Efficient Algorithms on Cocomparability Graphs Via Vertex Ordering Characterizations

Lalla Mouatadid

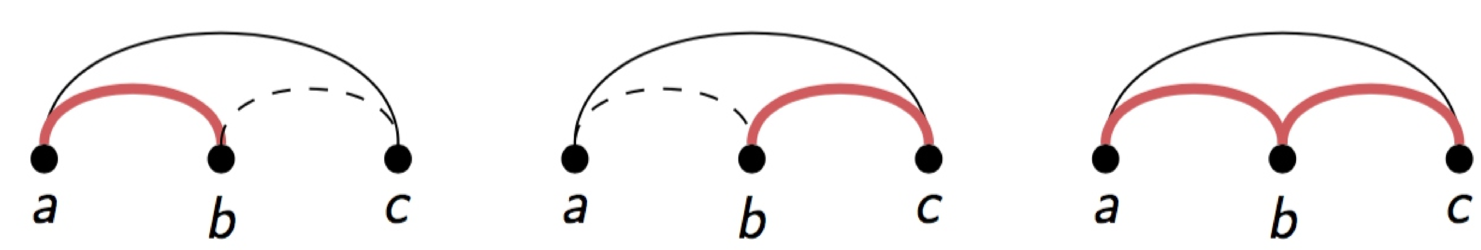
University of Toronto, Department of Computer Science

lalla@cs.toronto.edu

Joint work with Ekkehard KÖhler and Michel Habib

Cocomparability Graphs

- The complements of comparability graphs.
- Many-to-one mapping from posets to cocomparability graphs.
- Cocomparability graphs are a superclass to:
  - Co-graphs, trapezoid, permutation, interval graphs
- An ordering σ of **V** is a *cocomparability (cocomp) ordering* if for every triple **a < b < c** in σ:



- **G is a cocomp graph iff it admits a cocomp *ordering*.**

Induced Matching

- *Induced matching:* A matching where every pair of edges is at distance at least 2 in **G.**
- Induced Matching is NP-complete on bipartite graphs, even if the graphs have degree 3, or are planar.
- Given an ordering σ of **V(G),** construct an ordering on the vertices of **L(G)**, the line graph of **G,** such that for every pair $e_i = (a,b)$, $e_j = (u,v)$:

$$\text{Rule } (\bullet): e_i \prec_\bullet e_j \iff a \preceq_\sigma u \text{ and } b \preceq_\sigma v$$

$$\text{Rule } (\star): e_i \prec_\star e_j \iff \begin{cases} a \prec_\sigma u & \text{if } a \neq u \\ a = u \text{ and } b \prec_\sigma v & \text{o.w.} \end{cases}$$

The Algorithms

Max Weighted Independent Set (MWIS)

Input: A cocomparability graph **G=(V, E)**:

- Compute a cocomparability ordering σ.
- Scan σ left to right: compute an ordering τ, where vertices are inserted in τ in increasing order of their (updated) weight.
- Scan τ right to left to greedily collect a maximum weight independent set.

Max Weighted Induced Matching (MWIM)

Input: Given a cocomparability graph **G=(V, E)**:

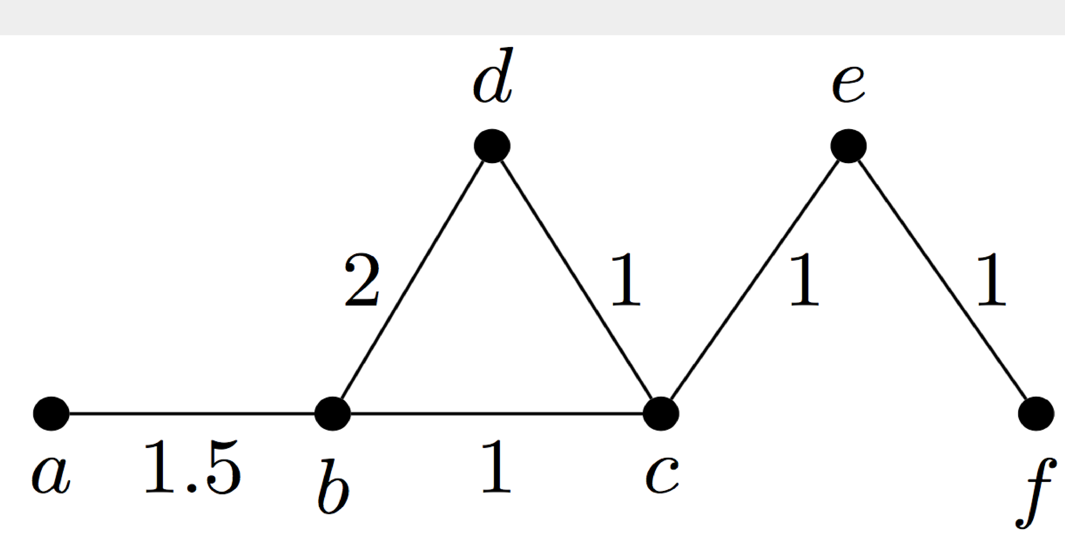
- Compute a cocomparability ordering σ.
- Using either Rule • or Rule ★, compute τ.
- Use the MWIS algorithm on τ to compute a maximum weight induced matching on **G.**

The Results:

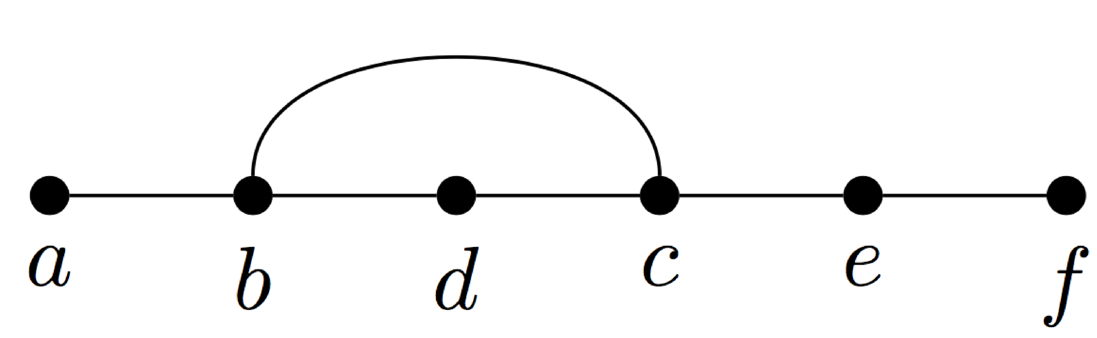
Let **S = {interval, split, threshold, cocomparability}**

- Given a vertex ordering of **G** in **S**, rules • and ★ compute vertex orderings of $\mathbf{L^2(G)}$.
- Every graph class in **S** is closed under $\mathbf{L^2(\bullet)}$.
- Maximum weighted independent set (MWIS) can be computed in O(m+n) time on **cocomparability** graphs.
- Maximum weighted induced matching (MWIM) can be computed in O(mn) time on **cocomparability** graphs.
- Both the MWIS and MWIM algorithms are robust.

