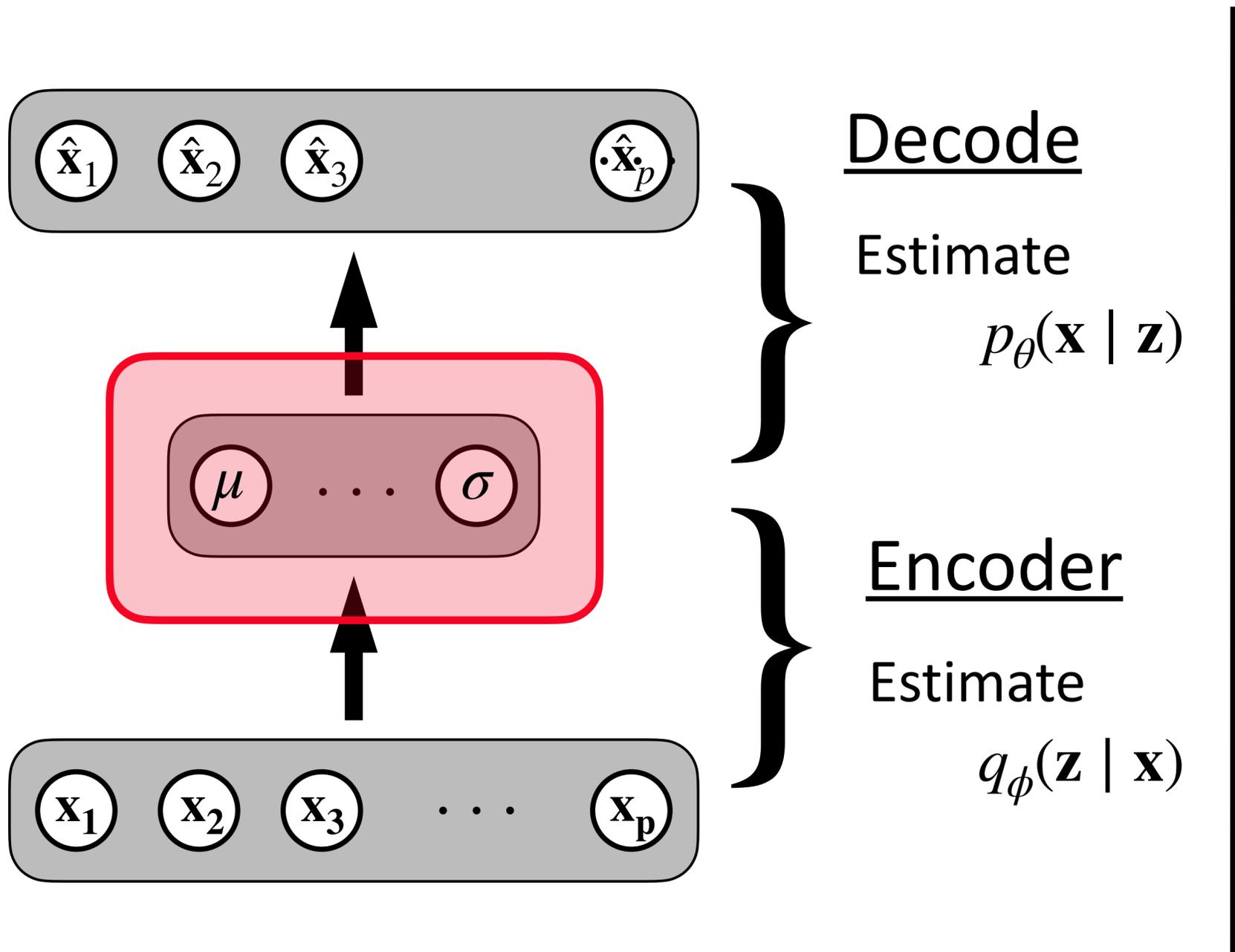


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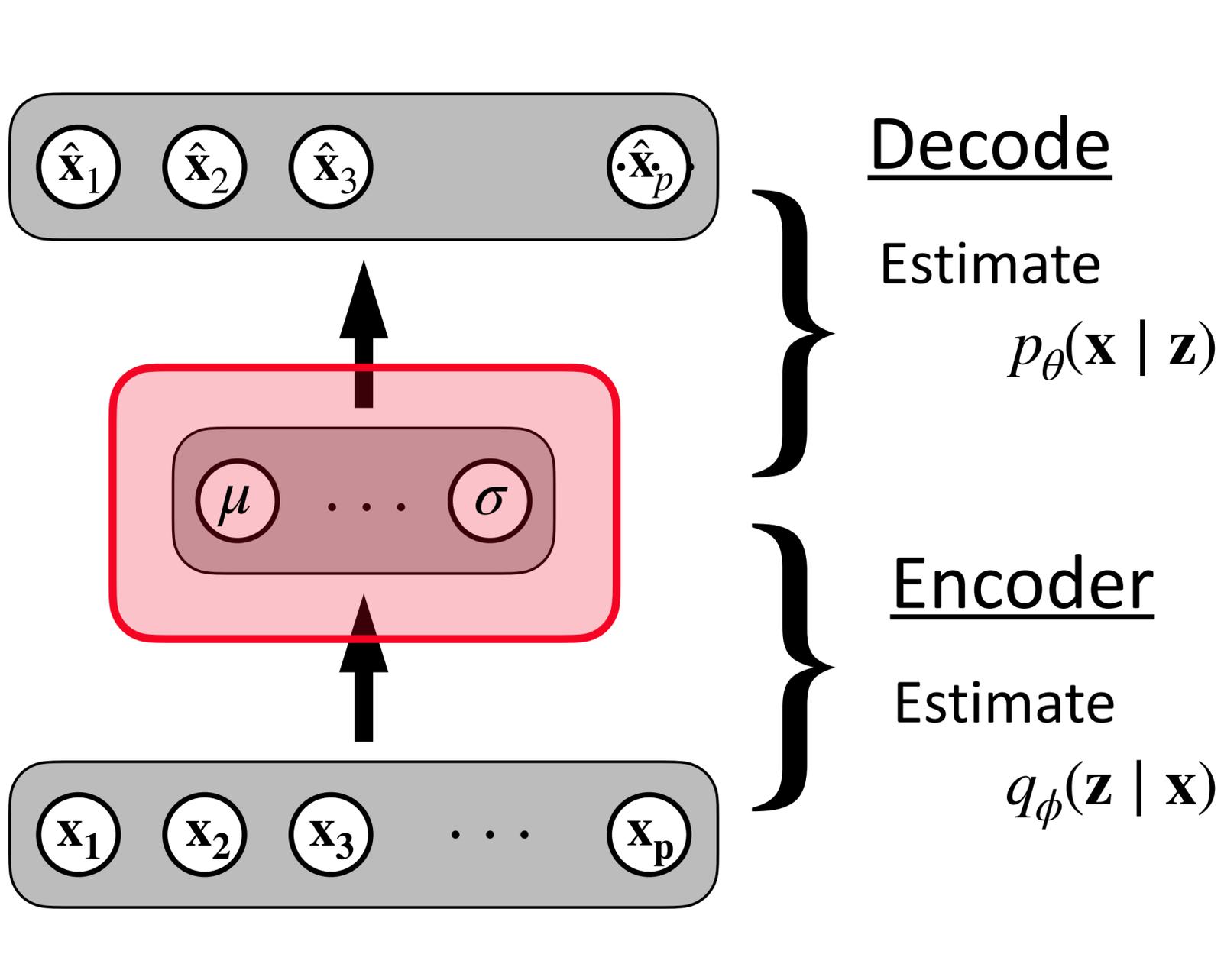


Variational Auto-encoders



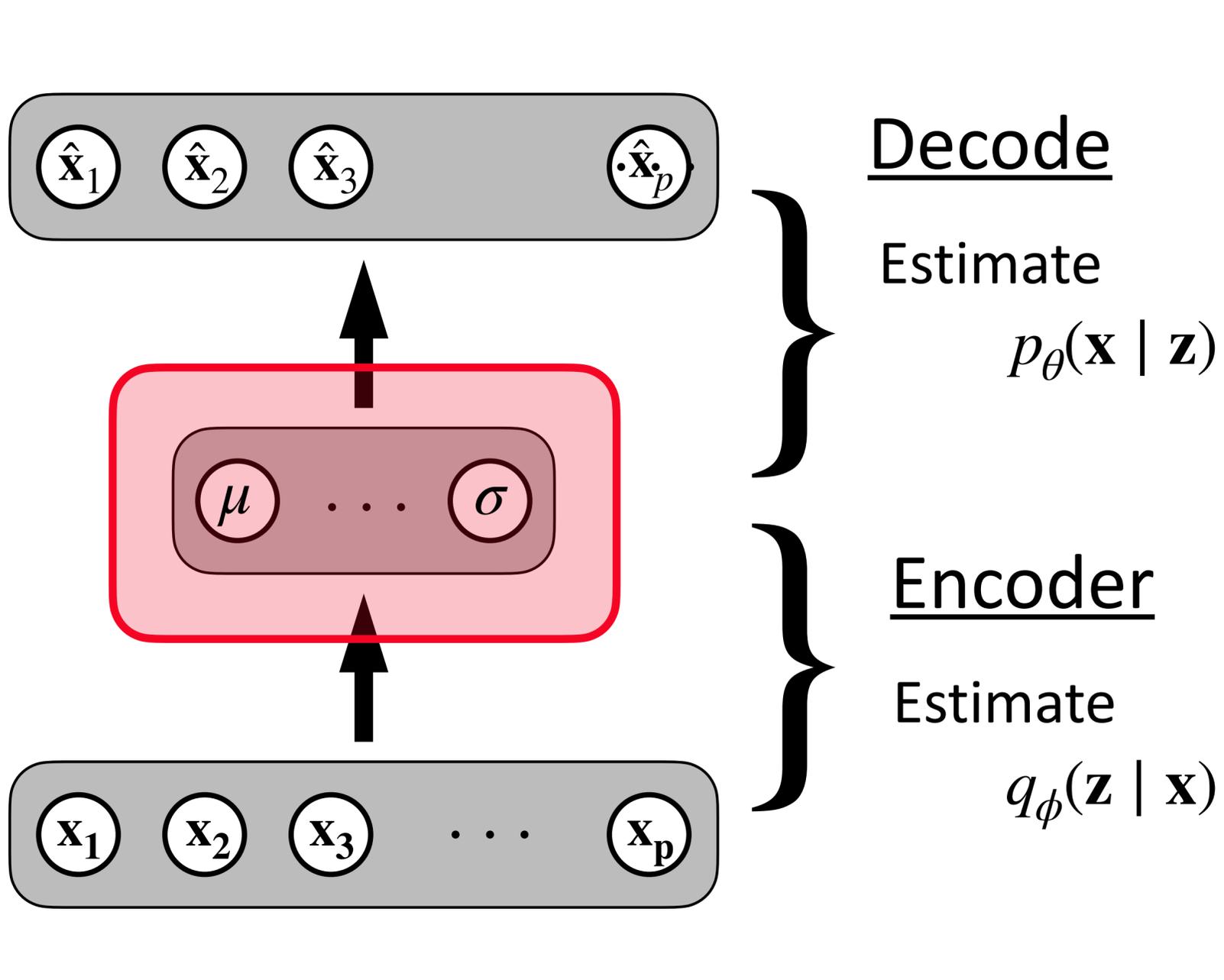
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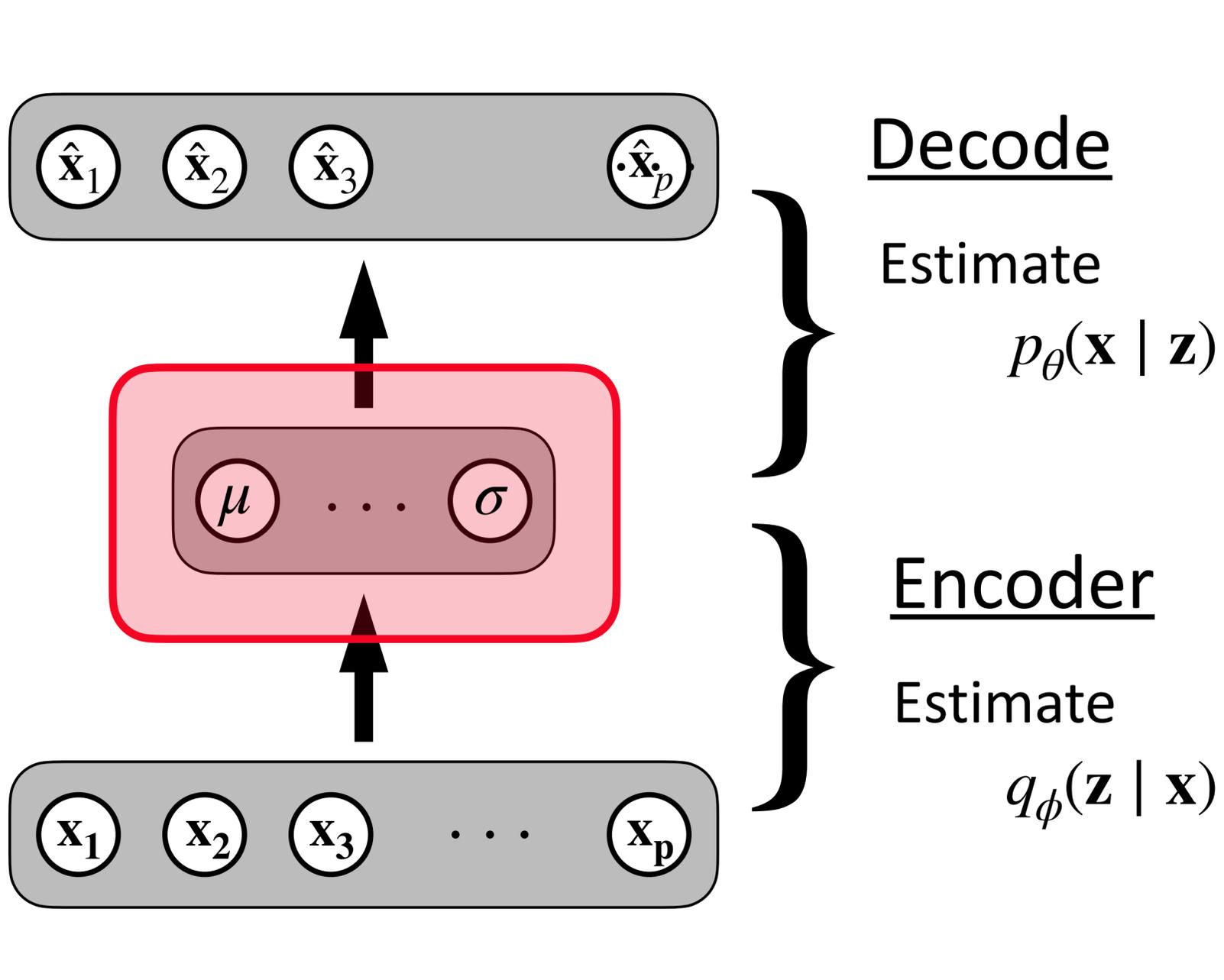
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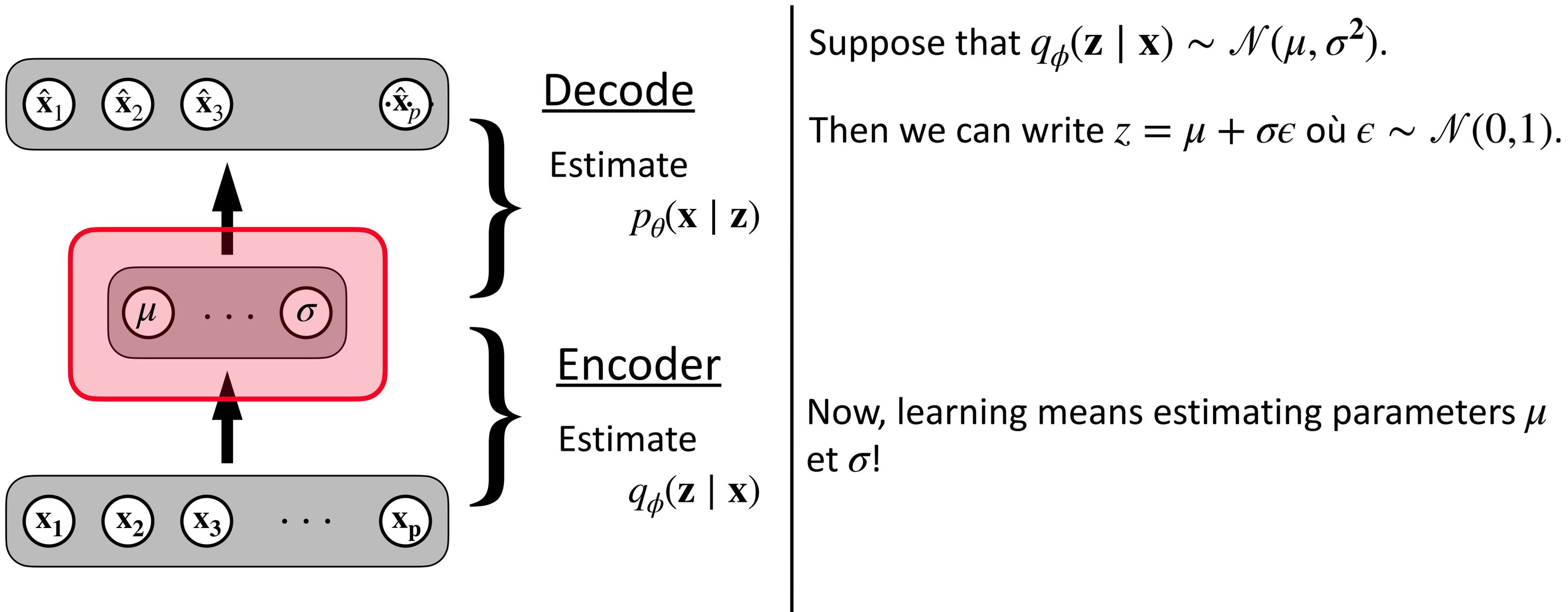


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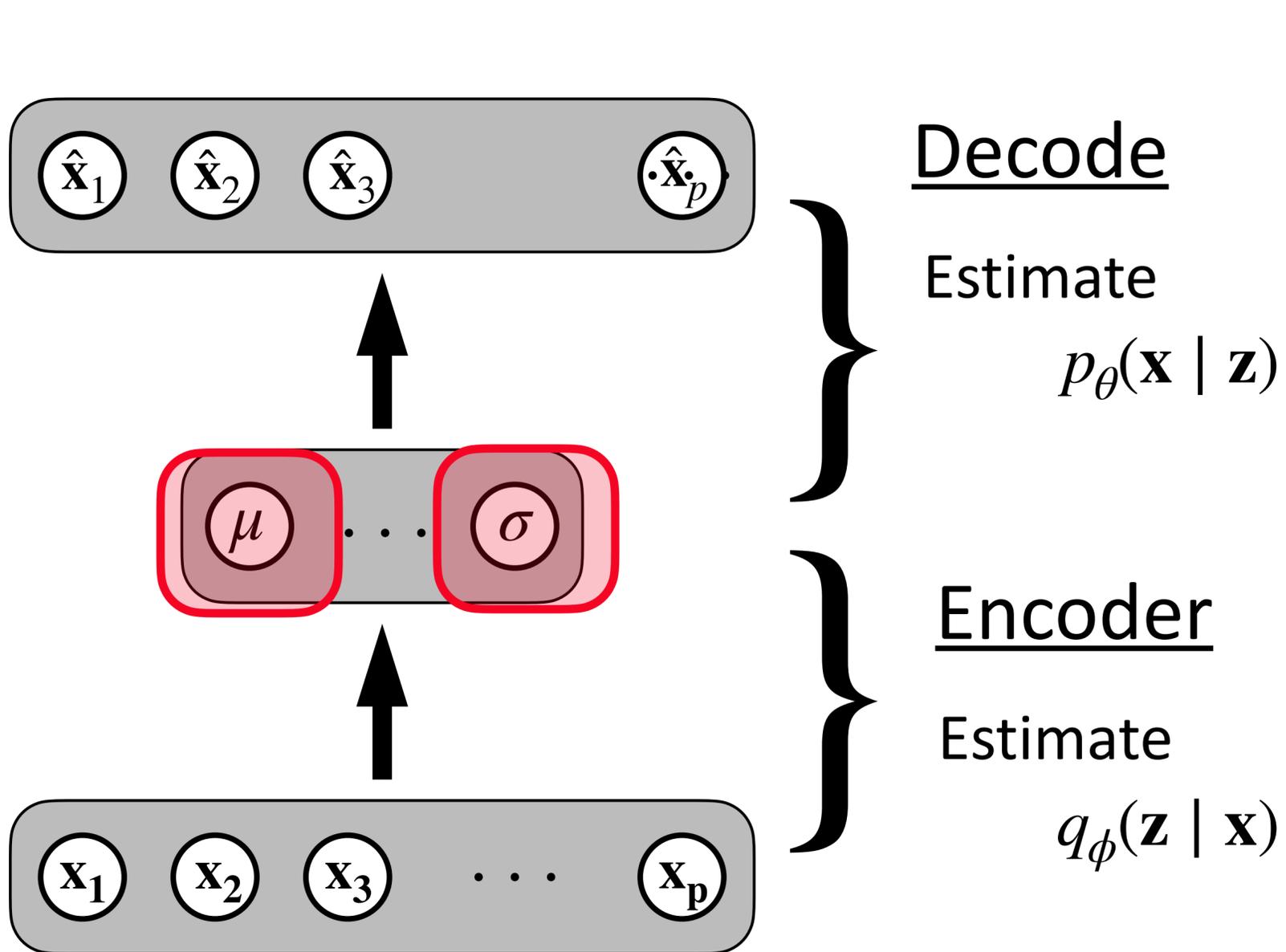
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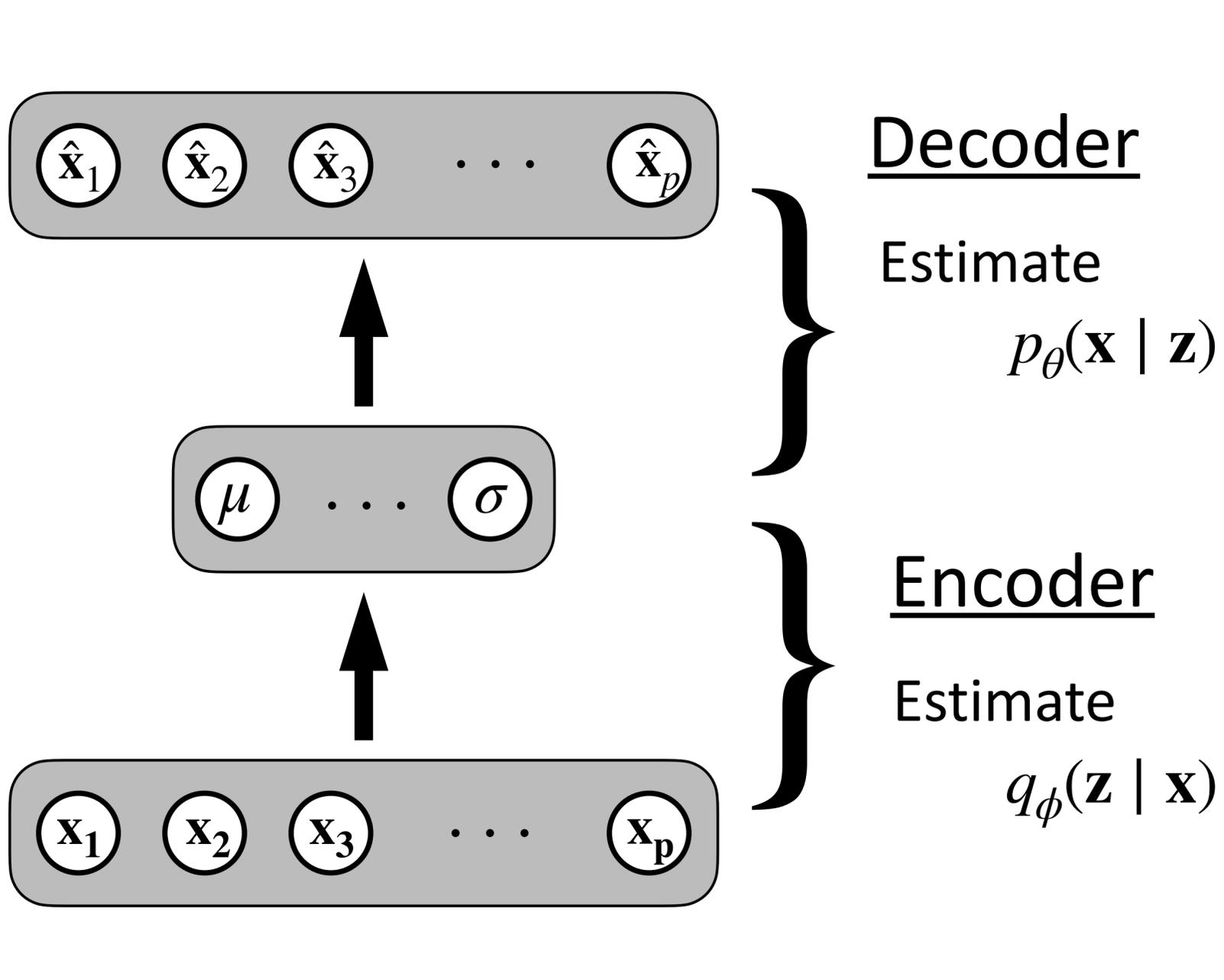


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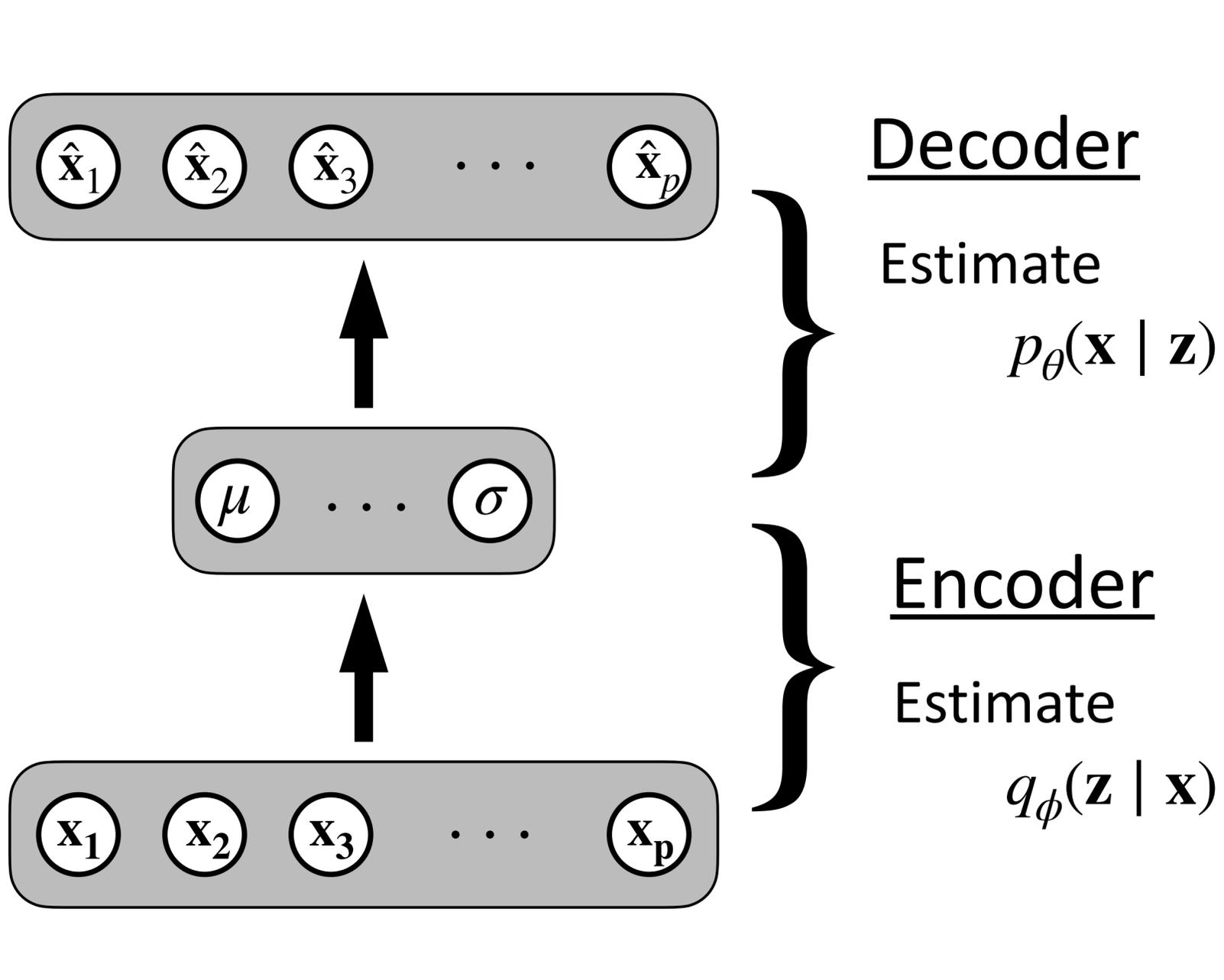
Now, learning means estimating parameters μ et σ !

Variational Auto-encoders



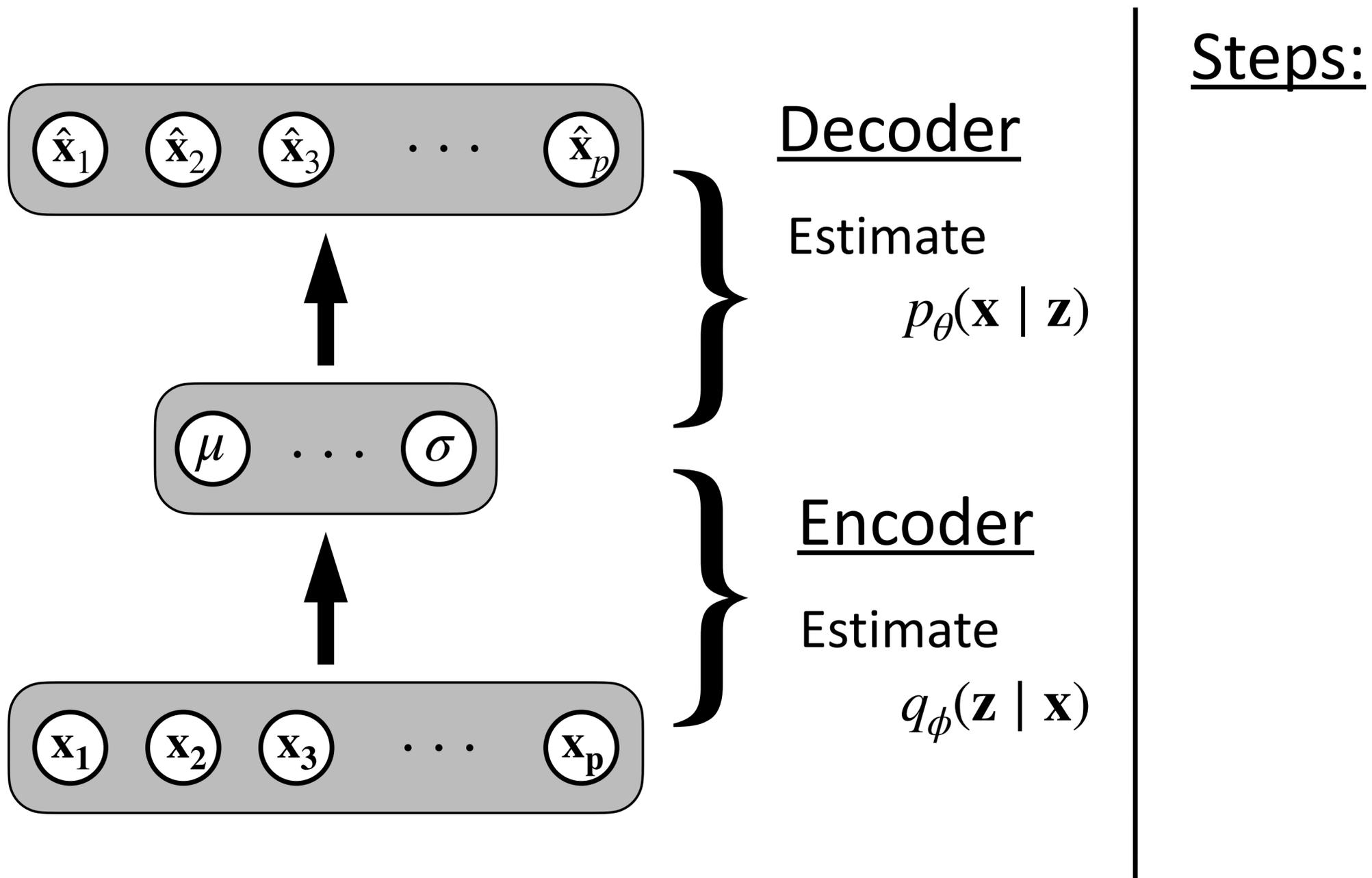
Variational Auto-encoders

VAEs are a way to train a generative model



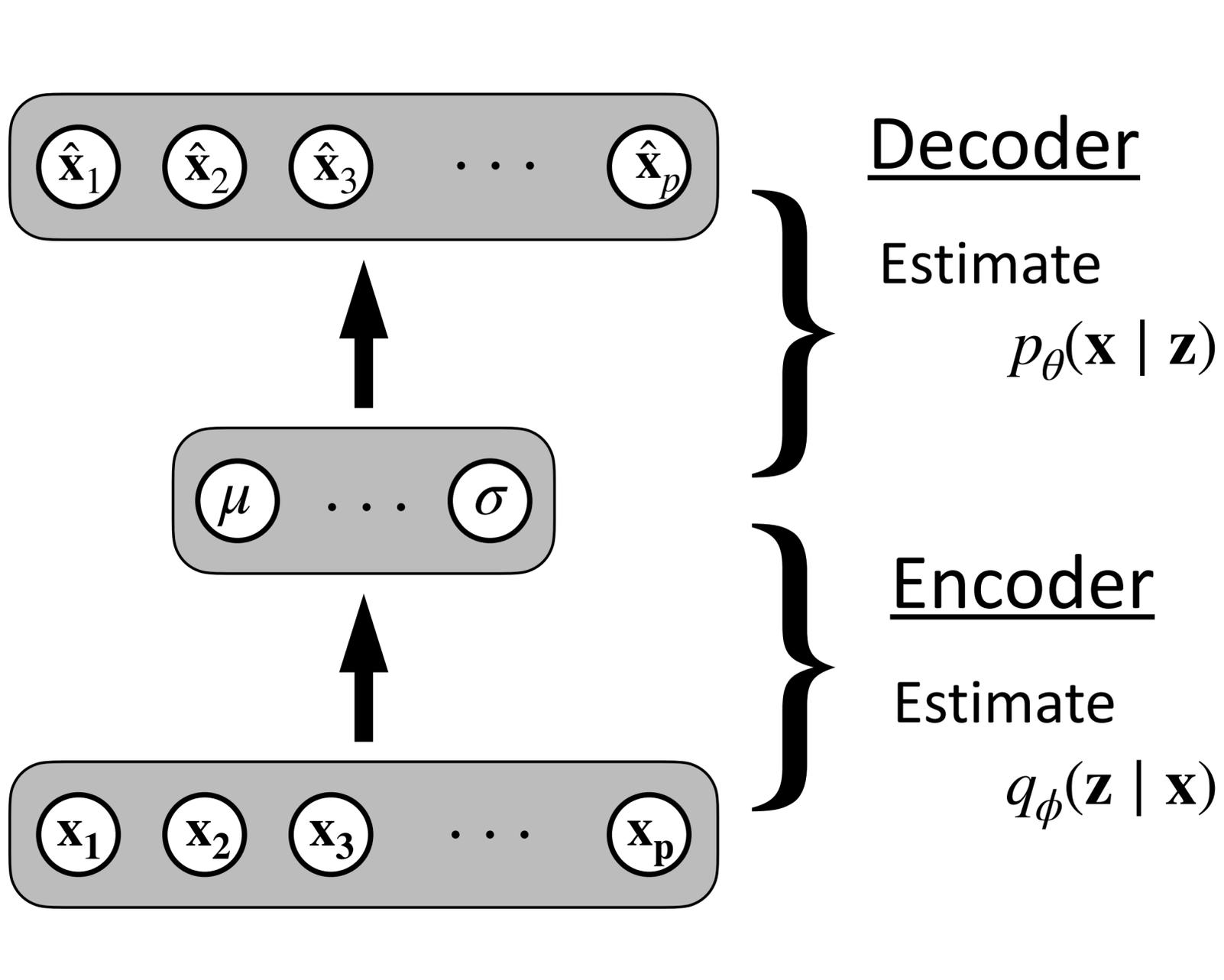
Variational Auto-encoders

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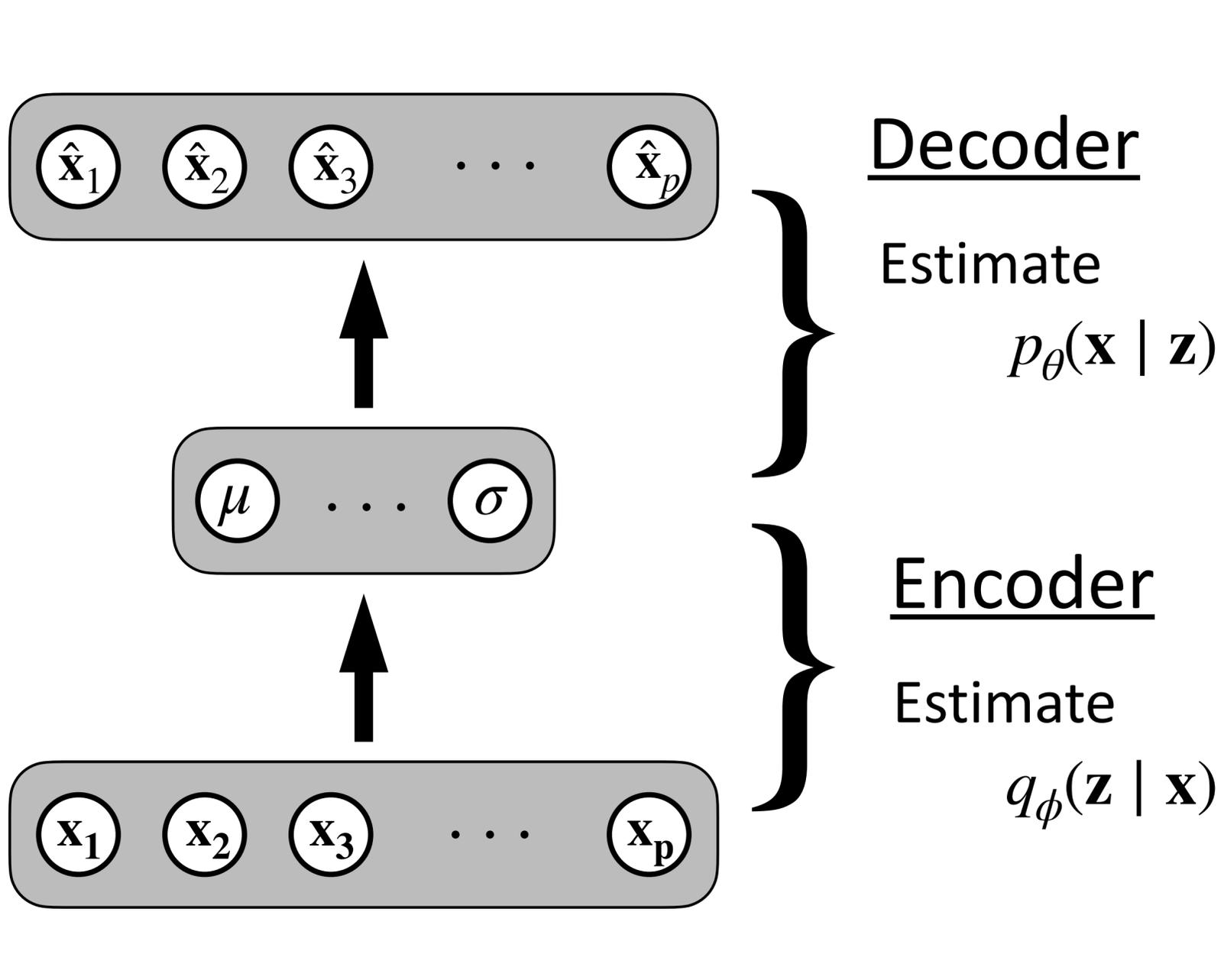


Steps:

1. Train the model to learn:

Variational Auto-encoders

VAEs are a way to train a generative model



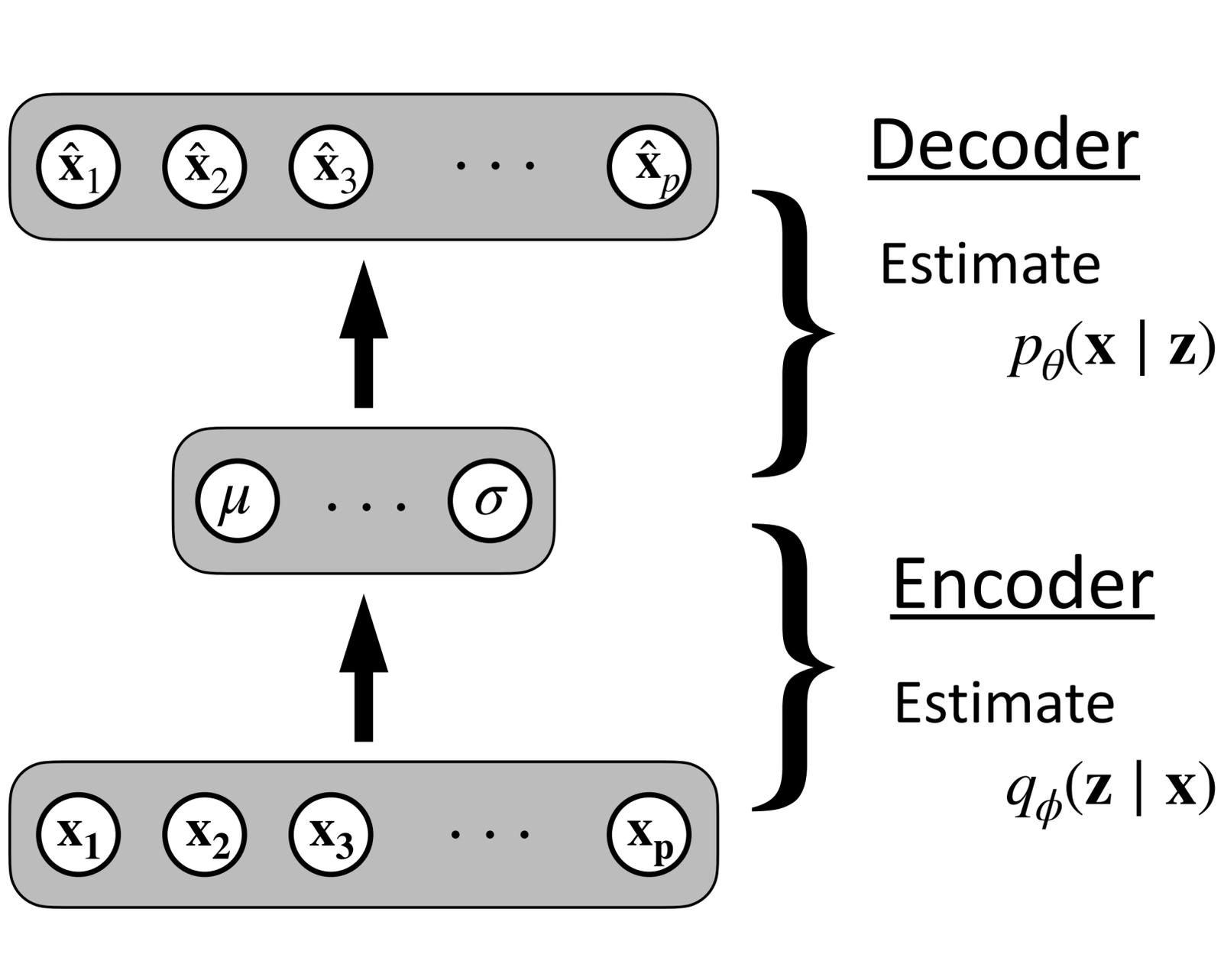
Steps:

1. Train the model to learn:

- $p_\theta(\mathbf{x} | \mathbf{z})$

Variational Auto-encoders

VAEs are a way to train a generative model



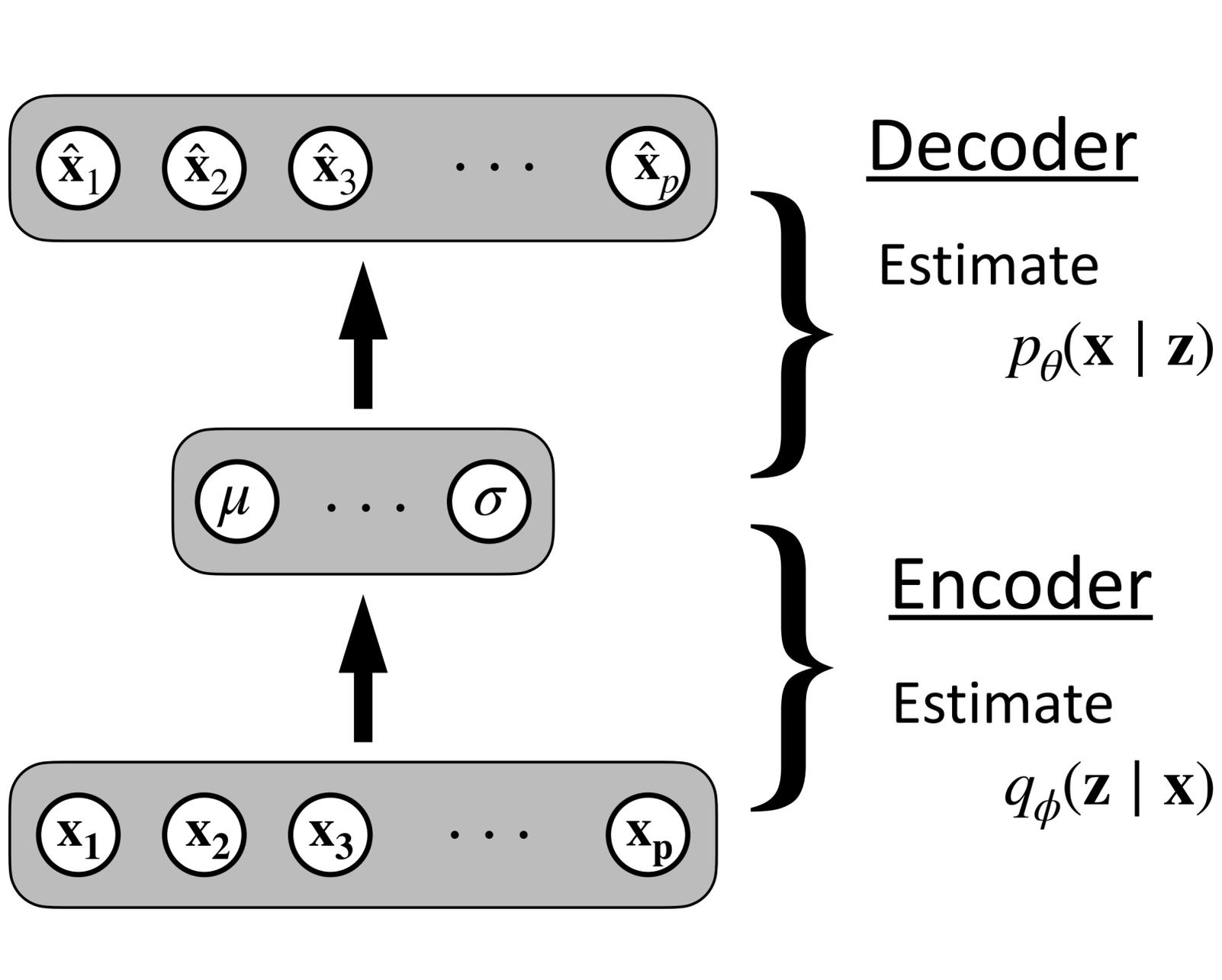
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Variational Auto-encoders

VAEs are a way to train a generative model



Steps:

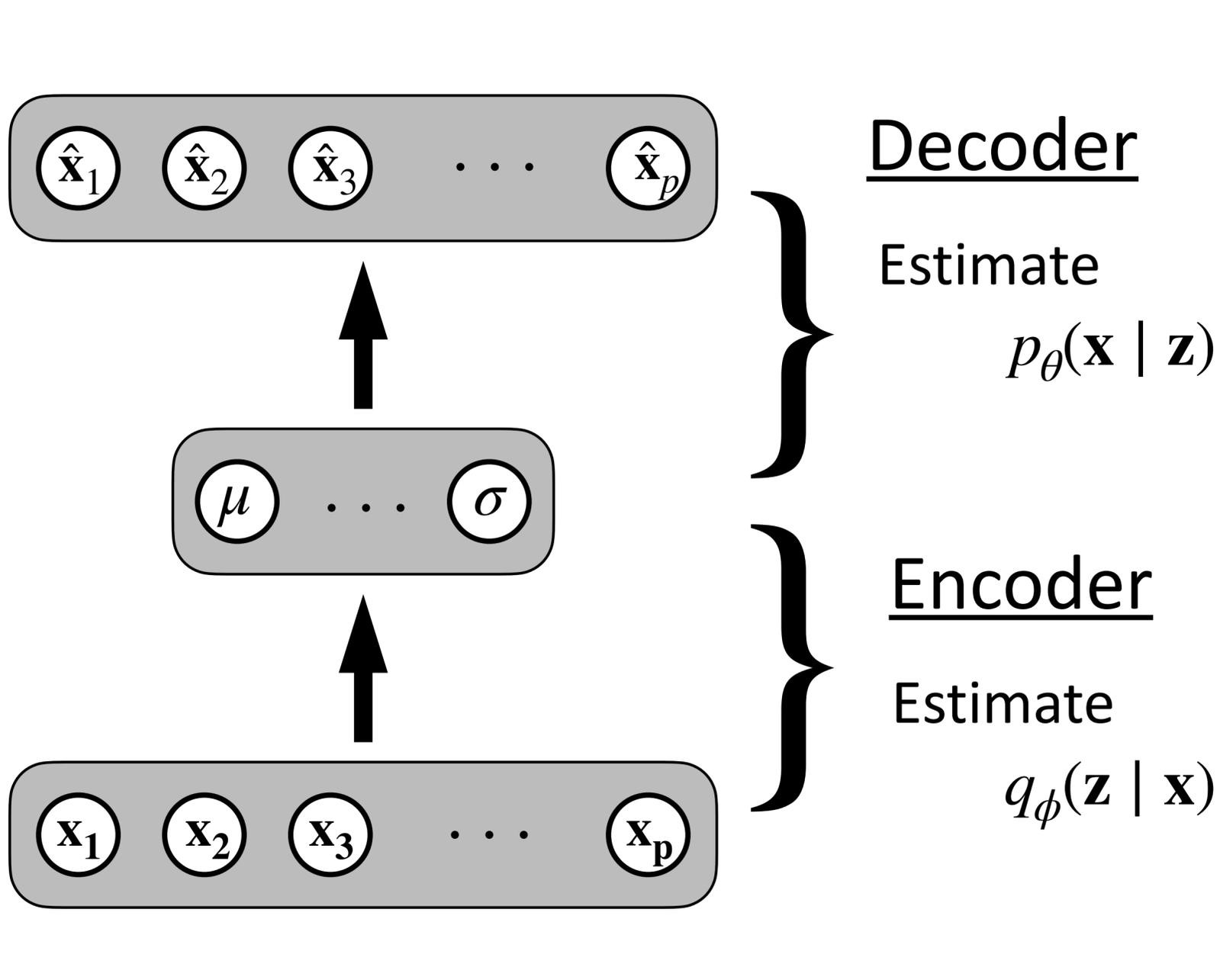
1. Train the model to learn:

- $p_\theta(\mathbf{x} | \mathbf{z})$
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2. Generate ϵ et obtain $z \sim P(z)$.

Variational Auto-encoders

VAEs are a way to train a generative model



Steps:

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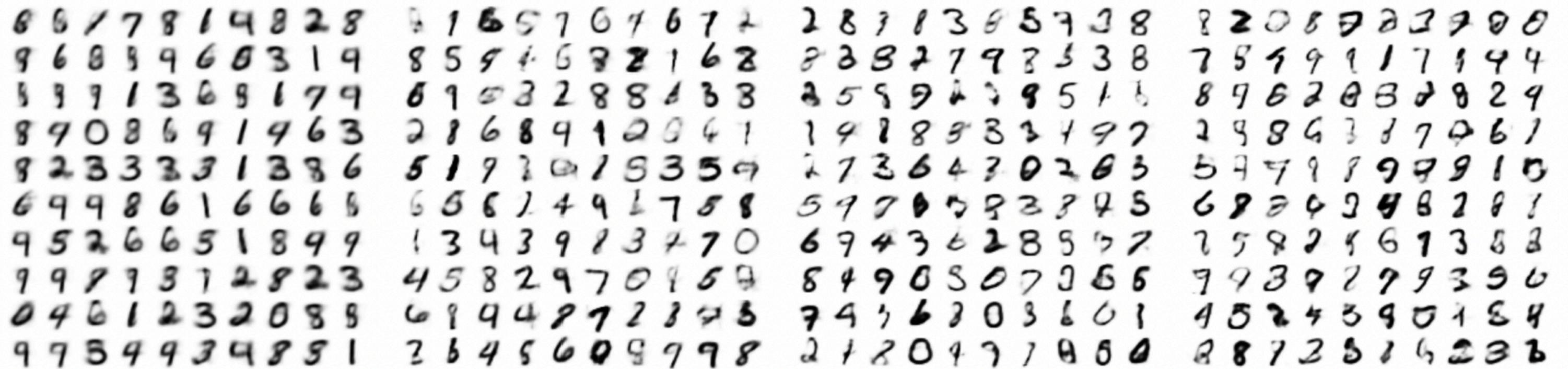
3. Obtain \mathbf{x} from $p_\theta(\mathbf{x} | \mathbf{z})$.

Variational Auto-encoders

Example: Generating numbers (range 0—9)

Variational Auto-encoders

Example: Generating numbers (range 0—9)



Multivariate Normal
(\mathbb{R}^2)

Multivariate Normal
(\mathbb{R}^5)

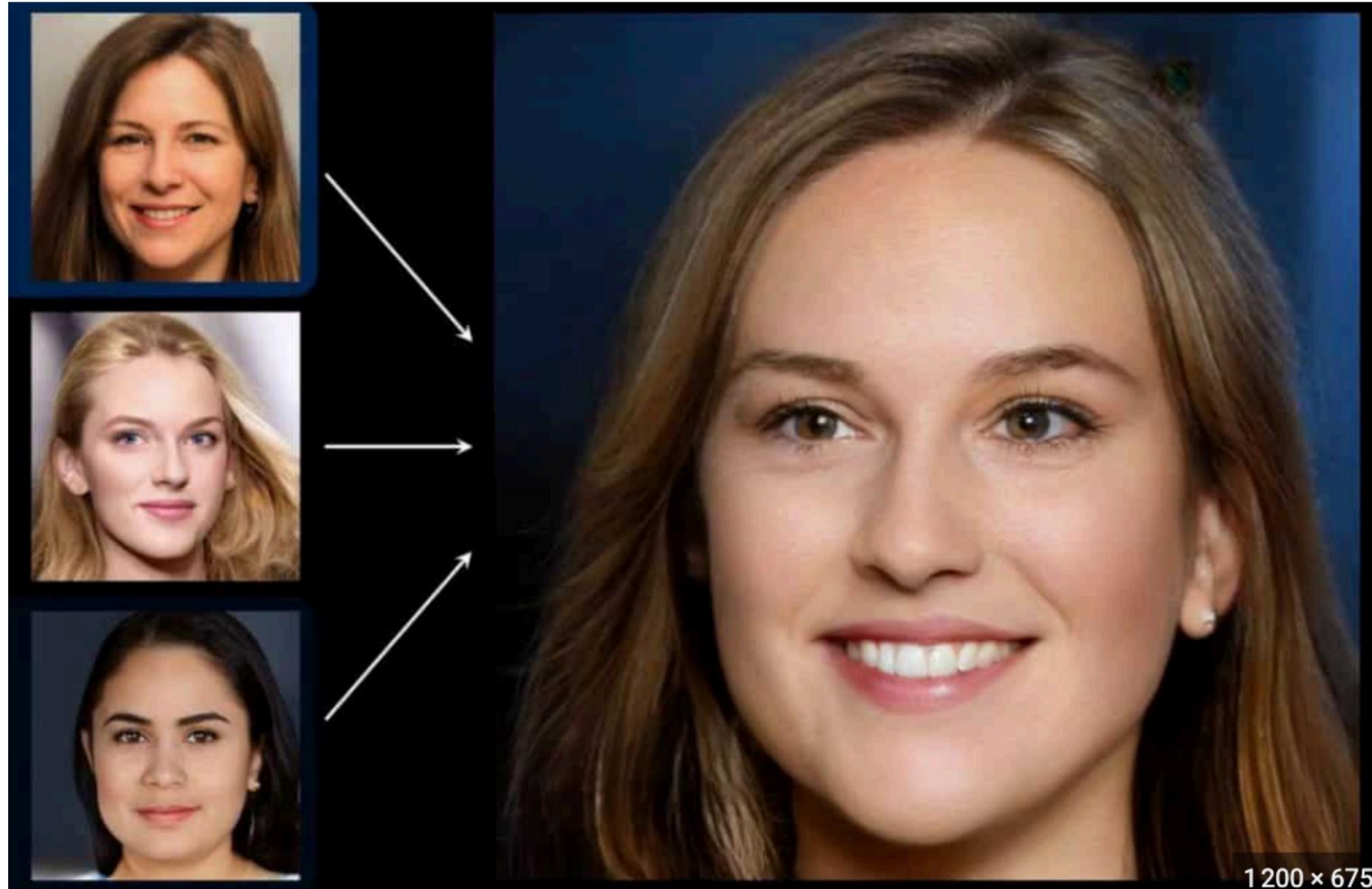
Multivariate Normal (\mathbb{R}^{10})

Multivariate Normal (\mathbb{R}^{20})

Generative Adversarial Networks (GANs)

GANs - Introduction

Well-known for image generation



Historical note

- Framework for learning a generative model (for example, an “inverted” CNN)
- Developed by researchers at Université de Montréal (2014)
- The first to generate high-quality complex images
- Have been (mostly?) replaced by diffusion models
- The study of GANs has provided insights into 2-player (min-max) optimization

GANs — Intuition

GANs — Intuition

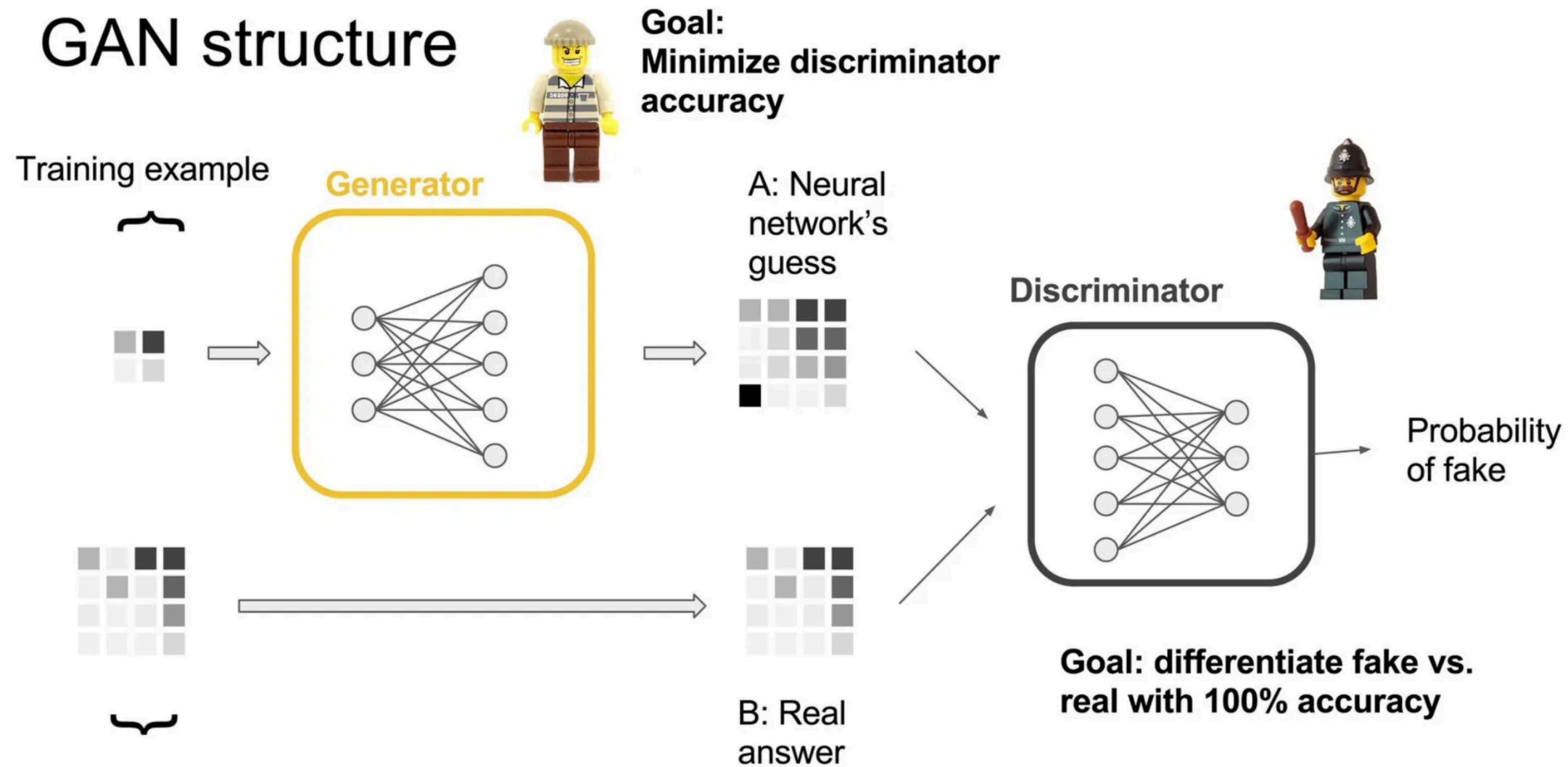
- **Two players (each is a neural network)**
 - **Generator: The first player. It learns to generate a (good) image**
 - **Discriminator: The second player, learns to recognize (discriminate) good images from bad images**

GANs — Intuition

- Two players (each is a neural network)
 - Generator: The first player. It learns to generate a (good) image
 - Discriminator: The second player, learns to recognize (discriminate) good images from bad images
- The game (at each round):
 - The second player receives an image. It must determine whether the image comes from the generator or from the training data
 - Depending on the response, the players update (their weights)

GANs — Introduction

Visually:



GANs - formalism

$$x \sim P_r$$

Data

GANs - formalism

$$\mathbf{x} \sim \mathbb{P}_r$$

Data

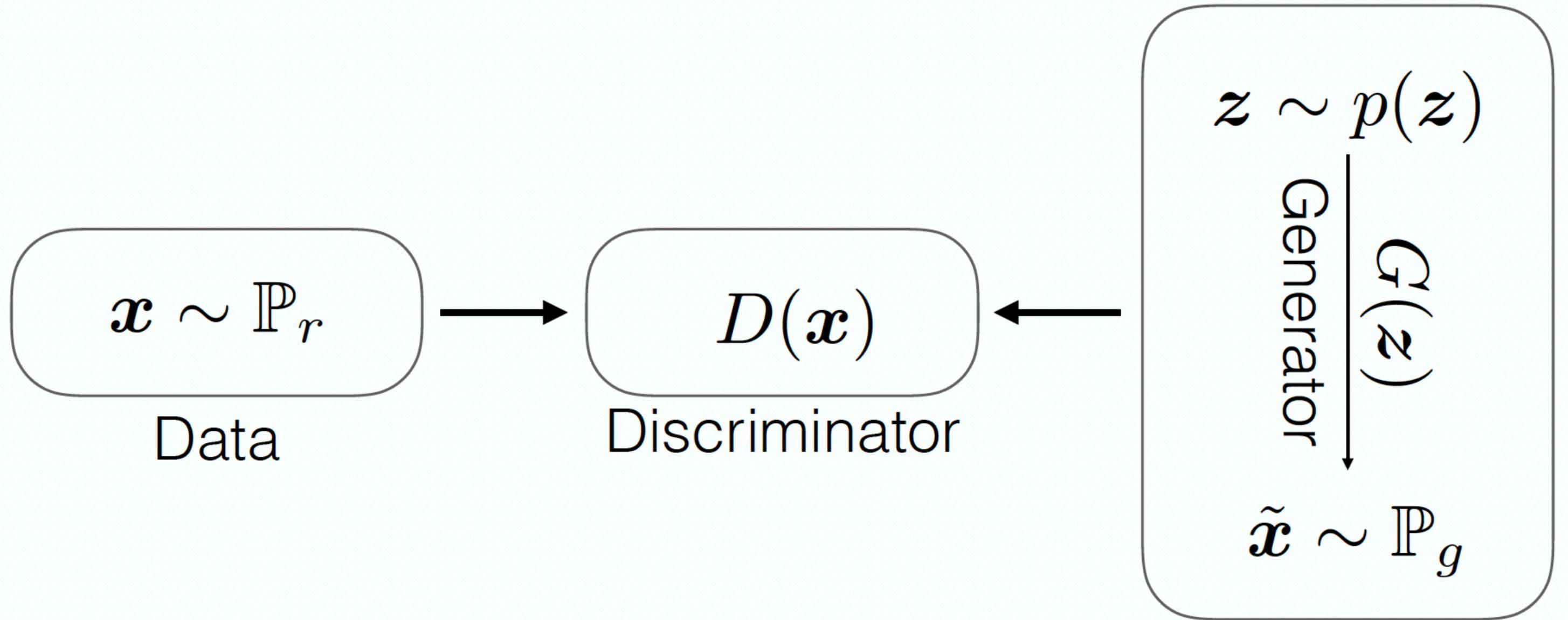
$$\mathbf{z} \sim p(\mathbf{z})$$

Generator

$$G(\mathbf{z})$$

$$\tilde{\mathbf{x}} \sim \mathbb{P}_g$$

GANs - formalism



GANs - objectives

GANs have two objectives (one for each player)

- The output of D is “1” for real and “0” for fake.

Objective of the discriminator

$$\max_D \mathbb{E}_{\mathbf{x} \sim \mathbb{P}_r} [\log(D(\mathbf{x}))] + \mathbb{E}_{\tilde{\mathbf{x}} \sim \mathbb{P}_g} [\log(1 - D(\tilde{\mathbf{x}}))].$$

Objective of the generator

$$\max_G \mathbb{E}_{\tilde{\mathbf{x}} \sim \mathbb{P}_g} [\log(D(\tilde{\mathbf{x}}))].$$

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\mathbb{P}_g Is the distribution generating data

$\tilde{\mathbf{x}} = G(\mathbf{z}), \quad \mathbf{z} \sim p(\mathbf{z})$ Where \mathbf{z} follows a normal (e.g.)

GANs - formalism

The game is framed as a two players game between the discriminator and the generator

$$\min_G \max_D \mathbb{E}_{\mathbf{x} \sim \mathbb{P}_r} [\log(D(\mathbf{x}))] + \mathbb{E}_{\tilde{\mathbf{x}} \sim \mathbb{P}_g} [\log(1 - D(\tilde{\mathbf{x}}))].$$

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Training GANs in practice alternates between training **D** and **G**

Algorithm 1 Minibatch stochastic gradient descent training of generative adversarial nets. The number of steps to apply to the discriminator, k , is a hyperparameter. We used $k = 1$, the least expensive option, in our experiments.

for number of training iterations **do**

for k steps **do**

- Sample minibatch of m noise samples $\{z^{(1)}, \dots, z^{(m)}\}$ from noise prior $p_g(z)$.
- Sample minibatch of m examples $\{x^{(1)}, \dots, x^{(m)}\}$ from data generating distribution $p_{\text{data}}(x)$.
- Update the discriminator by ascending its stochastic gradient:

$$\nabla_{\theta_d} \frac{1}{m} \sum_{i=1}^m \left[\log D(x^{(i)}) + \log \left(1 - D(G(z^{(i)})) \right) \right].$$

end for

- Sample minibatch of m noise samples $\{z^{(1)}, \dots, z^{(m)}\}$ from noise prior $p_g(z)$.
- Update the generator by descending its stochastic gradient:

$$\nabla_{\theta_g} \frac{1}{m} \sum_{i=1}^m \log \left(1 - D(G(z^{(i)})) \right).$$

end for

The gradient-based updates can use any standard gradient-based learning rule. We used momentum in our experiments.

GANs - Varia

Monet ↔ Photos



Monet → photo

Zebras ↔ Horses



zebra → horse

Summer ↔ Winter



summer → winter



photo → Monet



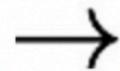
horse → zebra



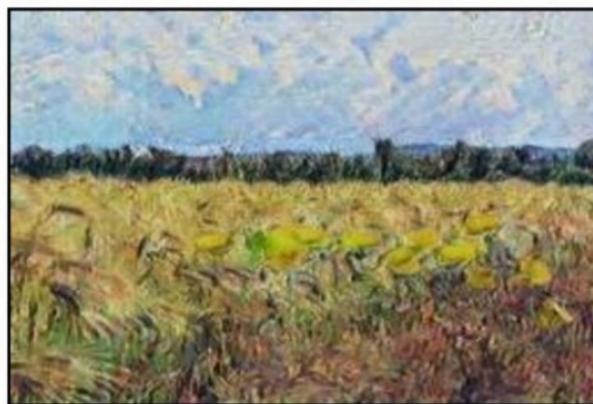
winter → summer



Photograph



Monet



Van Gogh



Cezanne



Ukiyo-e

DALL-E 2

(As an example of using a
diffusion model)

DALL-E



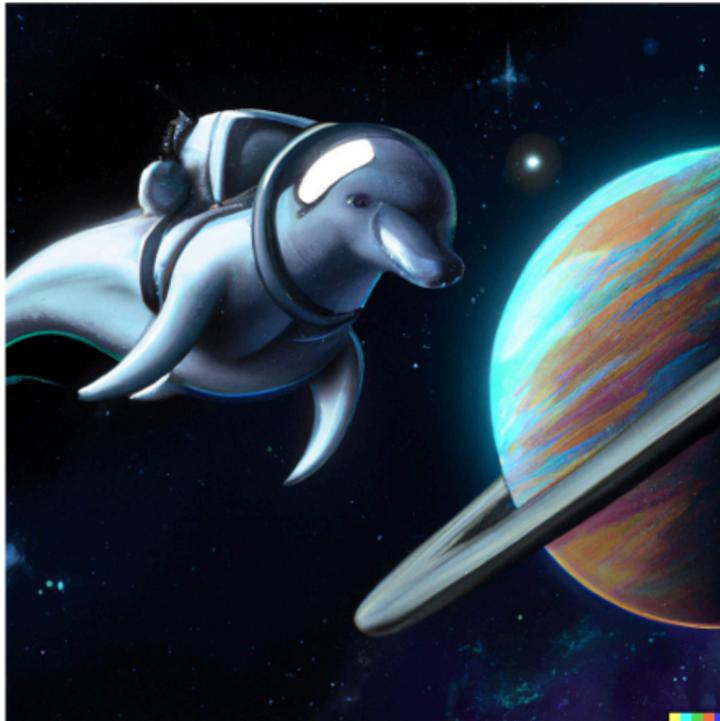
an espresso machine that makes coffee from human souls, artstation



panda mad scientist mixing sparkling chemicals, artstation



a corgi's head depicted as an explosion of a nebula



a dolphin in an astronaut suit on saturn, artstation



a propaganda poster depicting a cat dressed as french emperor napoleon holding a piece of cheese



a teddy bear on a skateboard in times square

DALL-E - Overview

“a corgi
playing a
flame
throwing
trumpet”

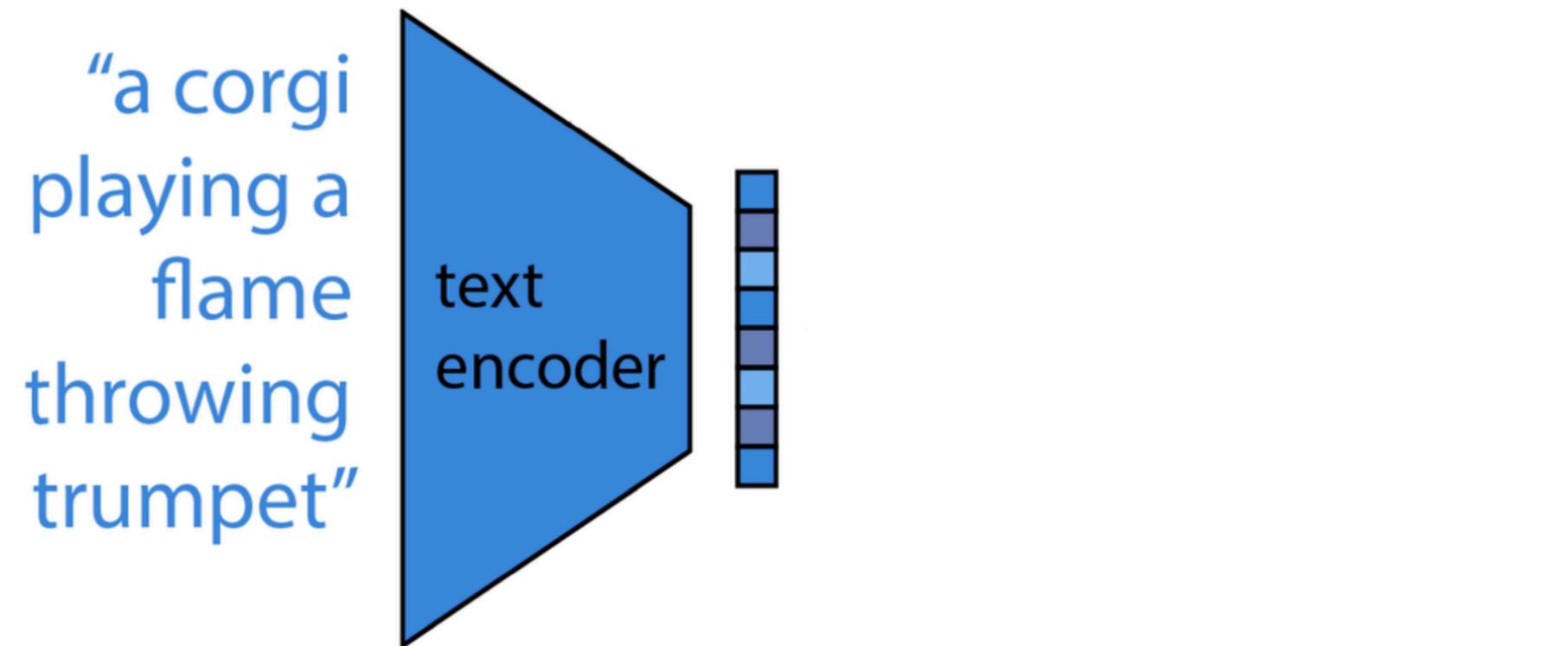


DALL-E - Overview

“a corgi
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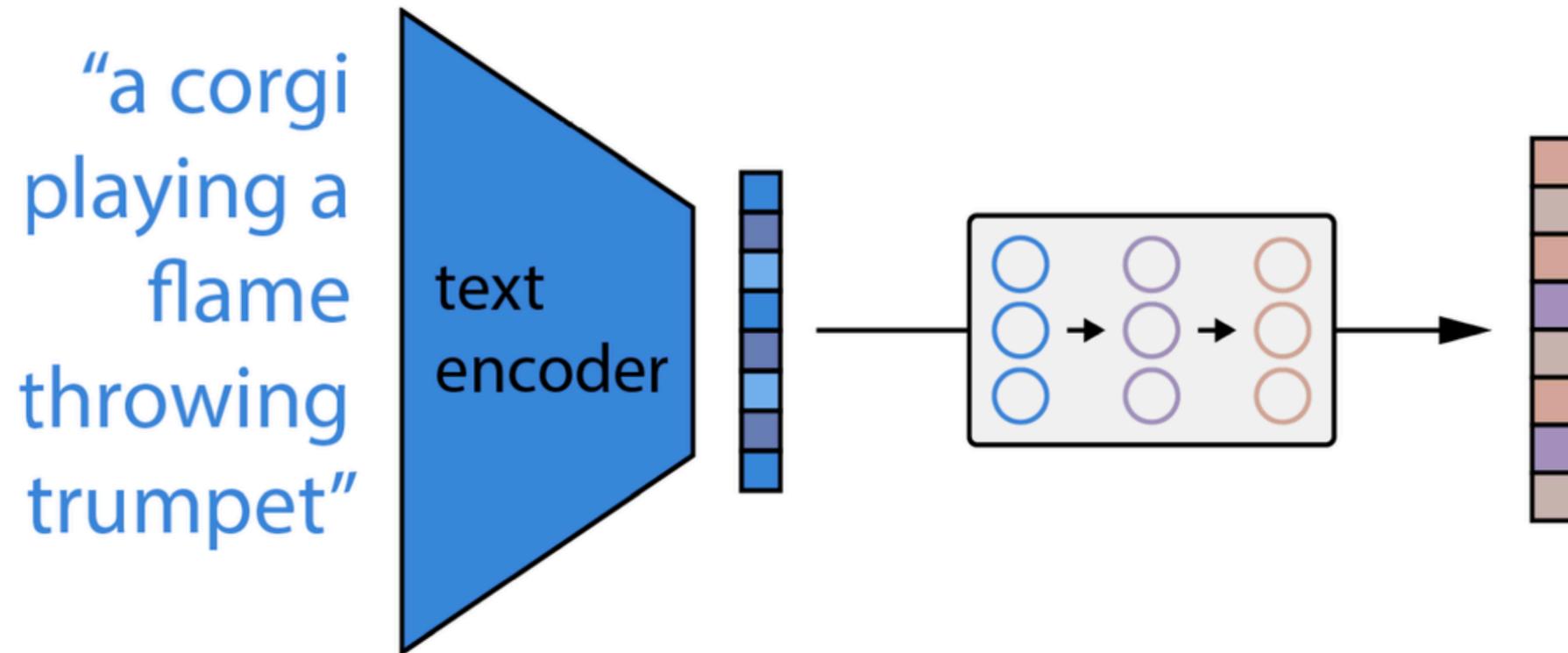
1. Input, a sentence (*prompt*) of the image we want to create

DALL-E - Overview



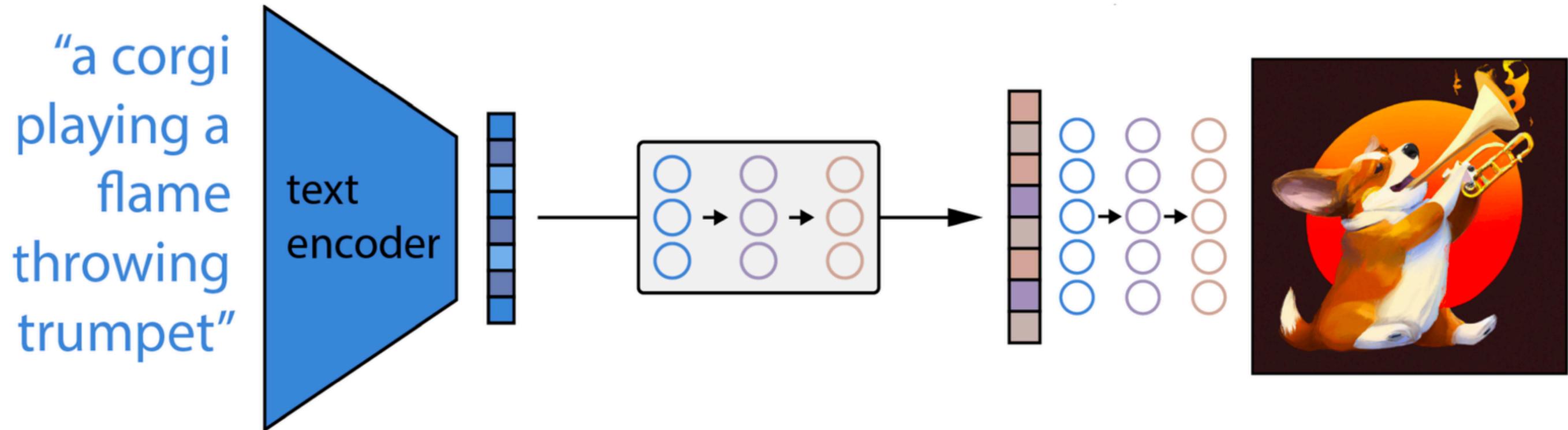
1. Input, a sentence (*prompt*) of the image we want to create
2. This prompt is encoded in a (latent) representation

DALL-E - Overview



1. Input, a sentence (*prompt*) of the image we want to create
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3. Transform this representation in an image representation (image space) — not shown

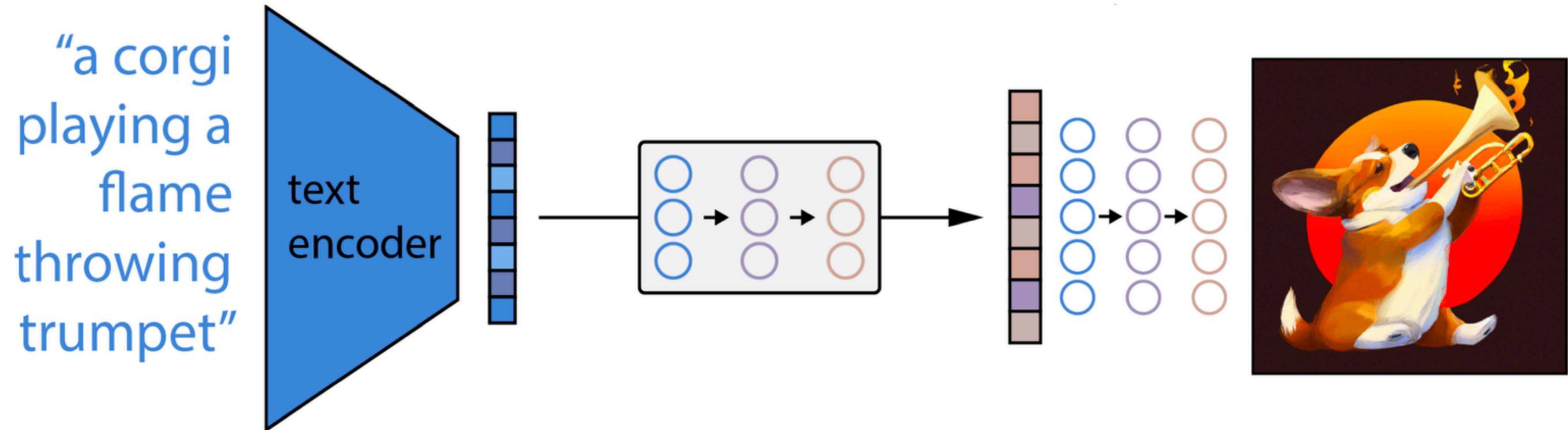
DALL-E - Overview



1. Input, a sentence (*prompt*) of the image we want to create
2. This prompt is encoded in a (latent) representation
3. Transform this representation in an image representation (image space) — not shown
4. Decode the image representation into an actual image

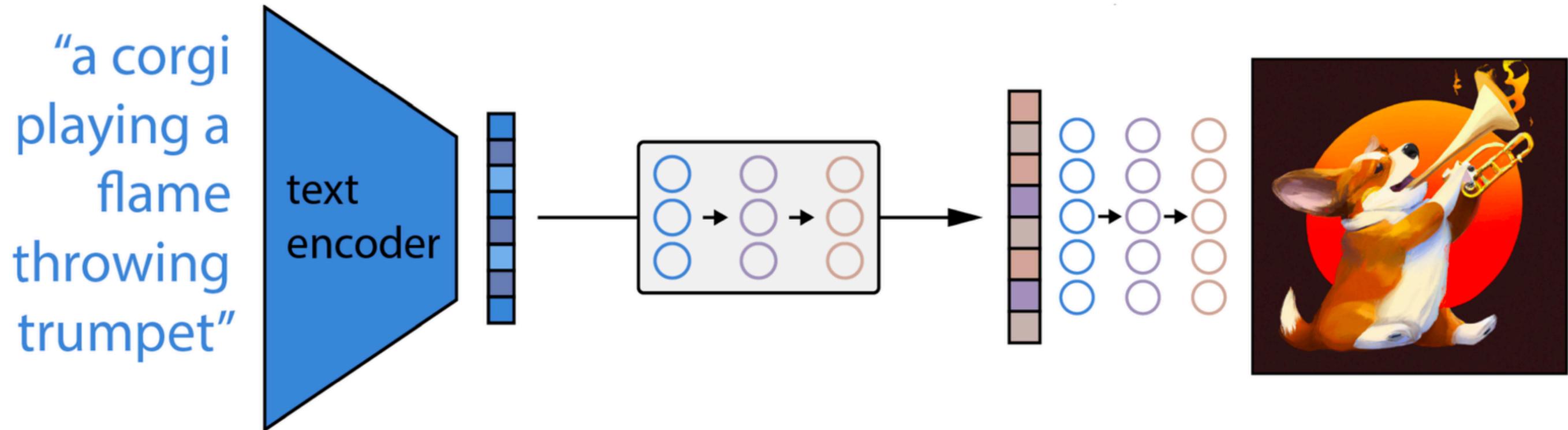
DALL-E - Overview

Each step uses specific techniques:



DALL-E - Overview

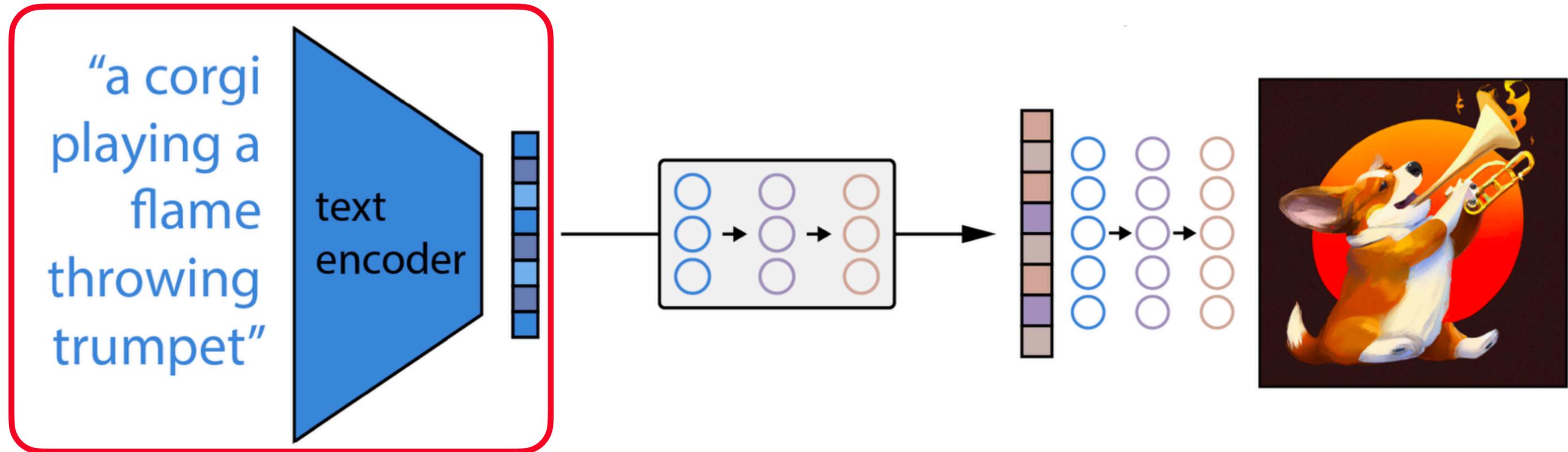
Each step uses specific techniques:



1. Contrastive Language-Image Pre-training (CLIP)

DALL-E - Overview

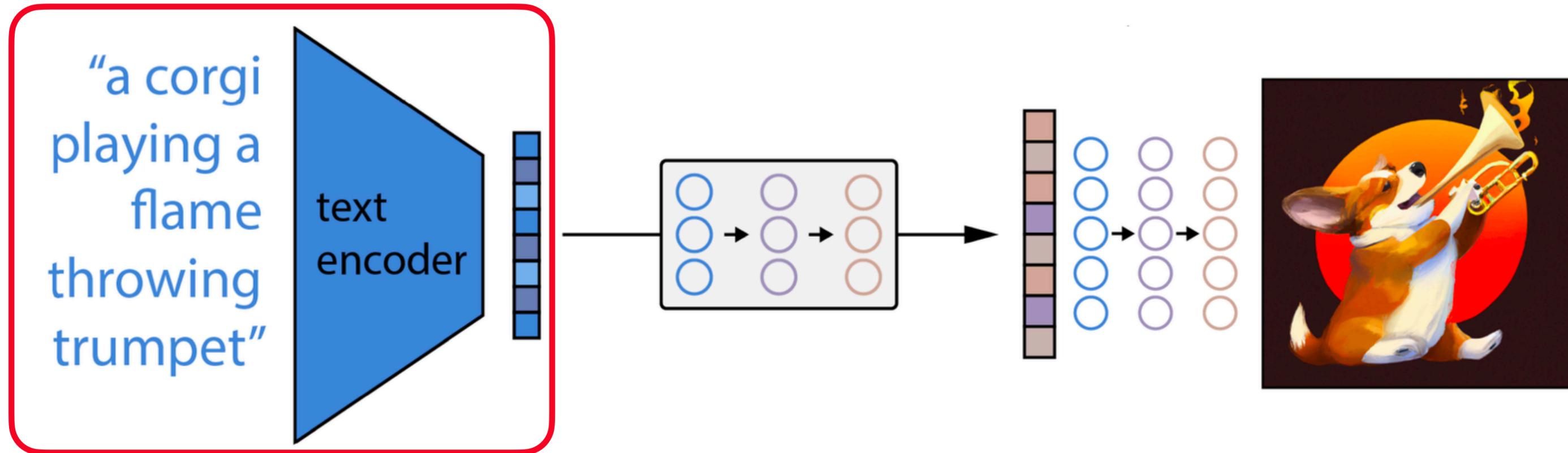
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DALL-E - Overview

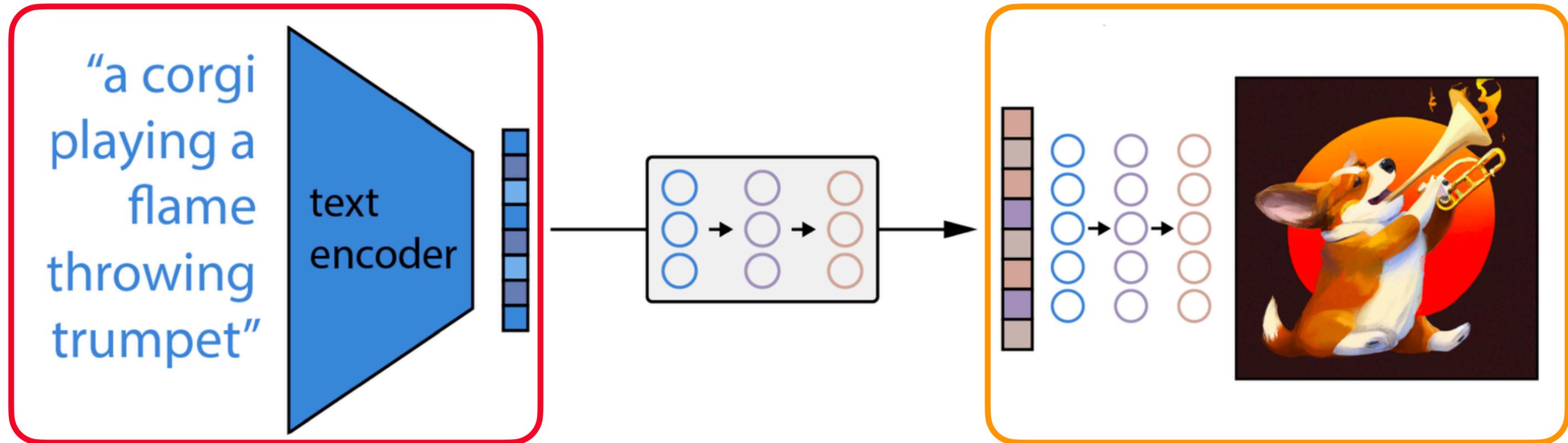
Each step uses specific techniques:



1. Contrastive Language-Image Pre-training (CLIP)
2. Generation of an image using a diffusion model

DALL-E - Overview

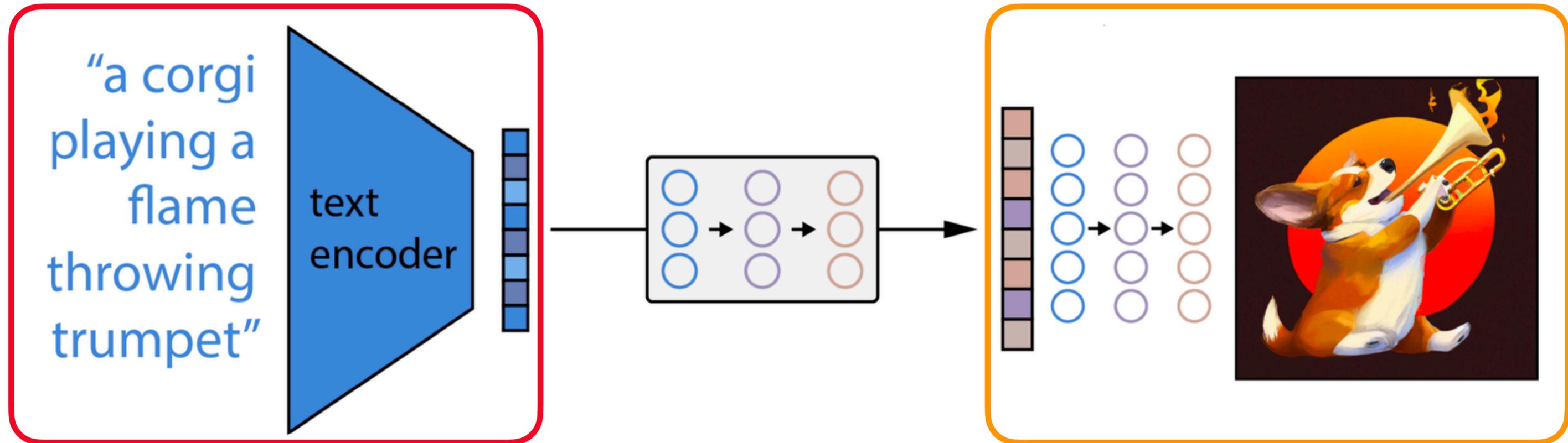
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DALL-E - Overview

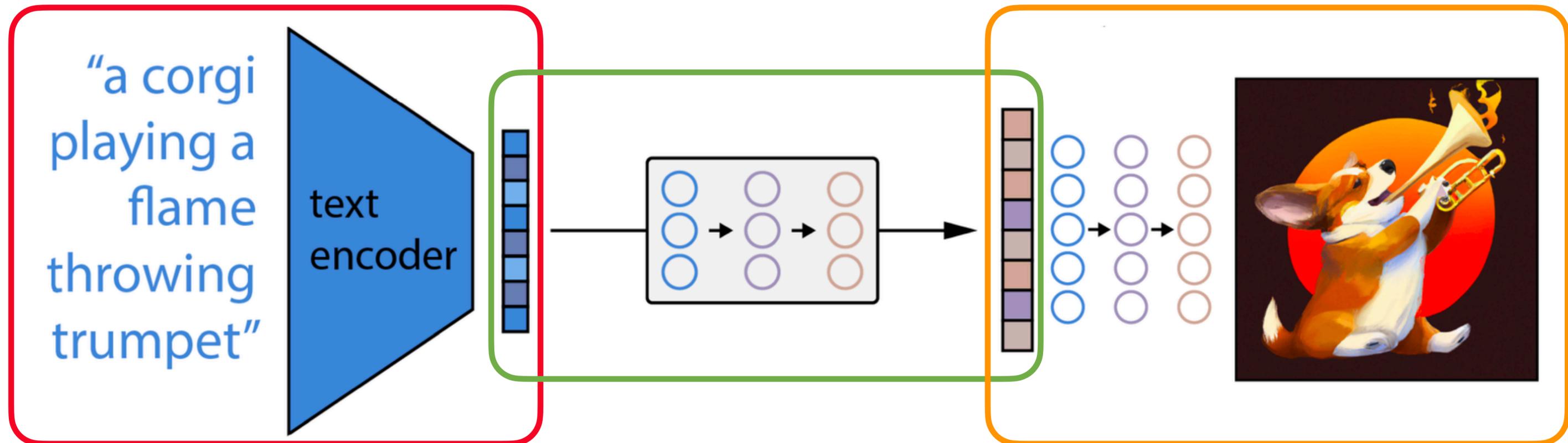
Each step uses specific techniques:



1. Contrastive Language-Image Pre-training (CLIP)
2. Generation of an image using a diffusion model
3. Learn the latent representations of text and images

DALL-E - Overview

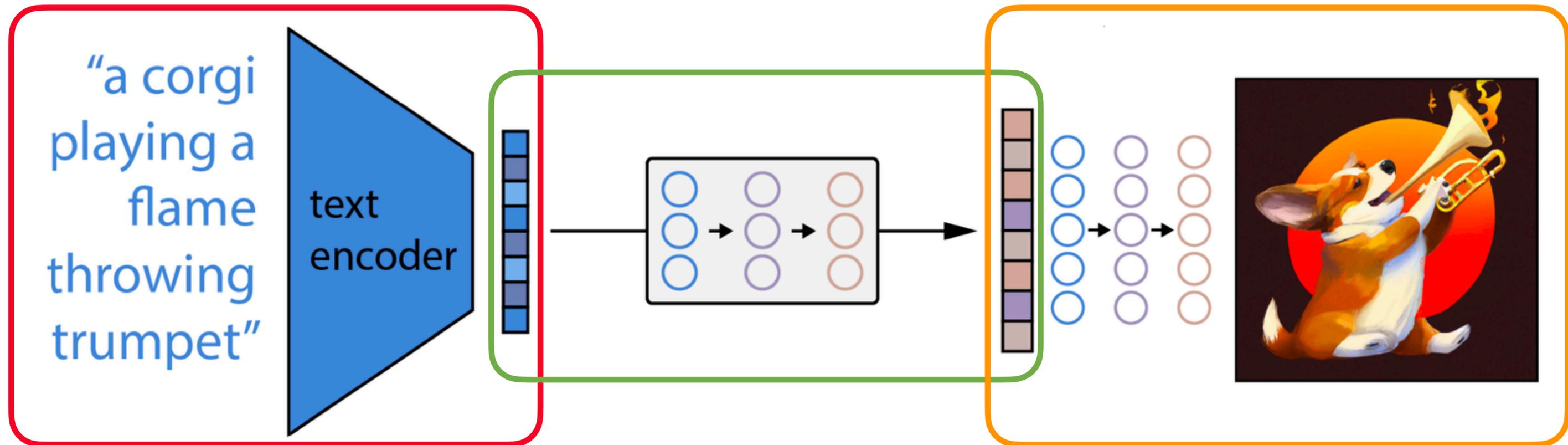
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DALL-E - Overview

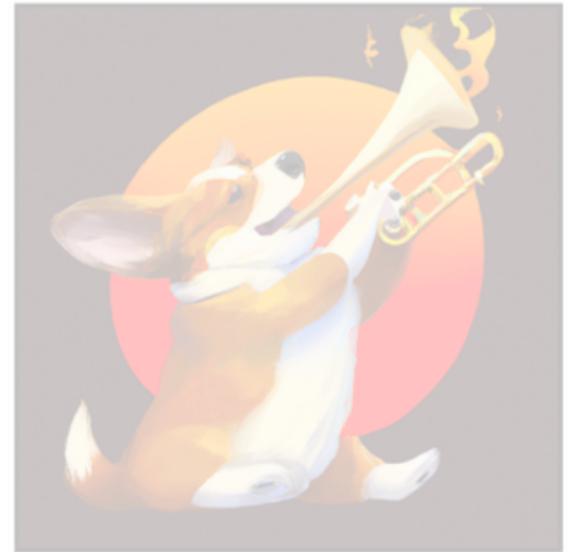
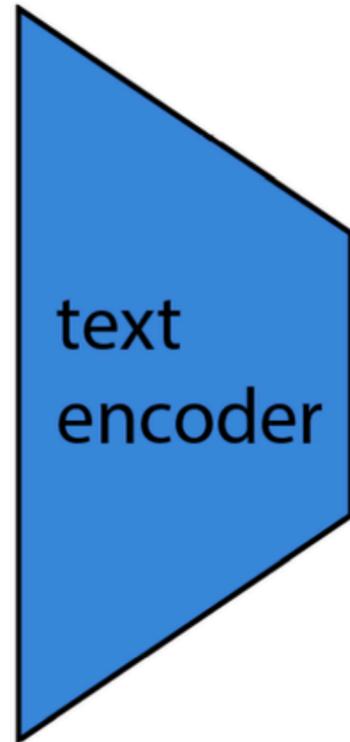
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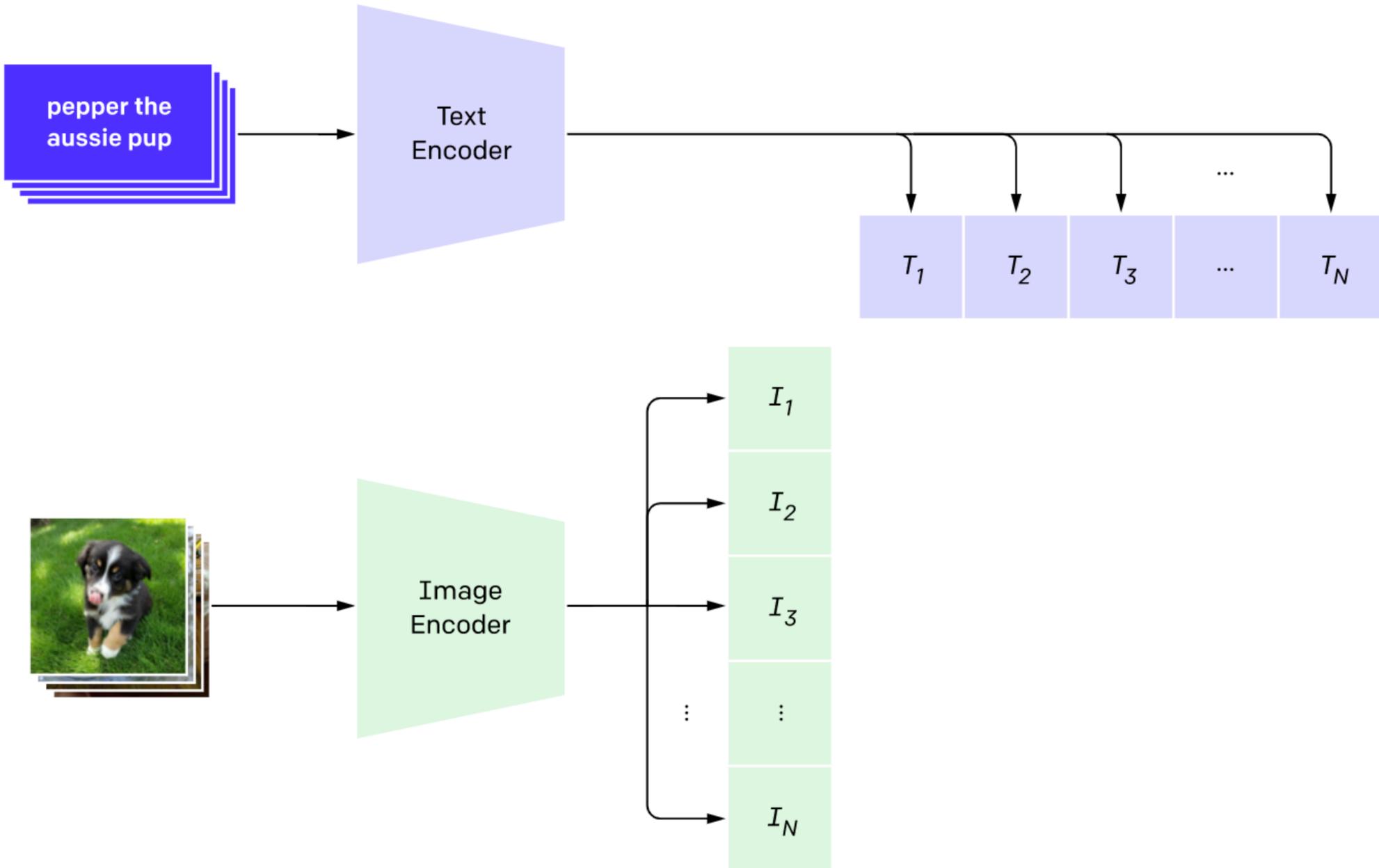
1. Contrastive Language-Image Pre-training (CLIP)
2. Generation of an image using a diffusion model
3. Learn the latent representations of text and images
4. Wrap-it up!

DALL-E - CLIP

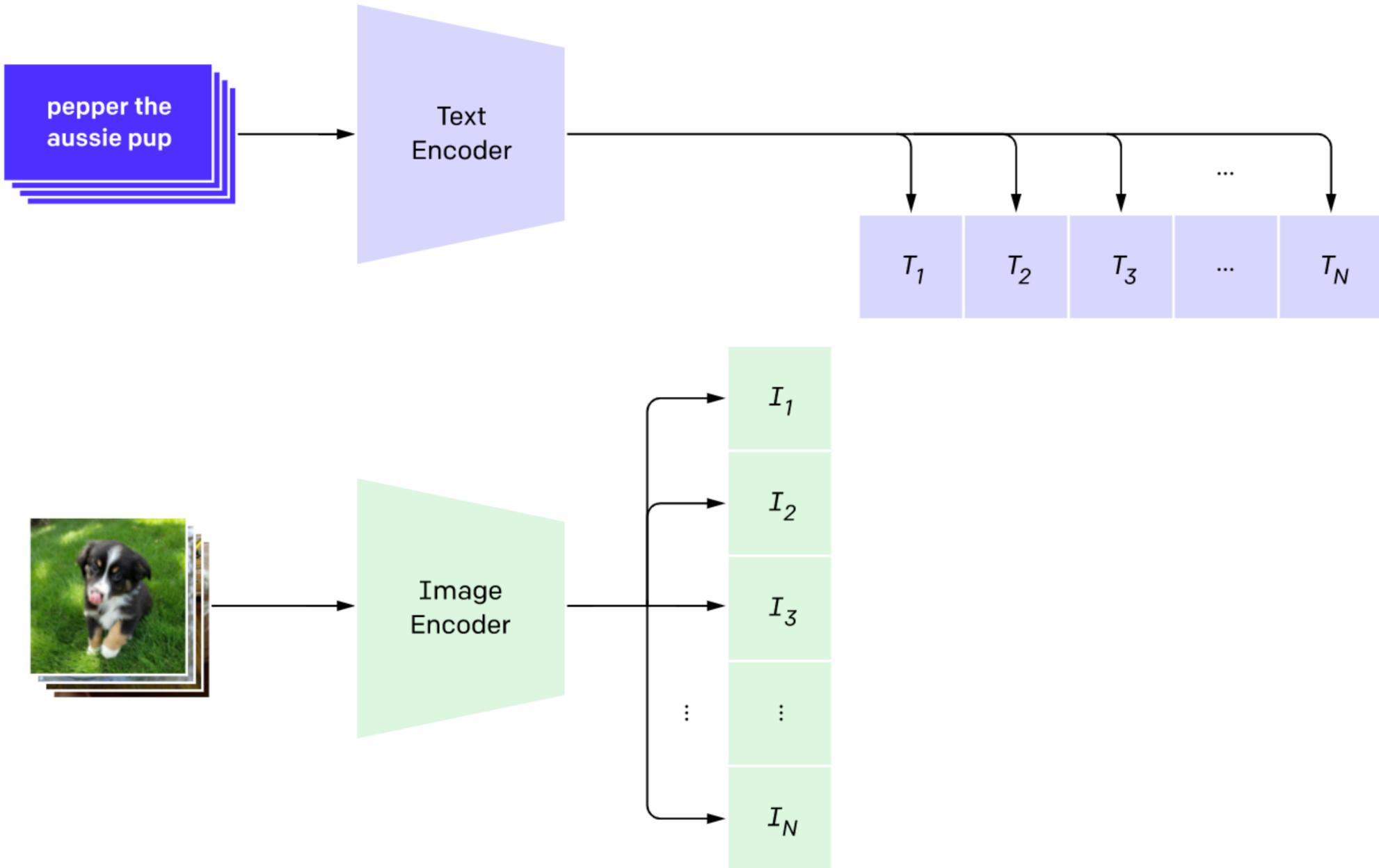
“a corgi
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DALL-E - CLIP

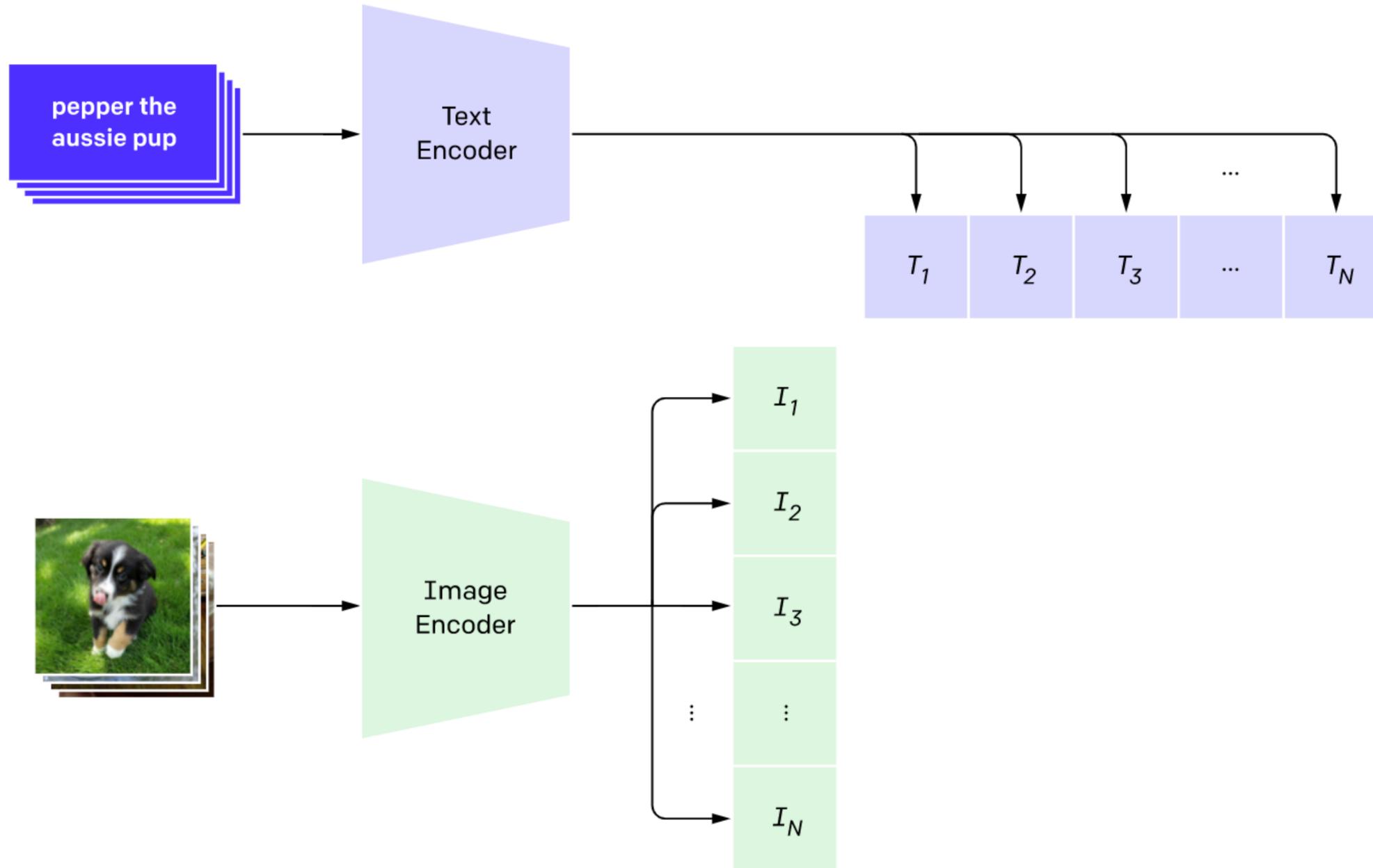


DALL-E - CLIP



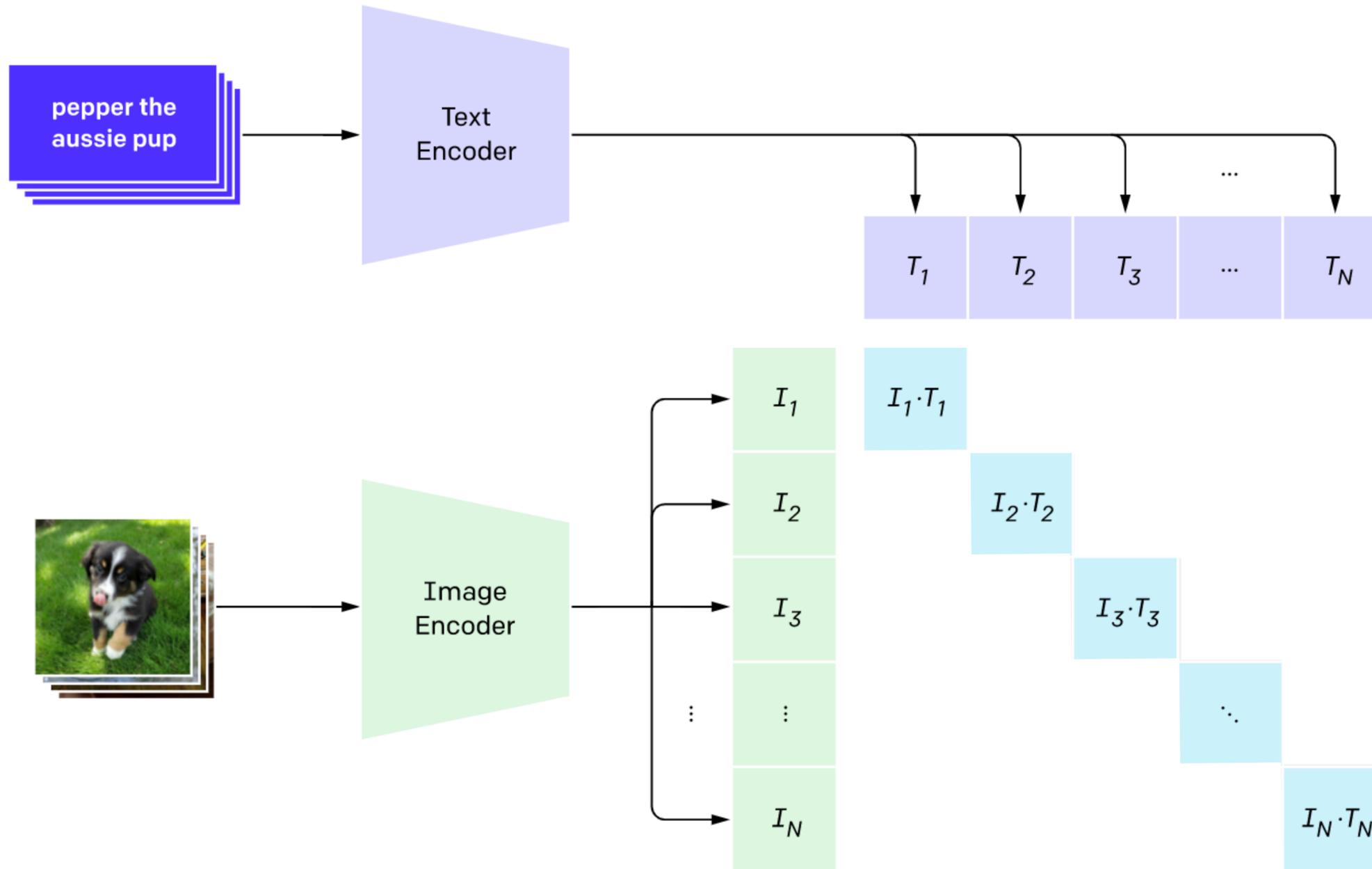
1. Learn latent representations of text T_i and images I_i

DALL-E - CLIP



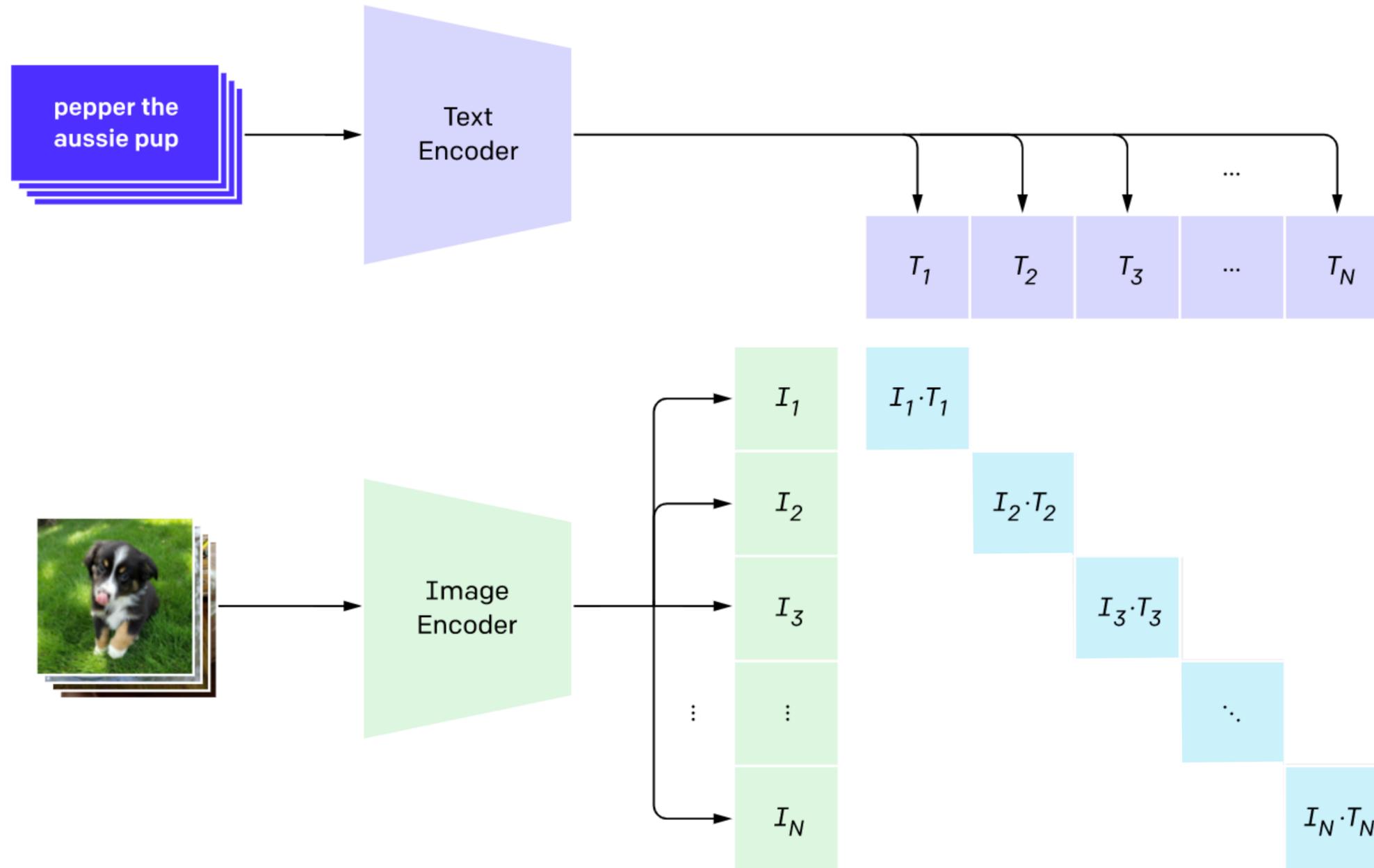
1. Learn latent representations of text T_i and images I_i
 - From a batch of size N

DALL-E - CLIP



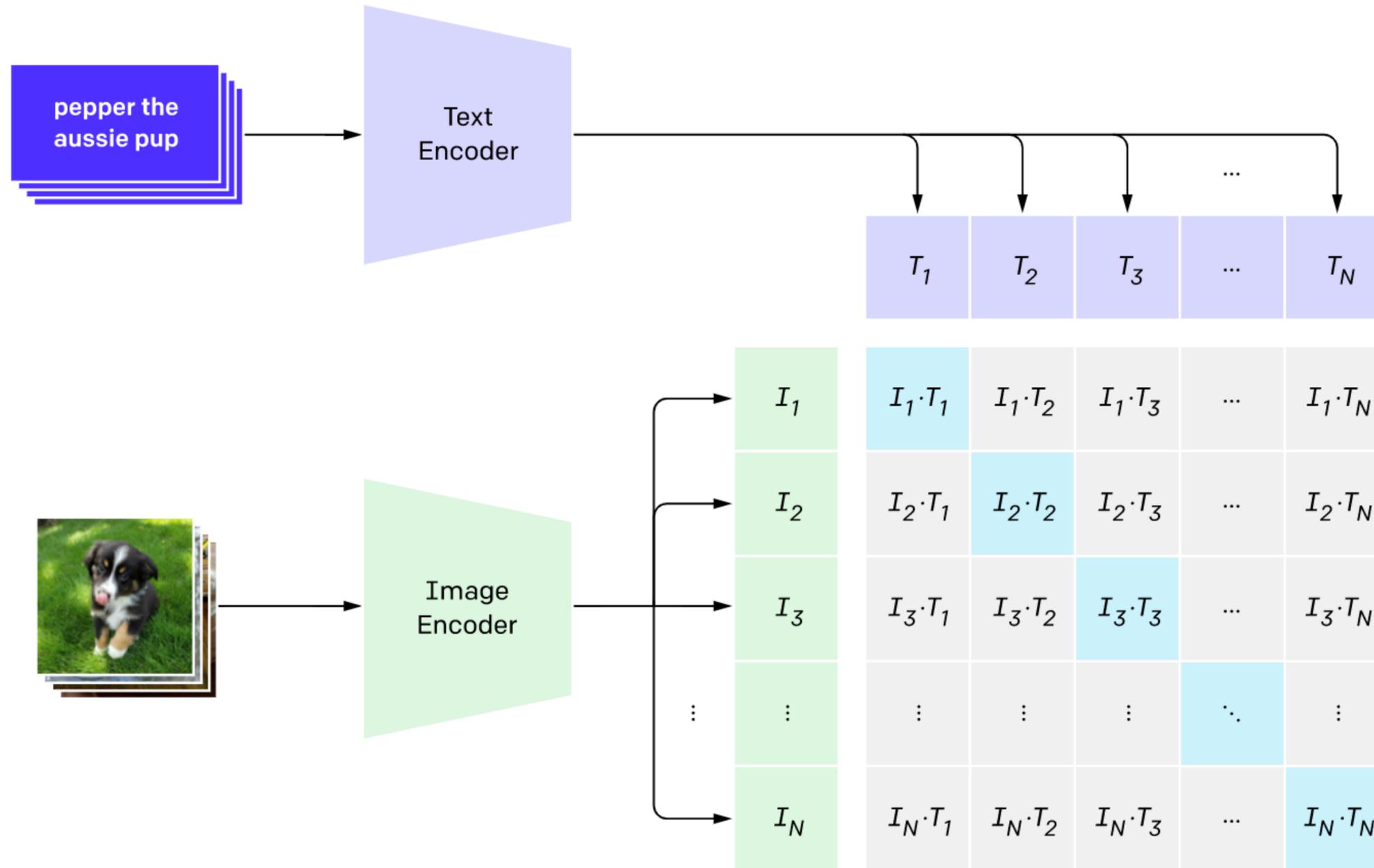
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DALL-E - CLIP



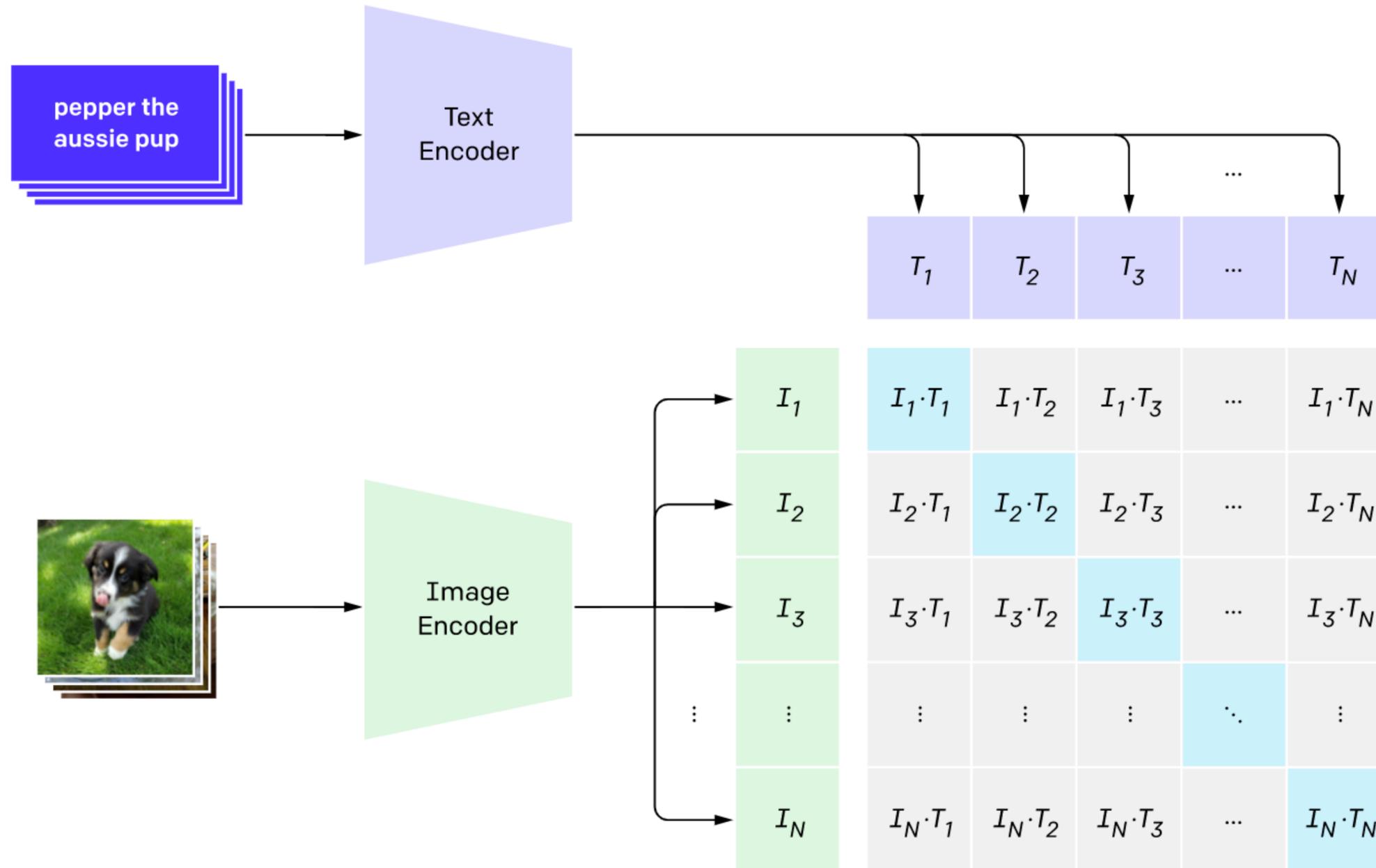
1. Learn latent representations of text T_i and images I_i
 - From a batch of size N
2. Similarity measure

DALL-E - CLIP



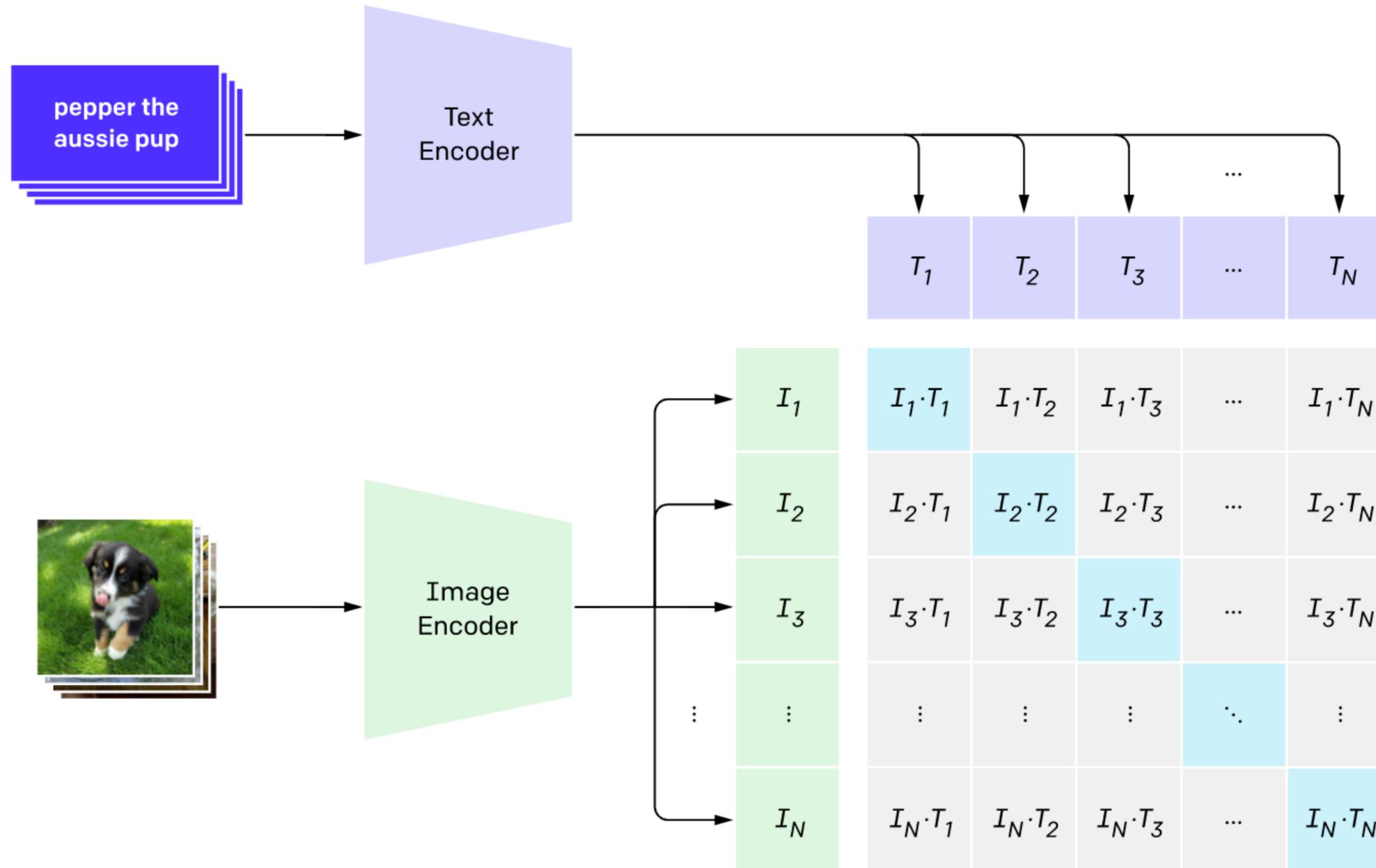
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DALL-E - CLIP



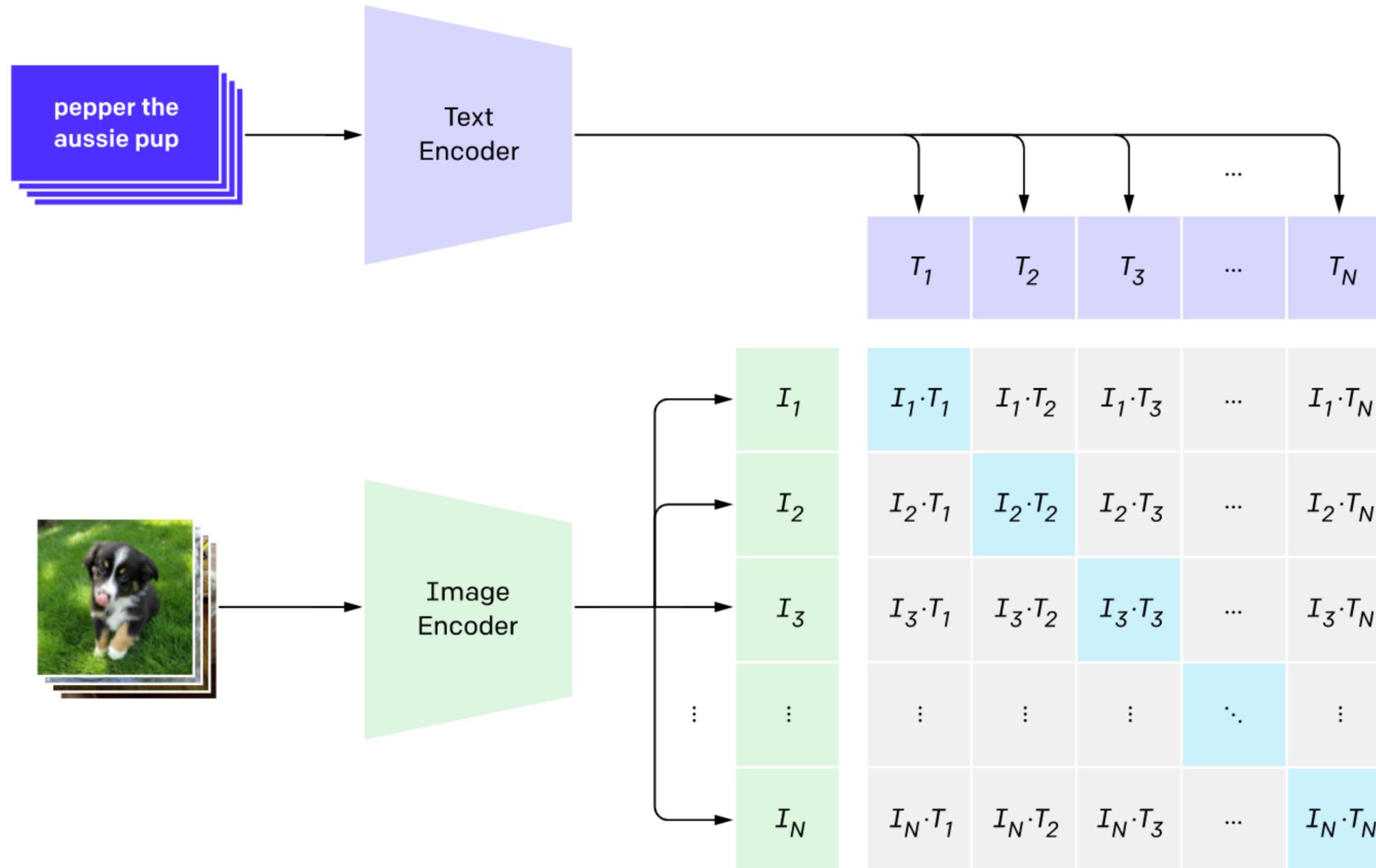
1. Learn latent representations of text T_i and images I_i
 - From a batch of size N
2. Similarity measure
 - $\max T_i I_i$

DALL-E - CLIP



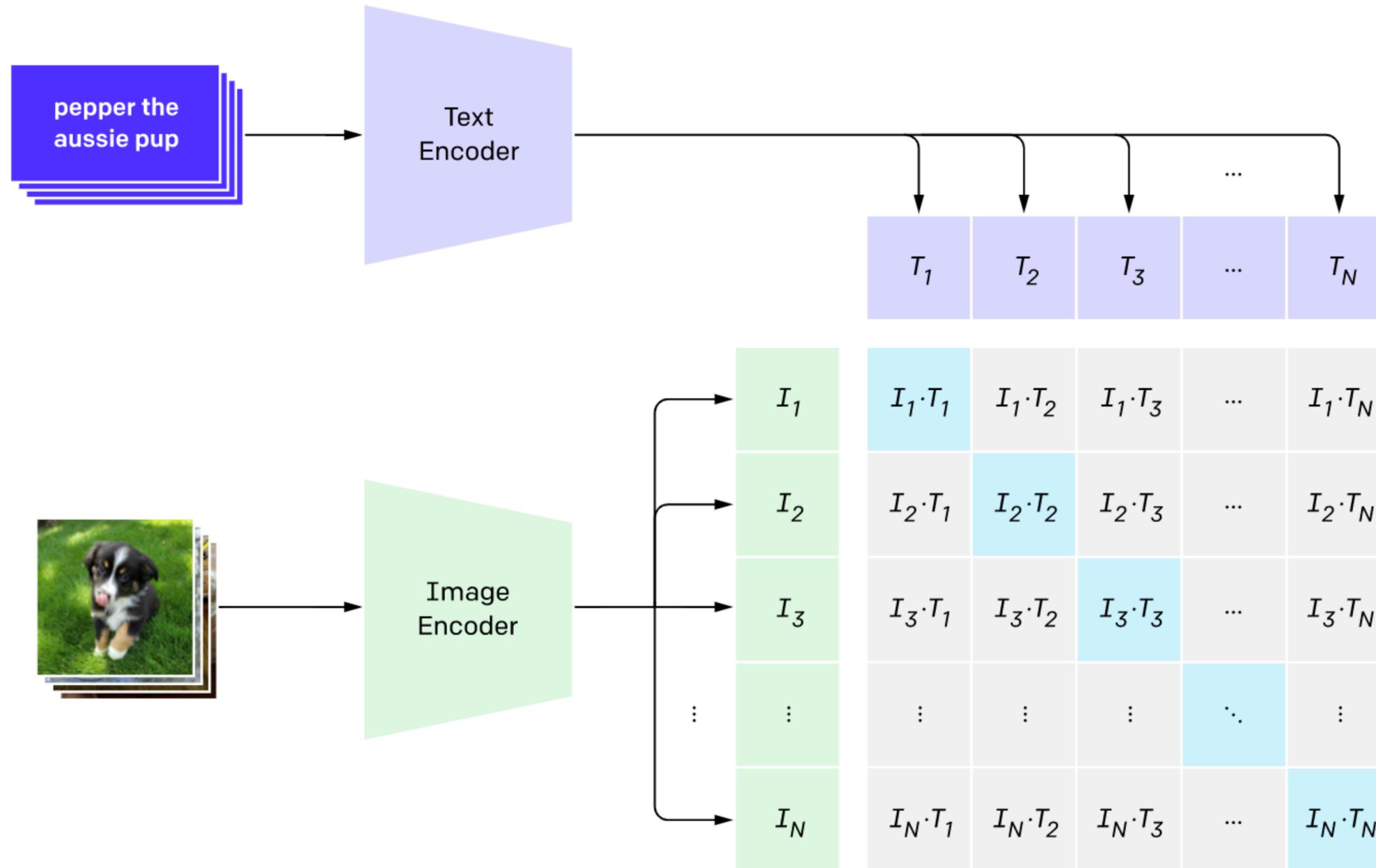
1. Learn latent representations of text T_i and images I_i
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2. Similarity measure
 - $\max T_i I_i$
3. Dissimilarity measure

DALL-E - CLIP



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 - From a batch of size N
2. Similarity measure
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3. Dissimilarity measure
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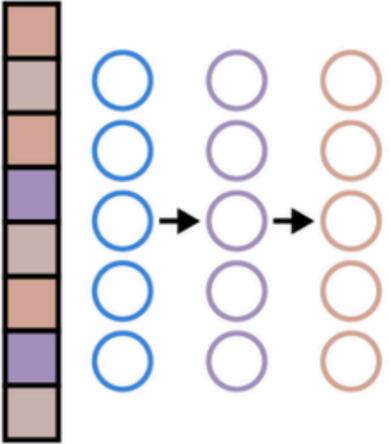
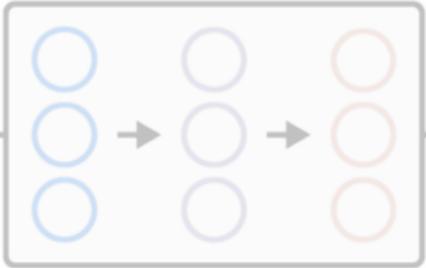
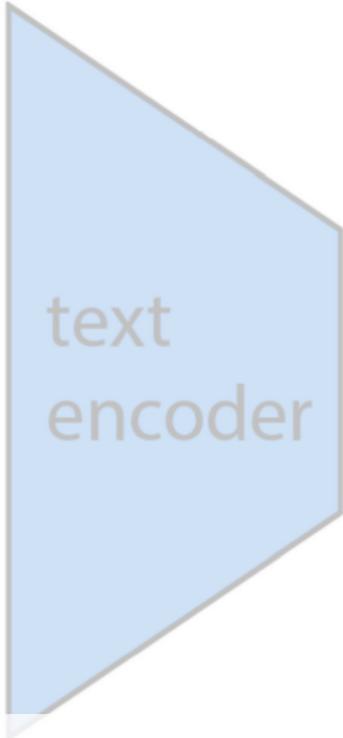
DALL-E - CLIP



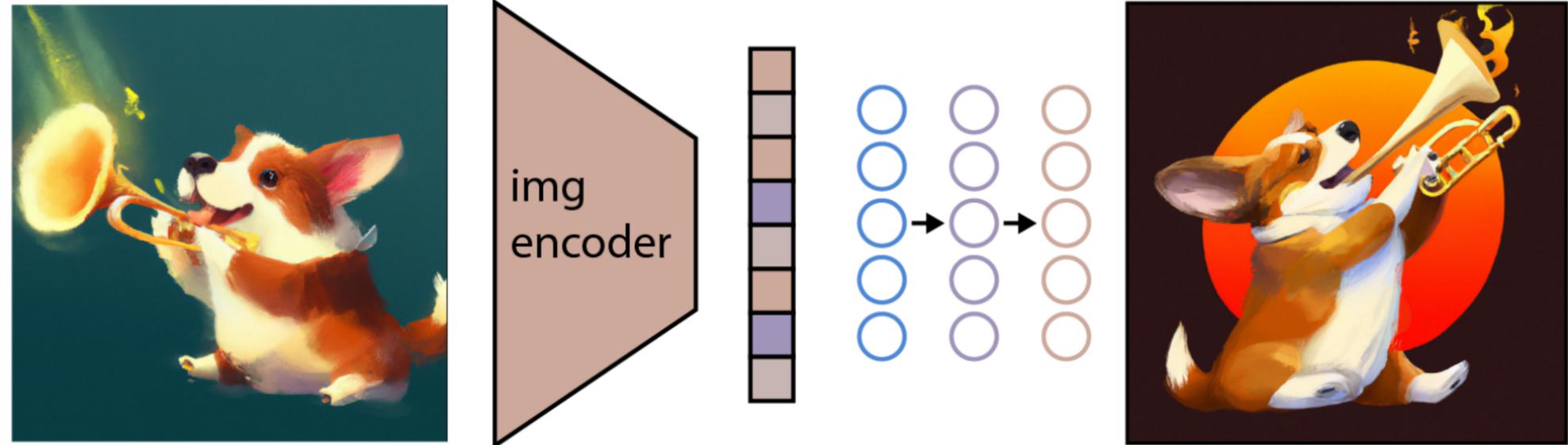
1. Learn latent representations of text T_i and images I_i
 - From a batch of size N
2. Similarity measure
 - $\max T_i I_i$
3. Dissimilarity measure
 - $\min T_i I_j \quad \forall j \neq i$
4. Maximize the similarities and minimize the dissimilarities

DALL-E - Diffusion Models

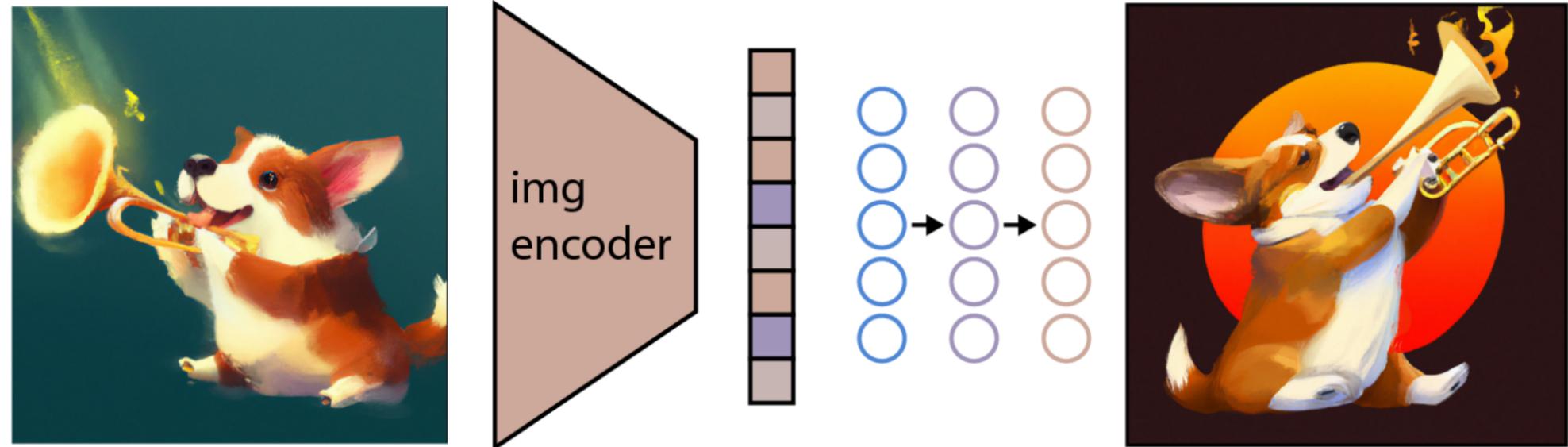
“a corgi playing a flame throwing trumpet”



DALL-E - Diffusion Models

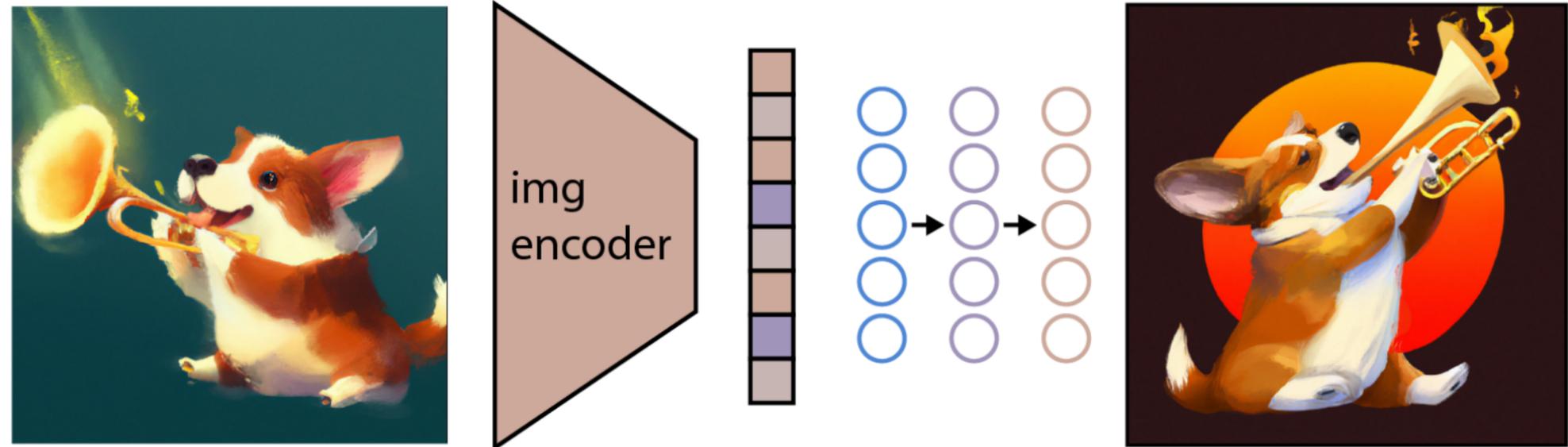


DALL-E - Diffusion Models



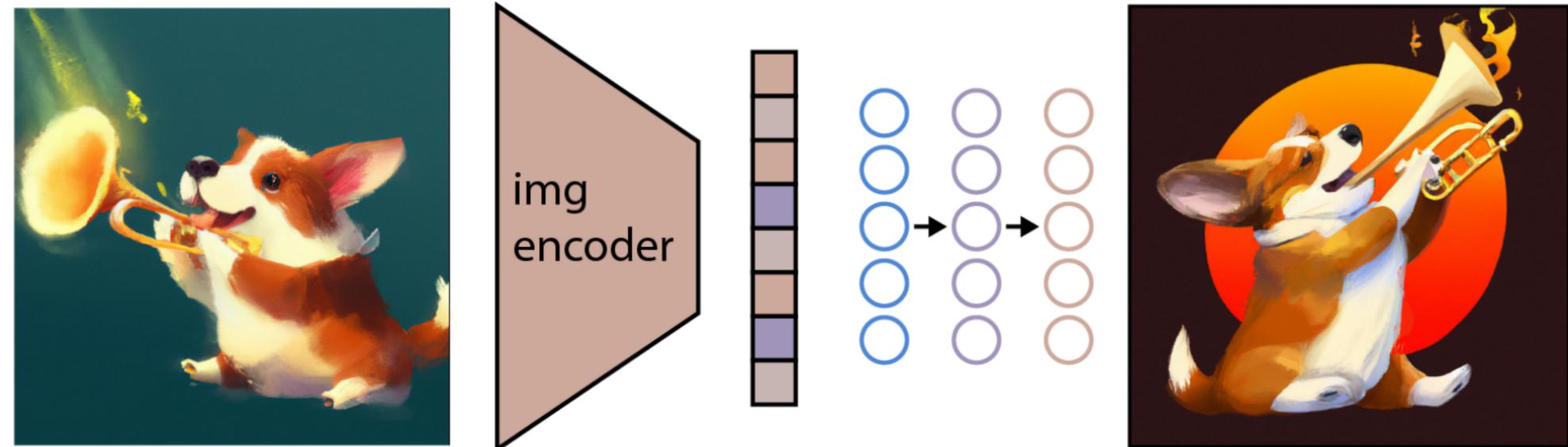
1. Contrastive Language-Image Pre-training (CLIP)

DALL-E - Diffusion Models



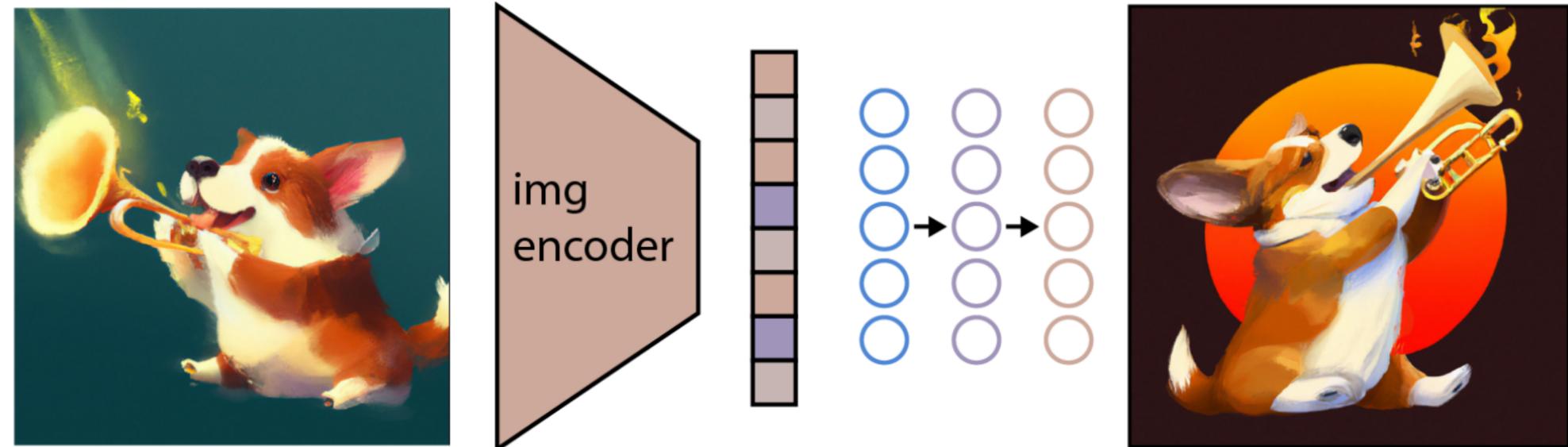
1. Contrastive Language-Image Pre-training (CLIP)
2. Generation of an image using a diffusion model

DALL-E - Diffusion Models



1. Contrastive Language-Image Pre-training (CLIP)
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DALL-E - Diffusion Models

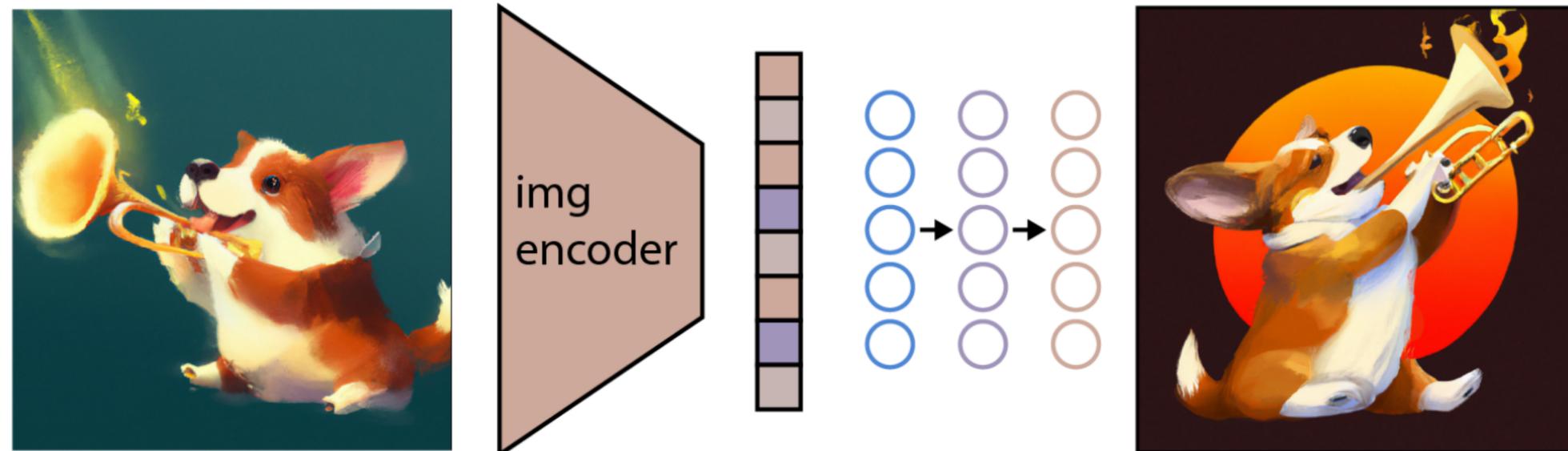


1. Contrastive Language-Image Pre-training (CLIP)
2. Generation of an image using a diffusion model
3. Learn the latent representations of text and images
4. Wrap-it up!

DALL-E - Diffusion Model

Idea:

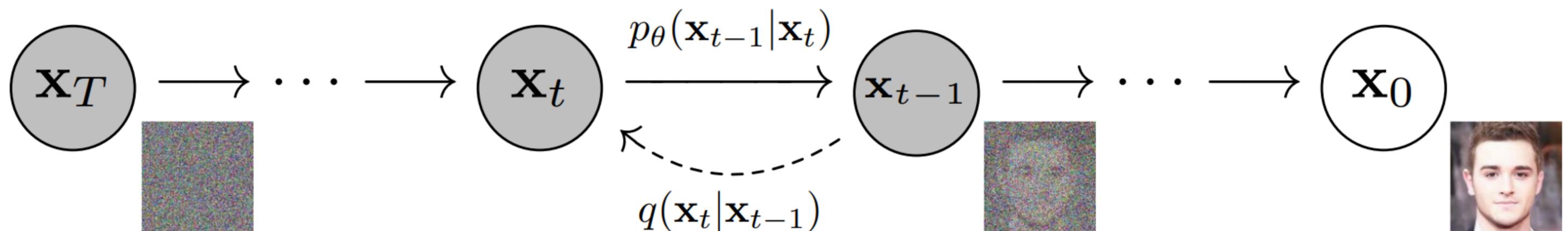
- From an image (left), generate other similar ones
- This is where diffusion models are used



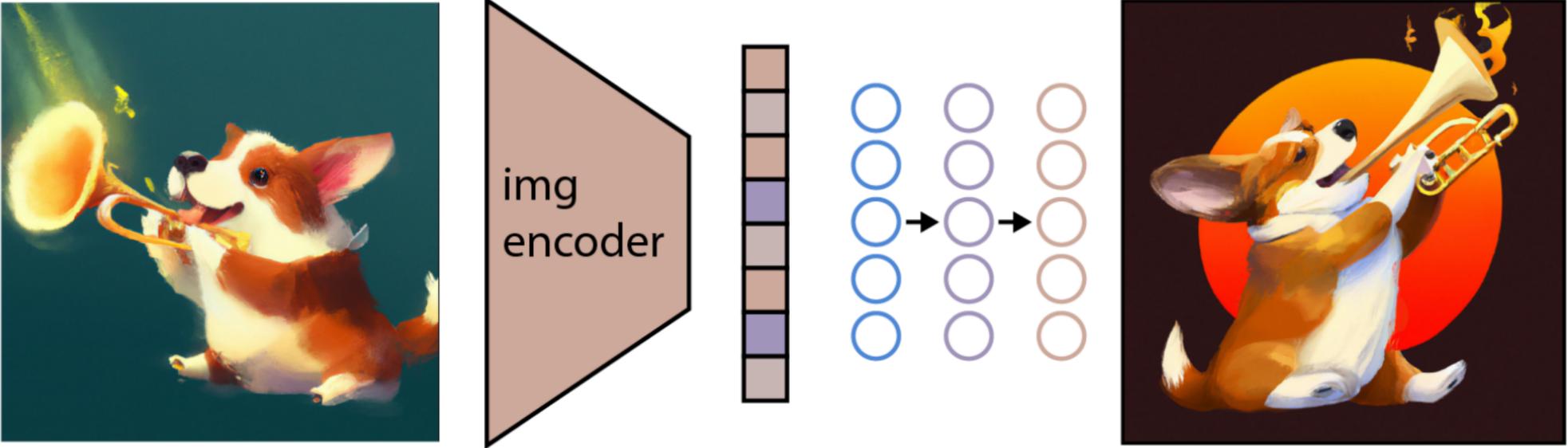
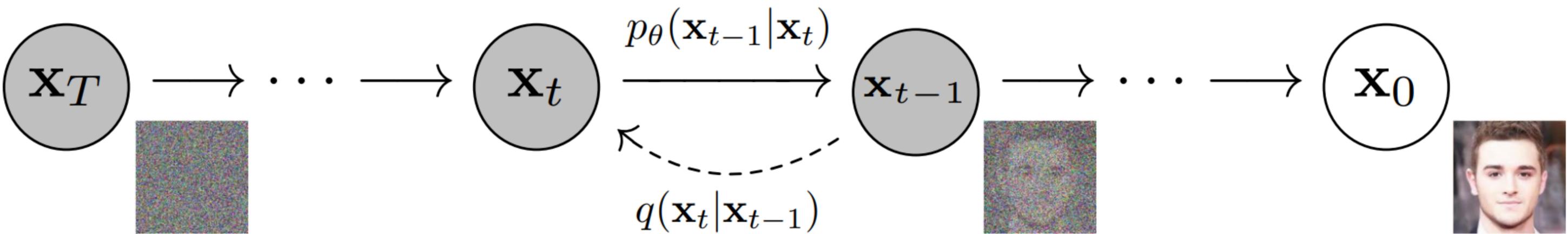
Diffusion Models

Idea:

- Add noise incrementally to an image until it is pure white noise
- Denoise the image to obtain the original image
- If we know the noise mechanism, starting from white noise, we can then generate an image

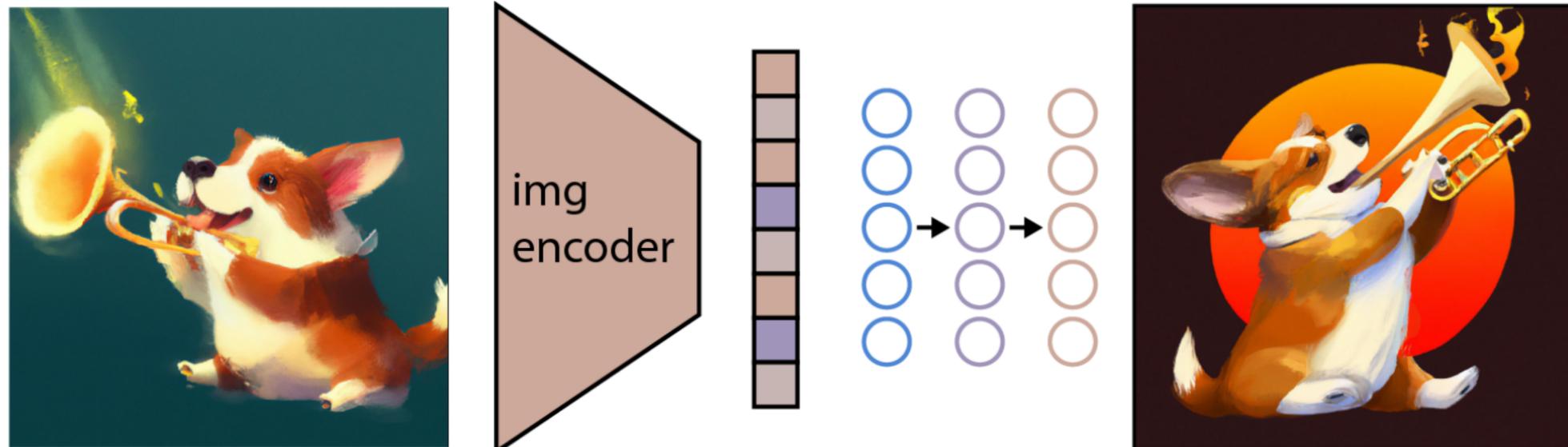
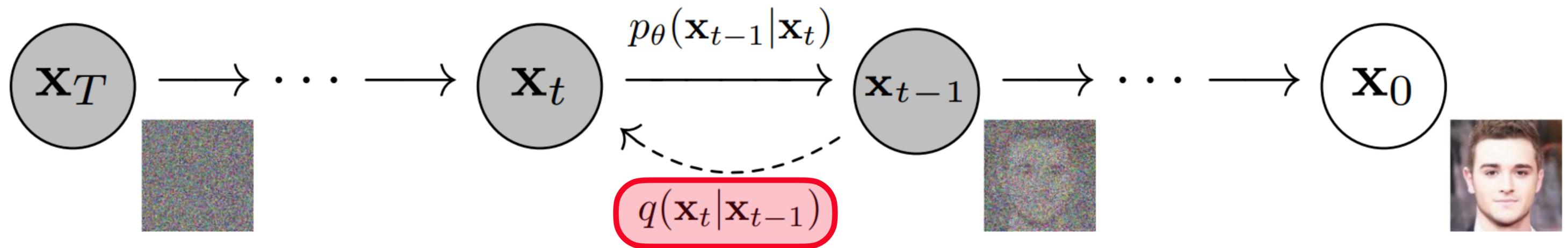


DALL-E - Diffusion Model

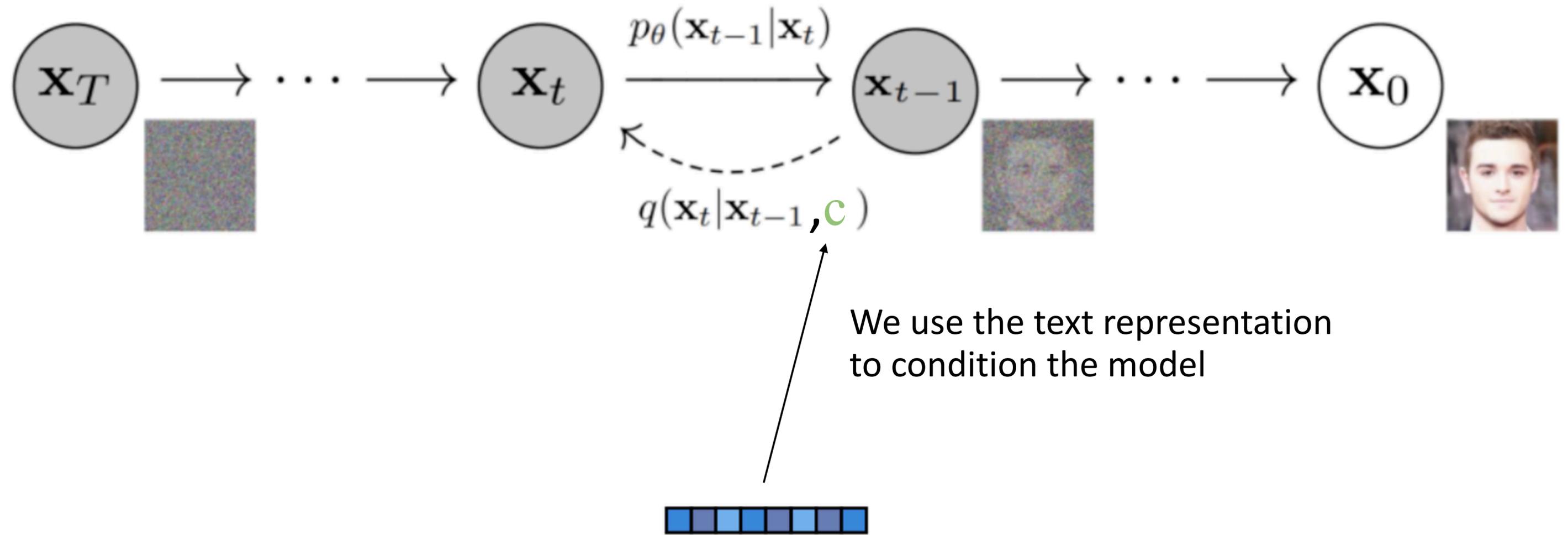


DALL-E - Diffusion Model

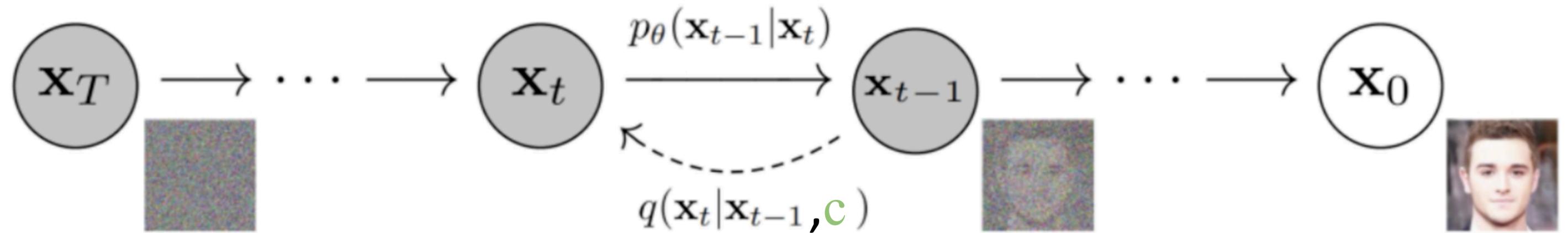
For text-to-image generation, we add information from the text
During the diffusion process



DALL-E



DALL-E



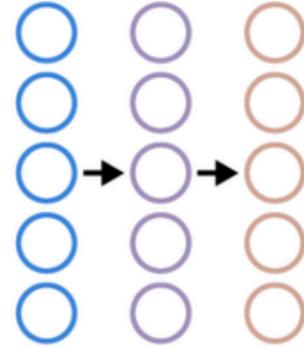
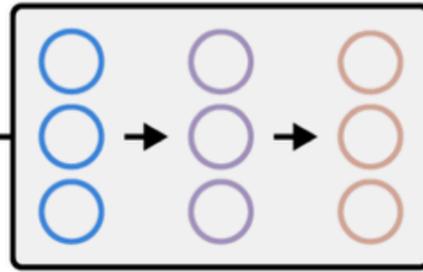
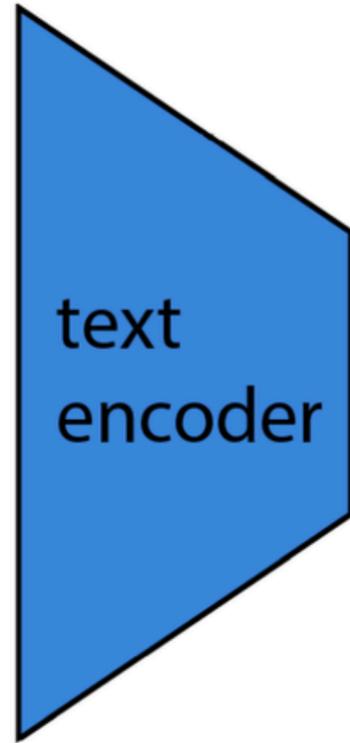
We use the text representation to condition the model



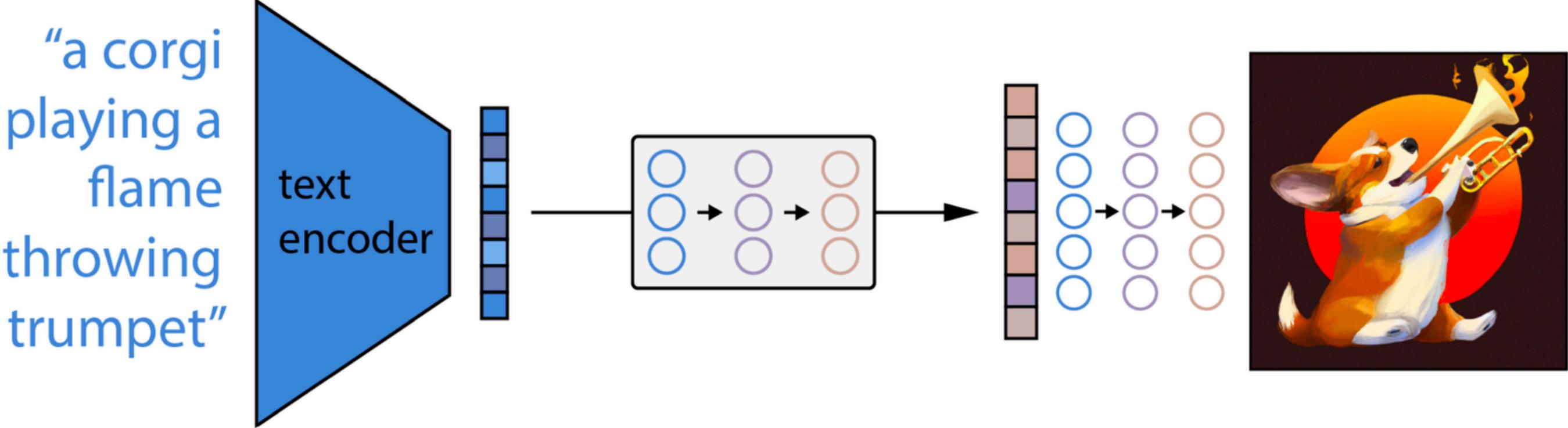
“Face face of a man with red hair”

DALL-E - Wrap-it up!

“a corgi
playing a
flame
throwing
trumpet”

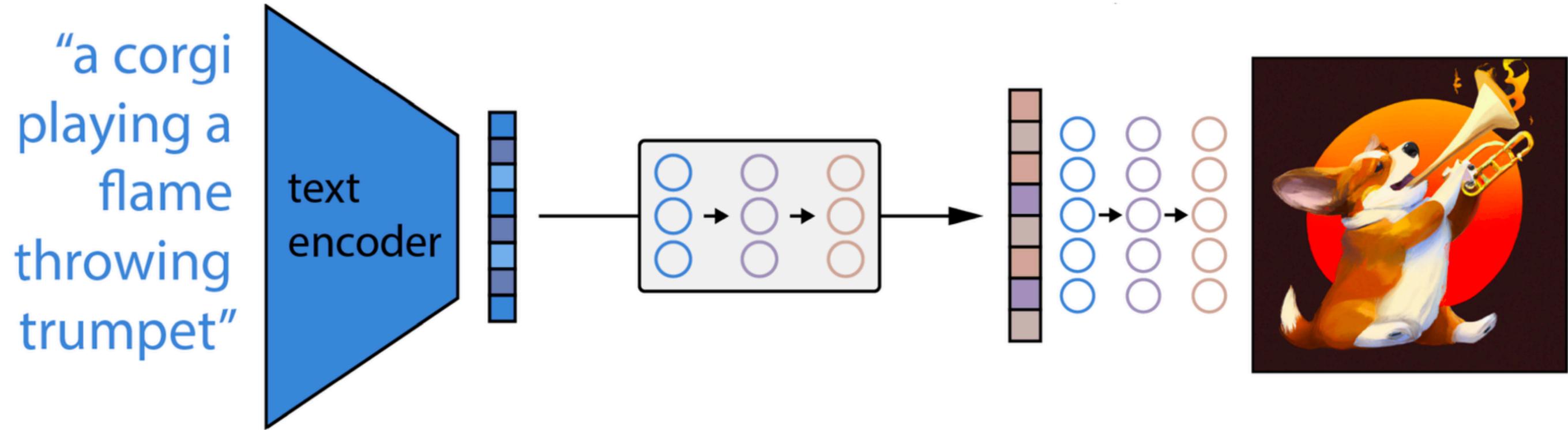


DALL-E - Wrap-it up!



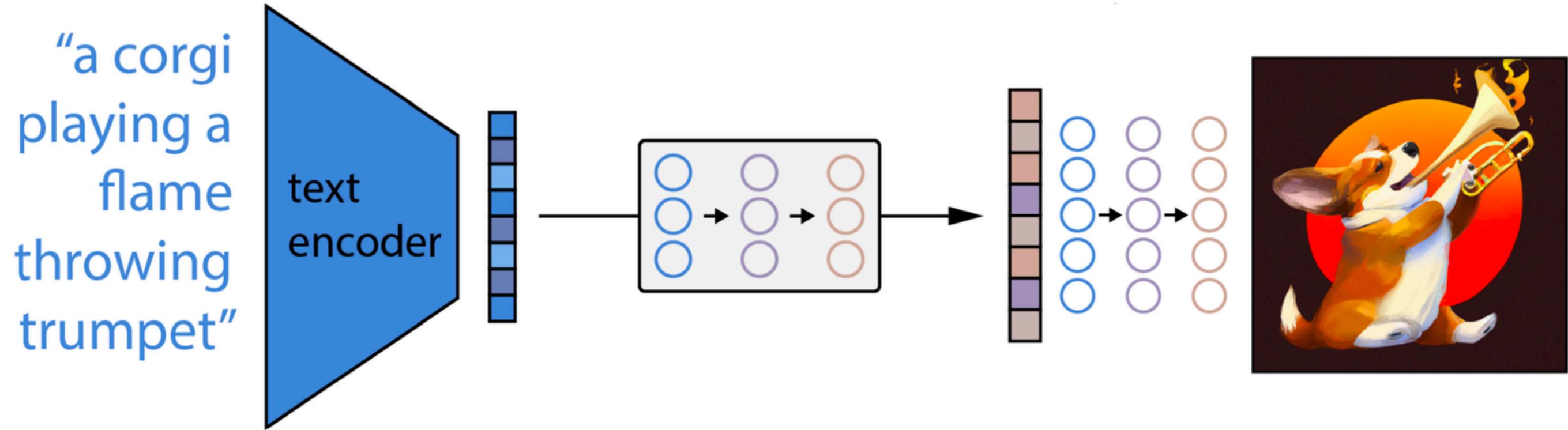
1. Contrastive Language-Image Pre-training (CLIP)

DALL-E - Wrap-it up!



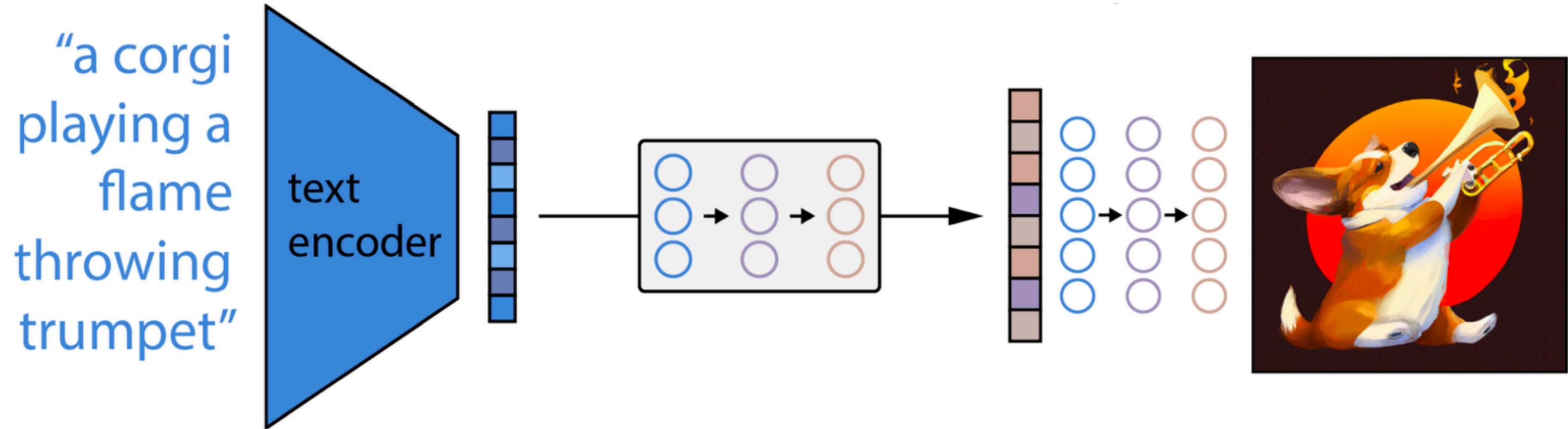
1. Contrastive Language-Image Pre-training (CLIP)
2. Generation of an image using a diffusion model

DALL-E - Wrap-it up!



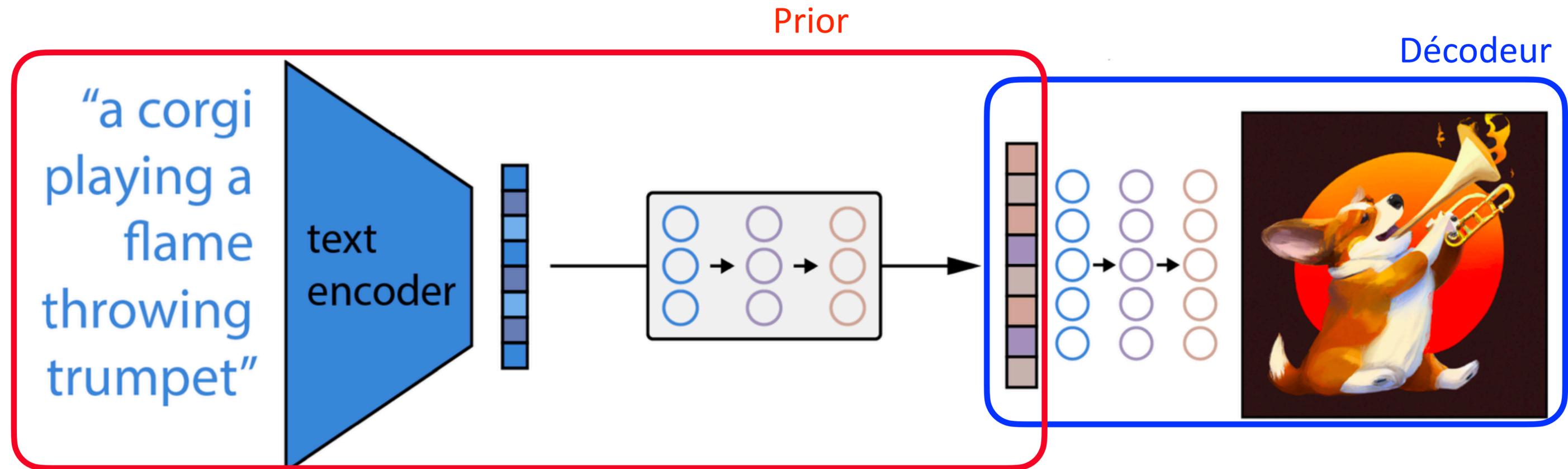
1. Contrastive Language-Image Pre-training (CLIP)
2. Generation of an image using a diffusion model
3. Learn the latent representations of text and images

DALL-E - Wrap-it up!



1. Contrastive Language-Image Pre-training (CLIP)
2. Generation of an image using a diffusion model
3. Learn the latent representations of text and images
4. Wrap-it up!

DALL-E - Wrap-it up!



Idea:

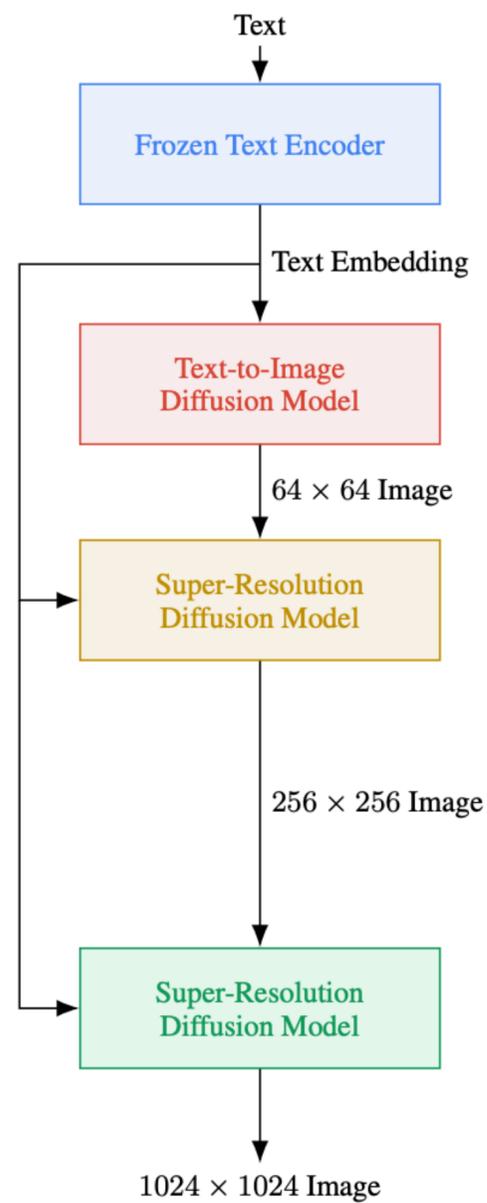
- Given (x, y) a tuple of an image x and text y .
- Given the representation of an image z .
- The distribution of the image given the text is:

$$P(x|y) = P(x, z_i|y) = P(x|z_i, y)P(z_i|y)$$

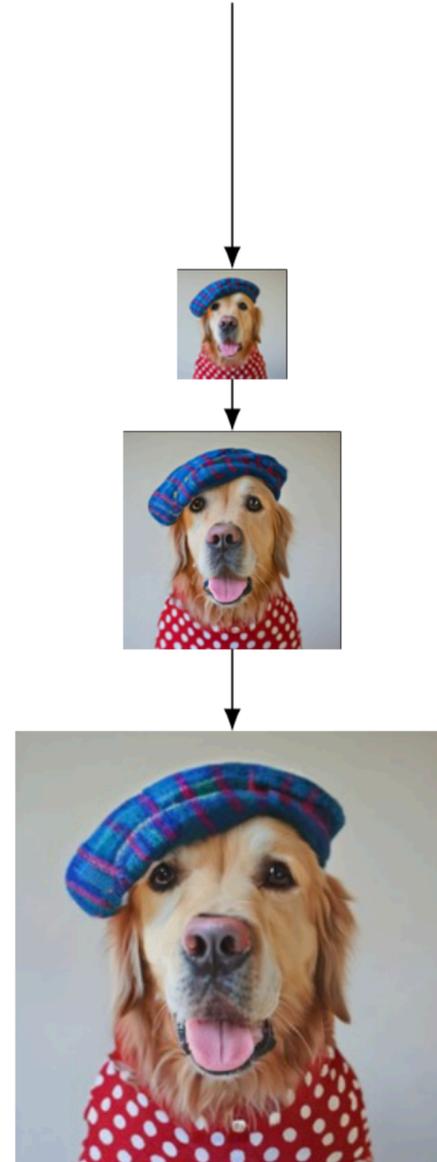
Imagen



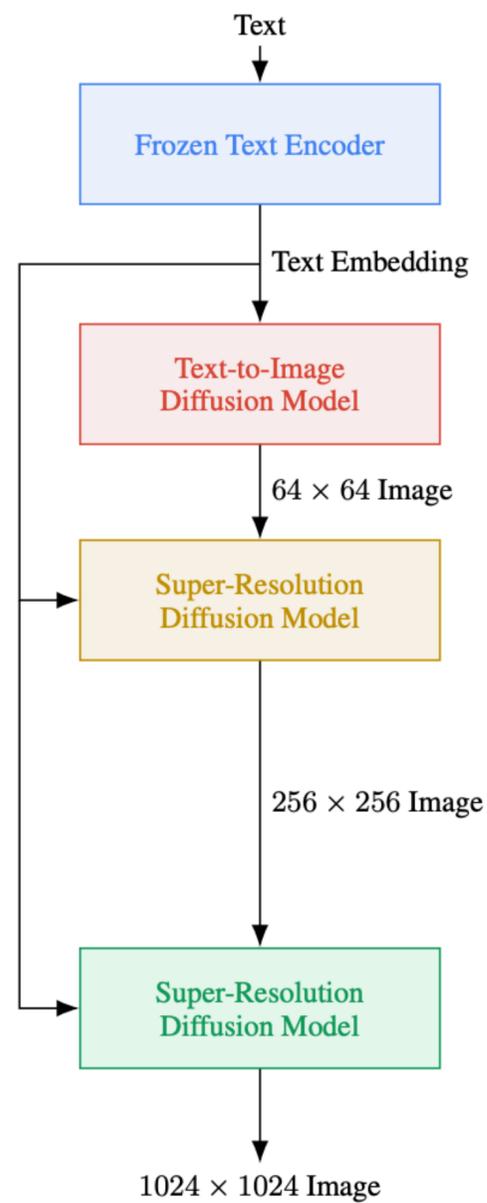
Architecture



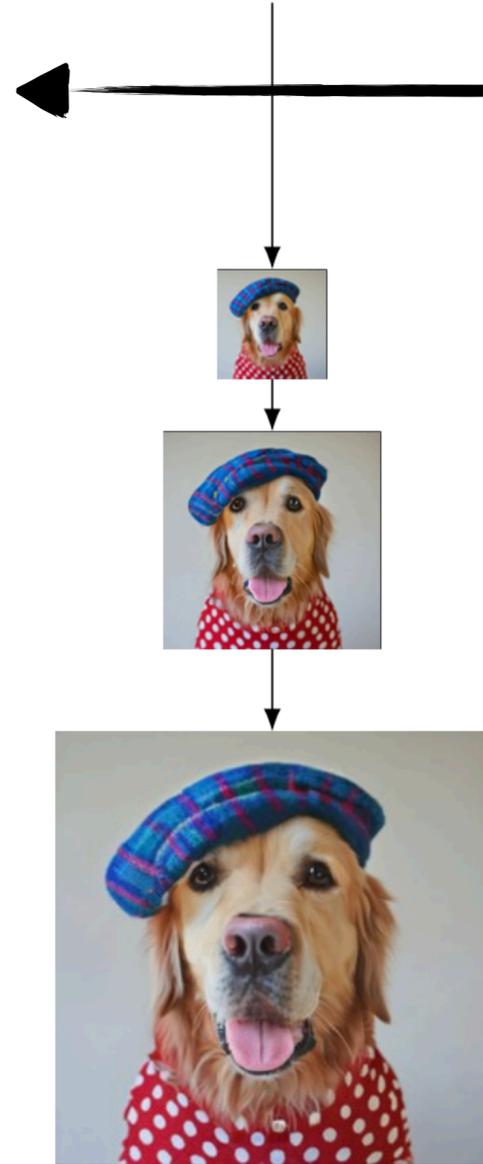
“A Golden Retriever dog wearing a blue checkered beret and red dotted turtleneck.”



Architecture

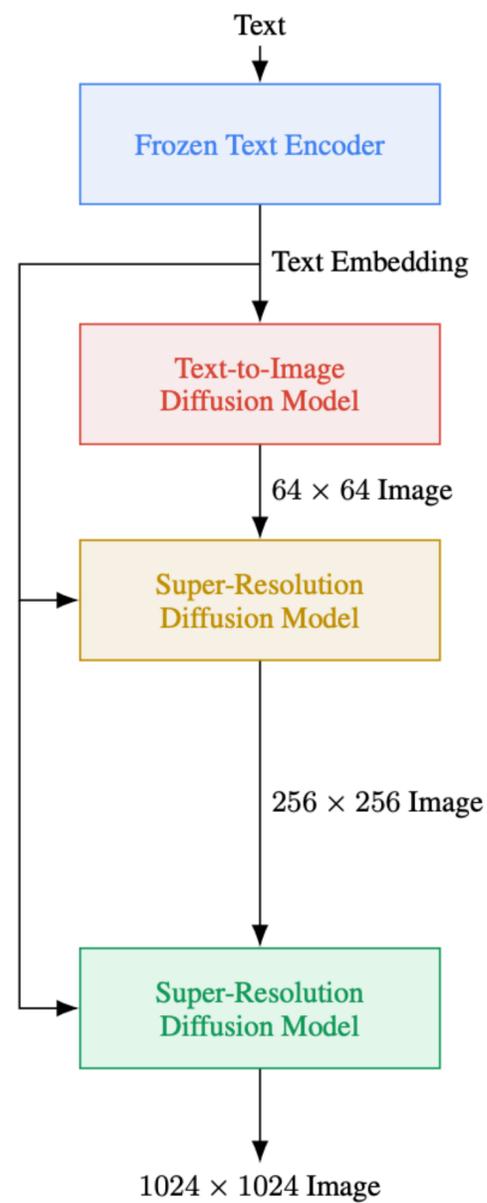


“A Golden Retriever dog wearing a blue checkered beret and red dotted turtleneck.”

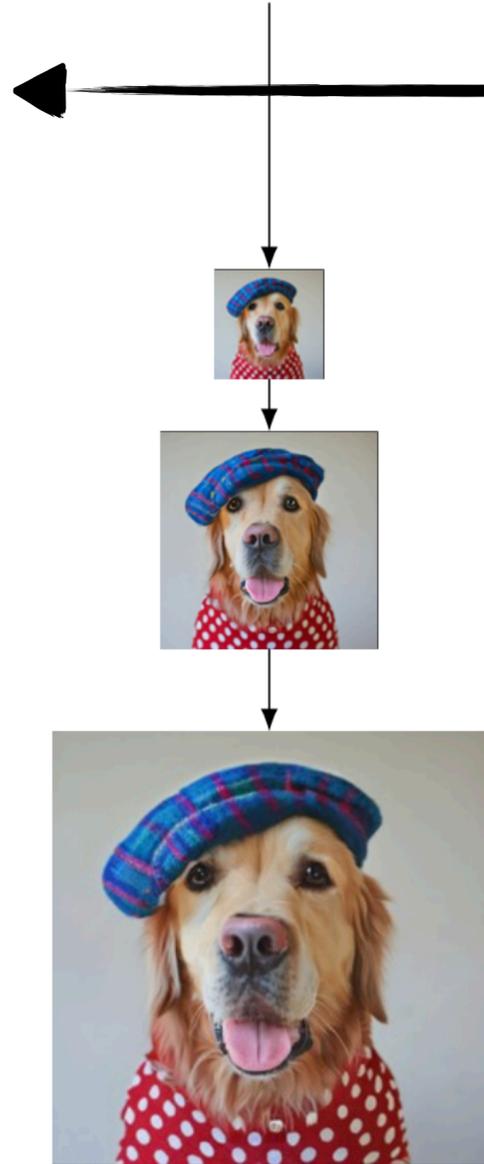


- Uses a fixed and pertained encoder

Architecture



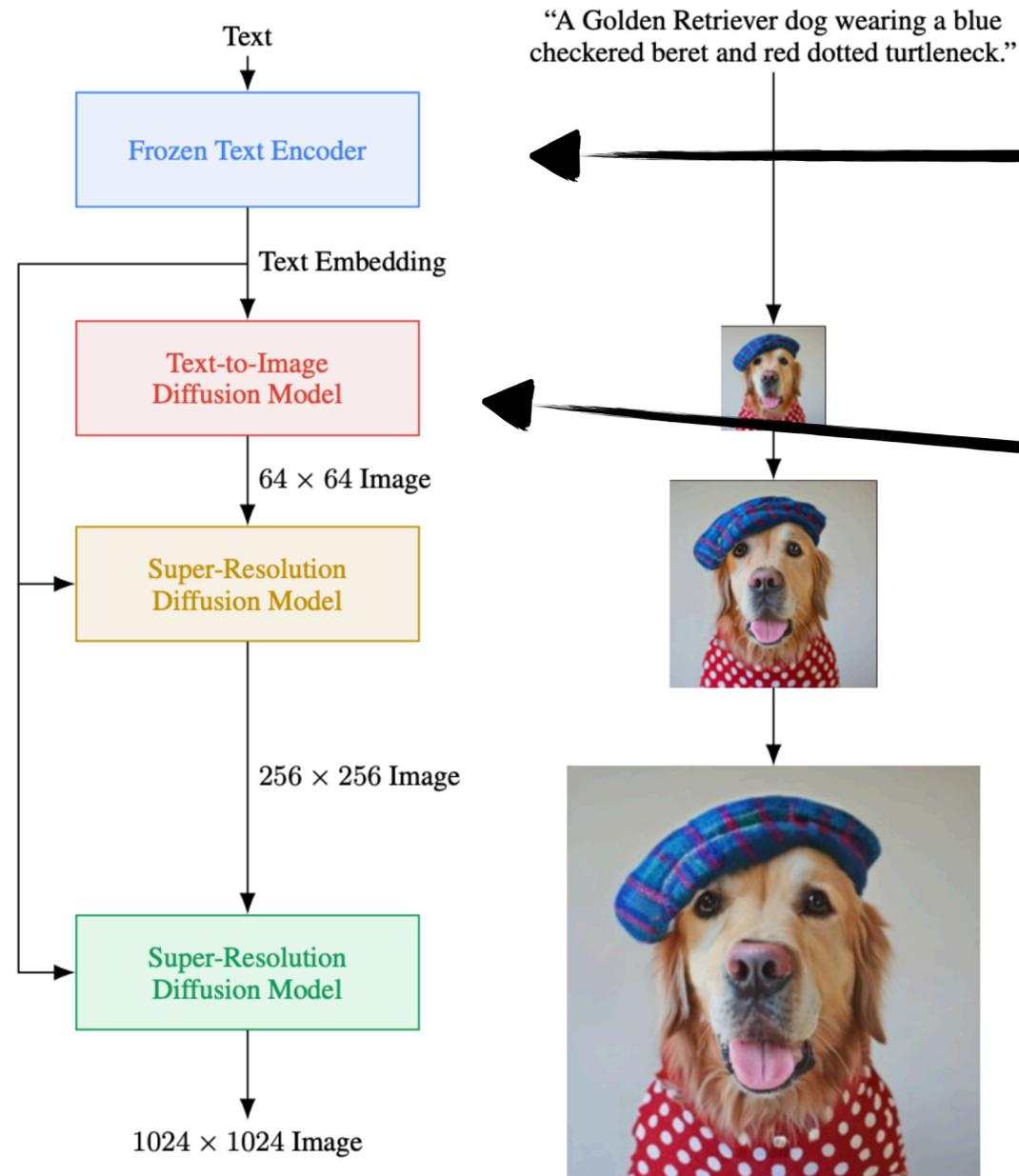
“A Golden Retriever dog wearing a blue checked beret and red dotted turtleneck.”



- Uses a fixed and pertained encoder

For example a transformer learned from a large-scale text dataset to predict the next word.
No image!

Architecture



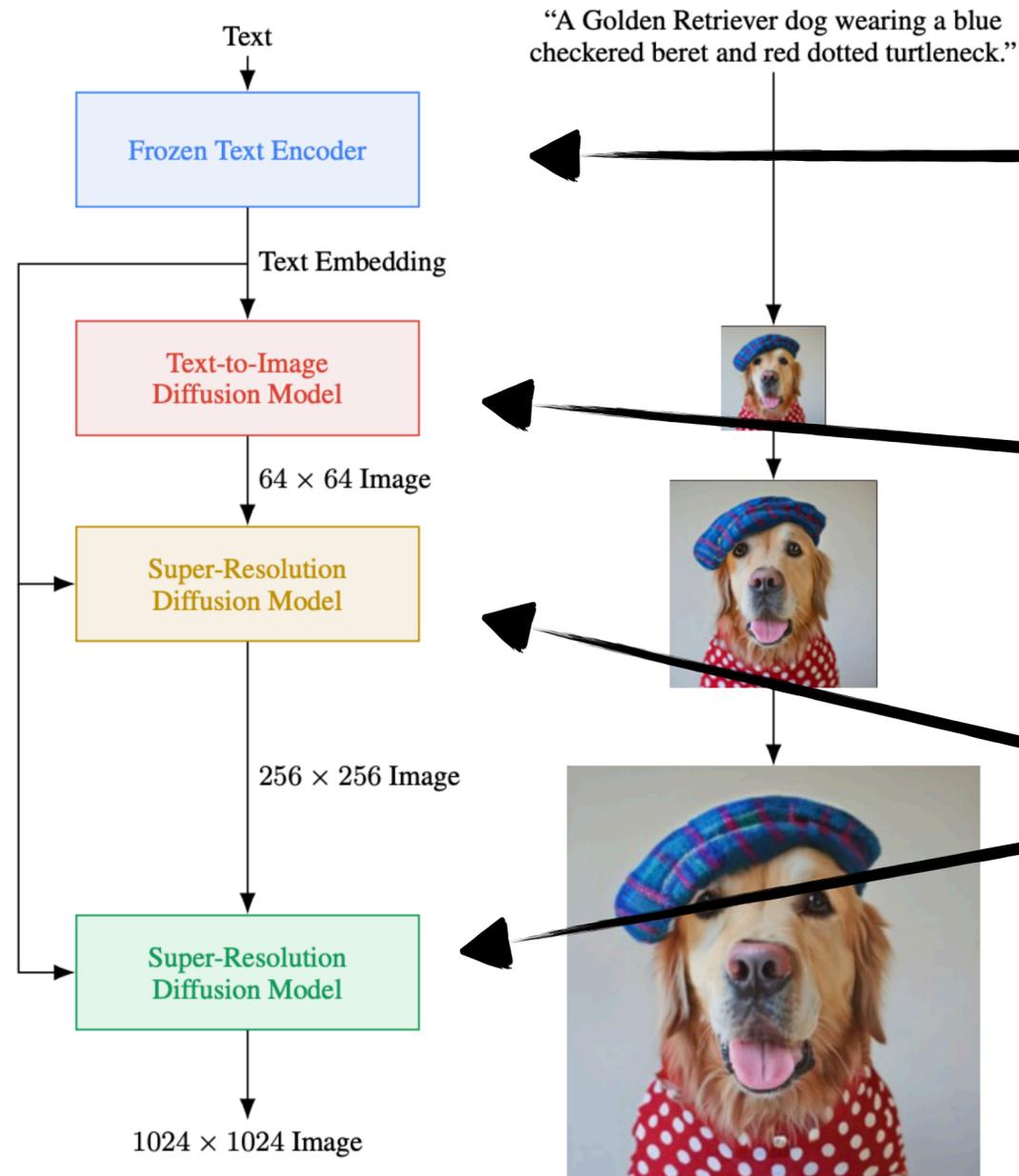
“A Golden Retriever dog wearing a blue checkered beret and red dotted turtleneck.”

- Uses a fixed and pertained encoder

For example a transformer learned from a large-scale text dataset to predict the next word.
No image!

- Followed by a diffusion model to obtain a first image

Architecture



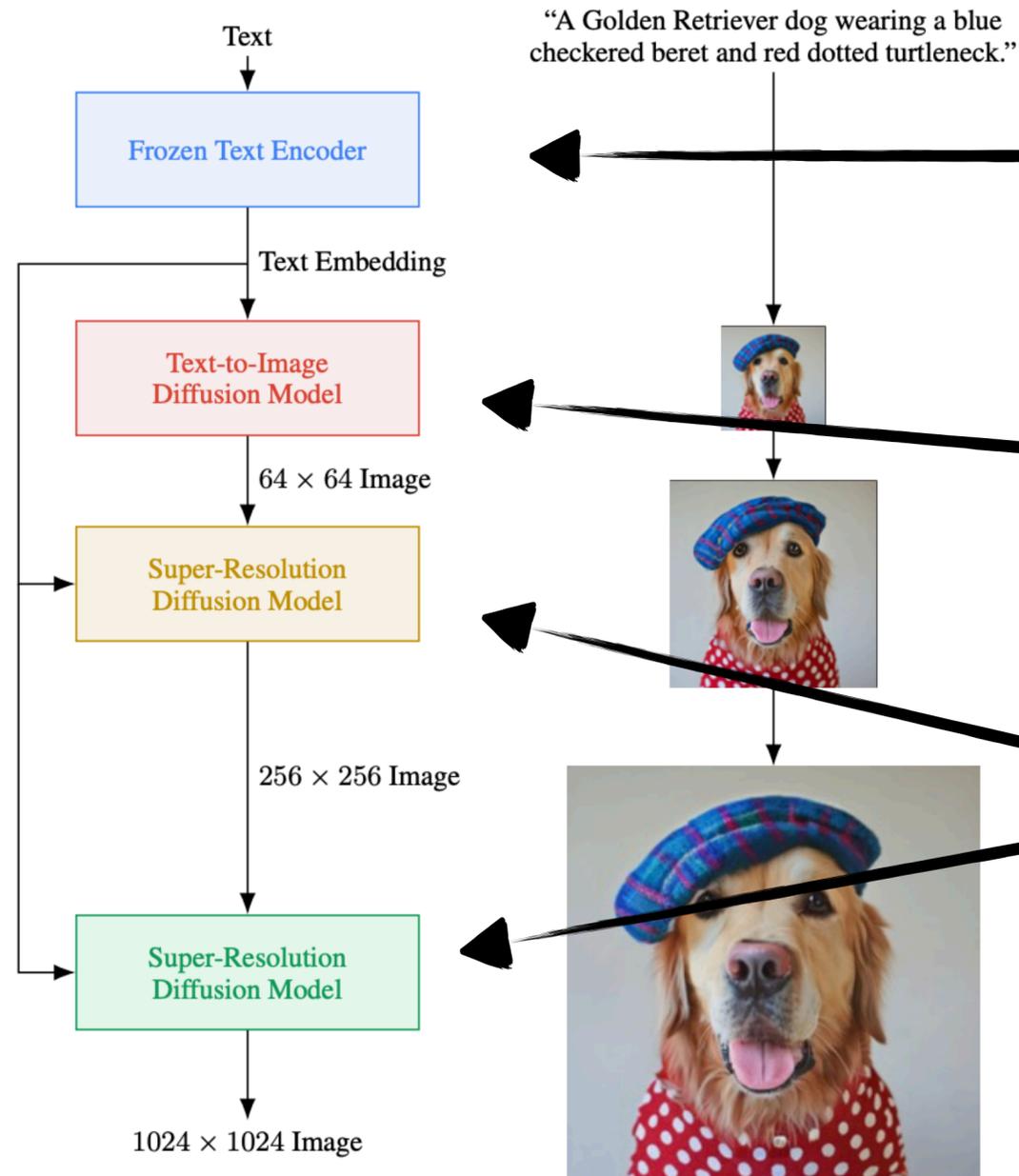
“A Golden Retriever dog wearing a blue checkered beret and red dotted turtleneck.”

- Uses a fixed and pertained encoder

For example a transformer learned from a large-scale text dataset to predict the next word.
No image!

- Followed by a diffusion model to obtain a first image
- Followed by a few other diffusion models to obtain images of higher and higher resolution

Architecture



“A Golden Retriever dog wearing a blue checked beret and red dotted turtleneck.”

- Uses a fixed and pertained encoder

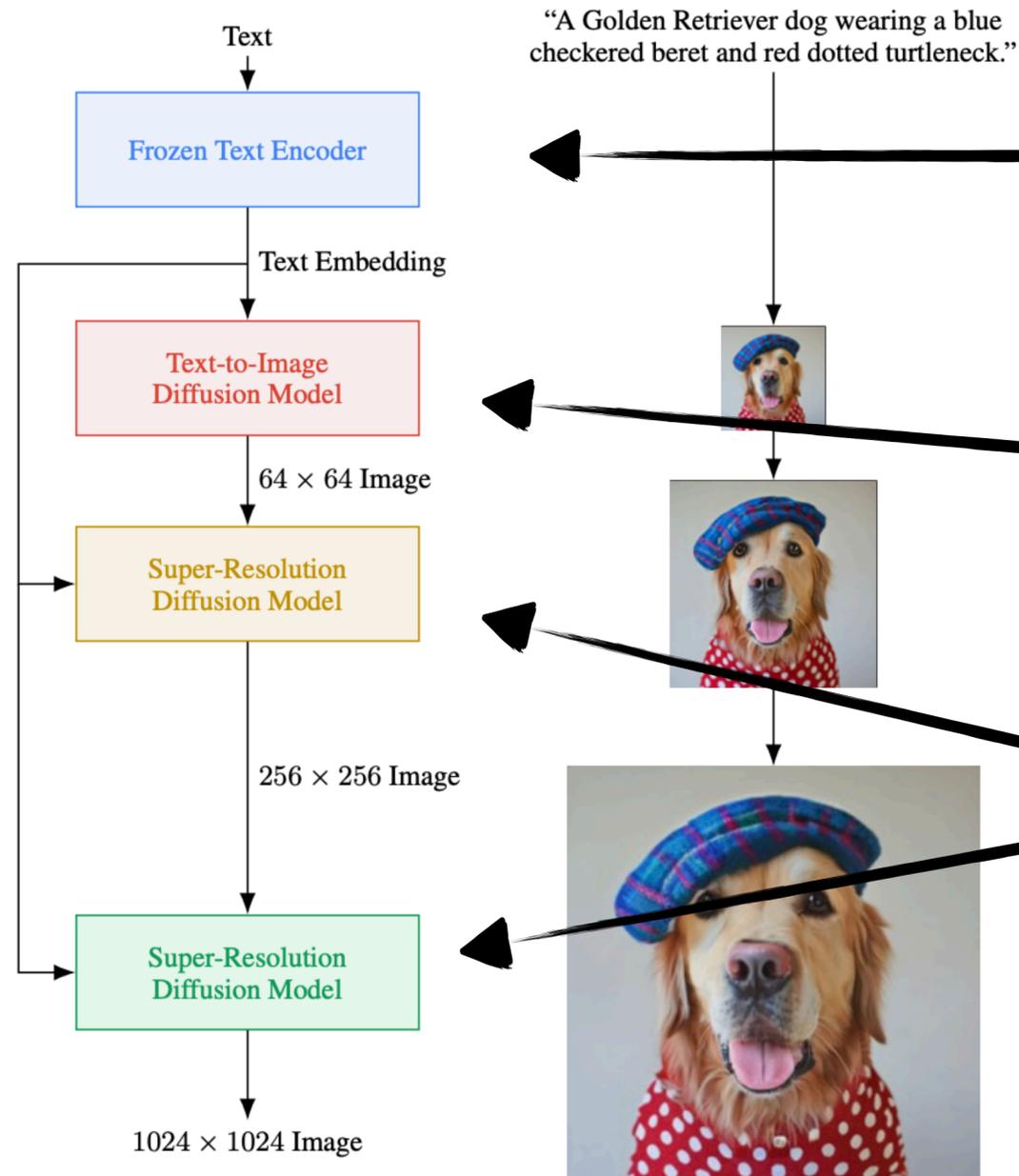
For example a transformer learned from a large-scale text dataset to predict the next word.
No image!

- Followed by a diffusion model to obtain a first image

- Followed by a few other diffusion models to obtain images of higher and higher resolution

- The diffusion models use an attention mechanism on the text representation

Architecture



"A Golden Retriever dog wearing a blue checkered beret and red dotted turtleneck."

- Uses a fixed and pertained encoder

For example a transformer learned from a large-scale text dataset to predict the next word.
No image!

- Followed by a diffusion model to obtain a first image

- Followed by a few other diffusion models to obtain images of higher and higher resolution

- The diffusion models use an attention mechanism on the text representation

- The diffusion moles are parametrized using a U-Net

« Classifier-free » guidance

- The diffusion model is trained using two objectives
 1. Generate images from the text
 2. (Also) Generate images
 - This allows to obtain high-quality images (1) that are diversified (2)
- Imagen proposes a method to ensure pixels don't saturate during diffusion (somewhat similar problem to clipping in RNNs)

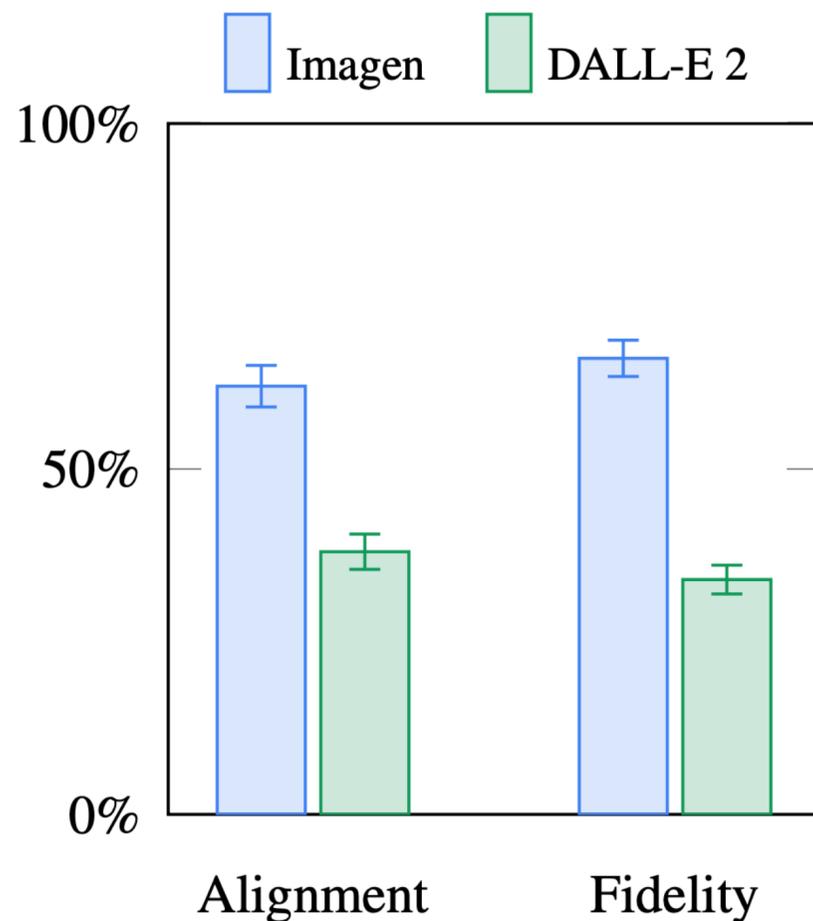
Avant Imagen



Imagen

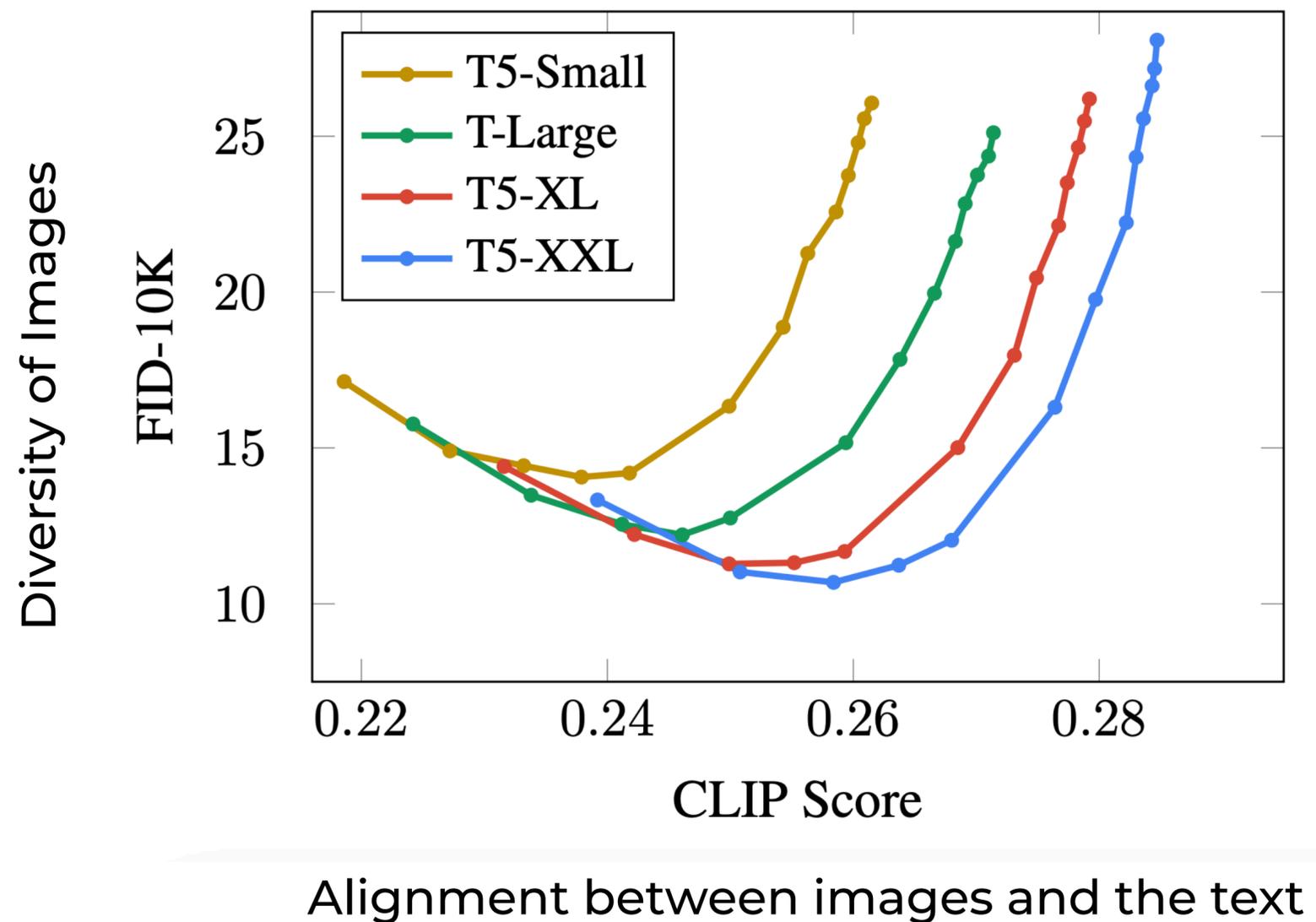


Compared to Dall-E 2

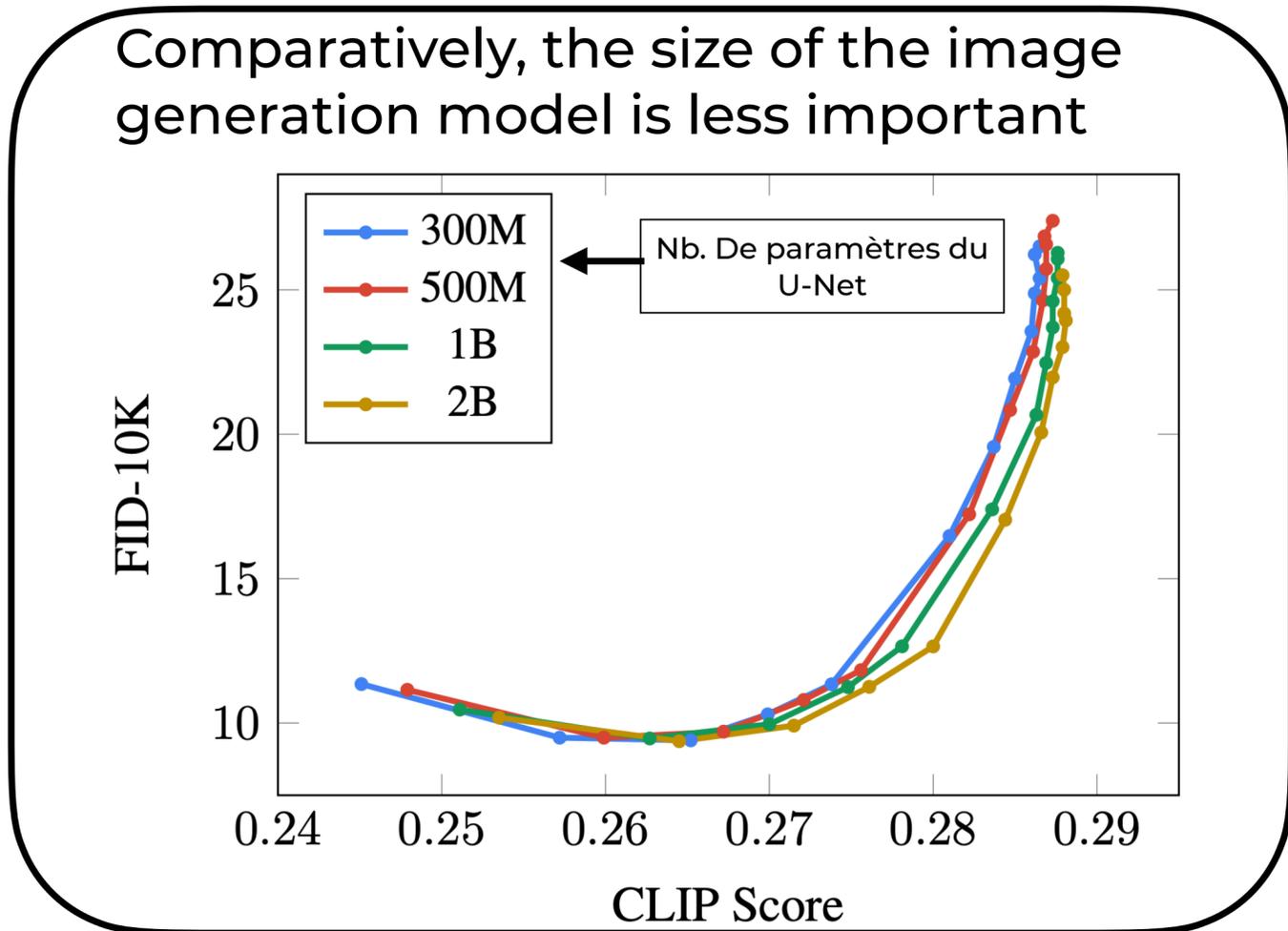


- Better empirical performance (according to a human study)
- “Alignment” -> “Does the caption accurately describe the above image”
- “Fidelity” -> “Which image is more photorealistic”
- Users a simpler architecture (no CLIP)

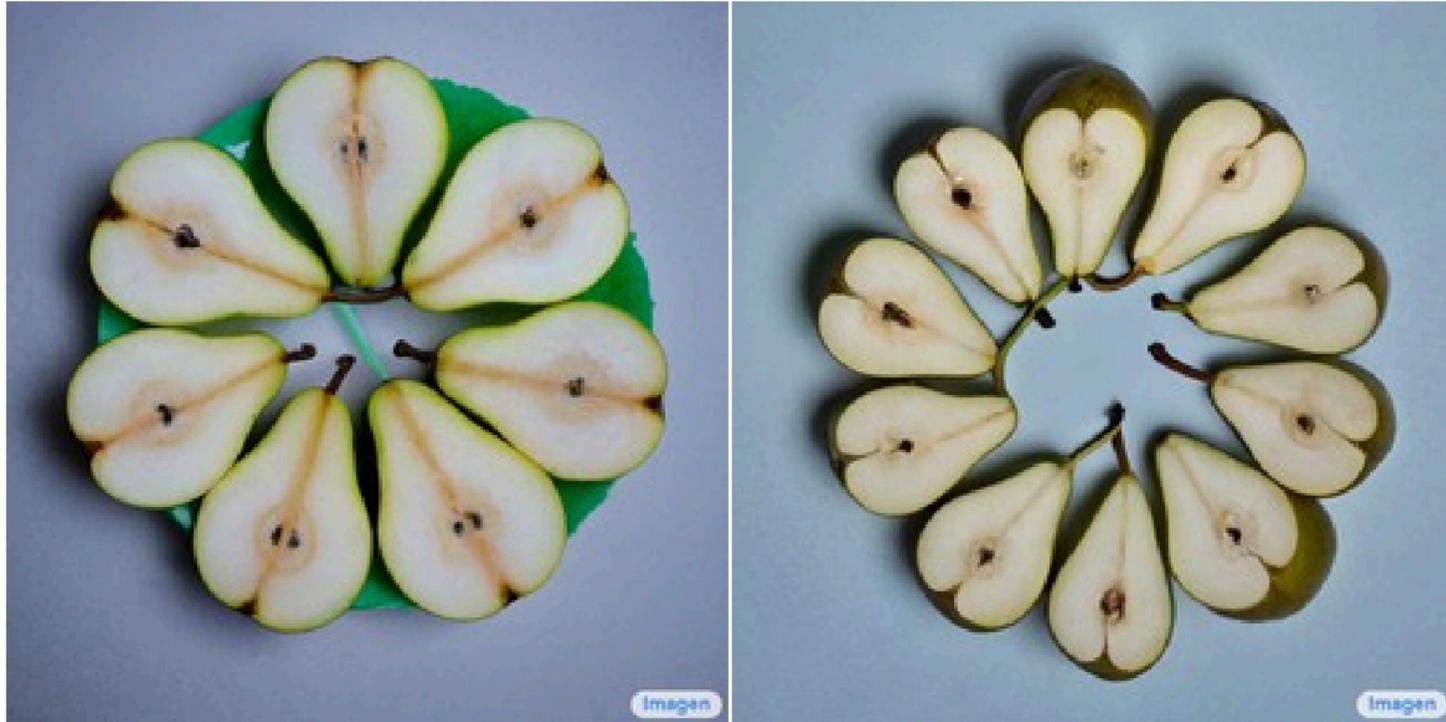
The size of the text encoder is an important hyper-ammeter



T5 XXL (2.6B)
- Trans. Encodeur-Decodeur
- Uses only the encoder



Still far from perfect...



A pear cut into seven pieces arranged in a ring.



One cat and two dogs sitting on the grass.

- E.g., operations that require counting and logic remain difficult