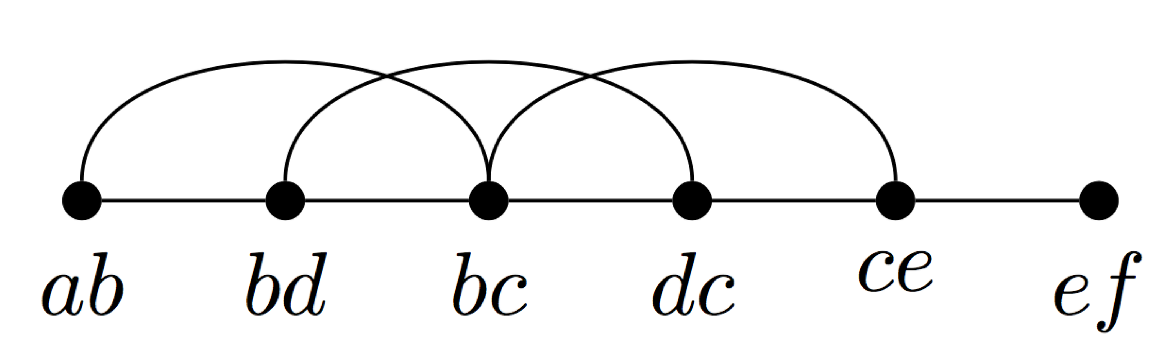
Example



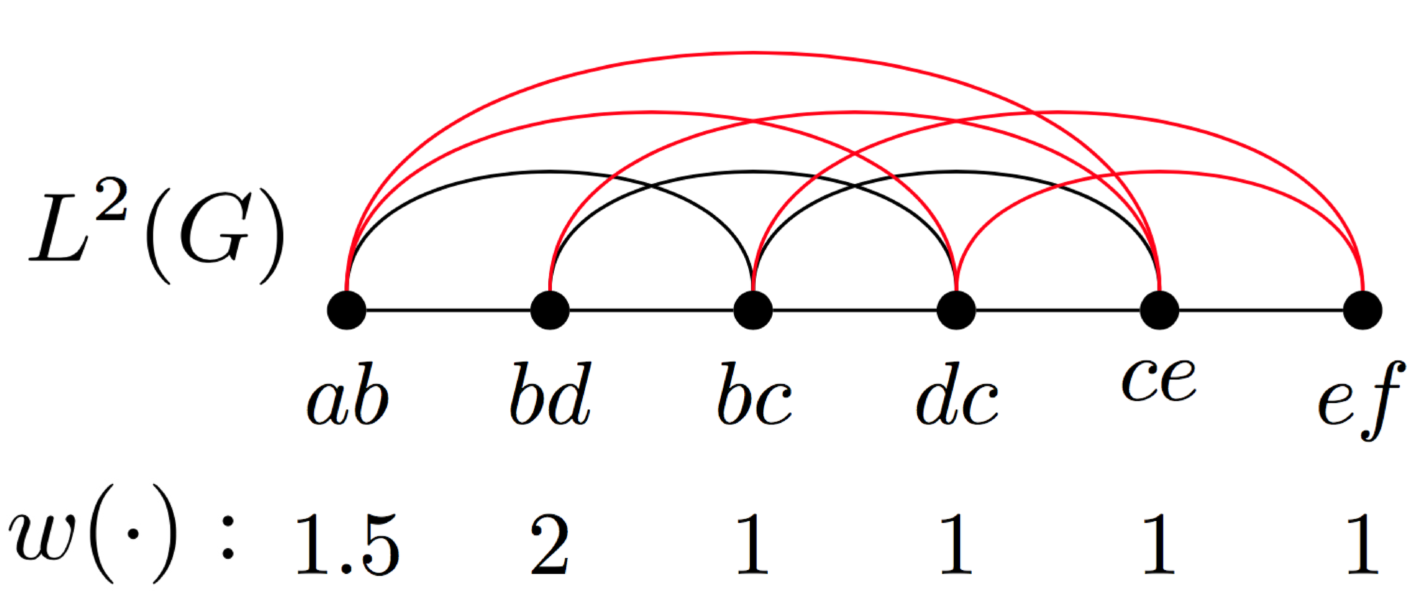
$G = (V, E, w)$



σ, a cocomparability ordering of G



$L(G)$



e_i	u	S_{e_i}	$w(S_{e_i})$	τ_i
$e_1 = ab$	-	$\{e_1\}$	1.5	e_1
$e_2 = bd$	-	$\{e_2\}$	2	e_1, e_2
$e_3 = bc$	-	$\{e_3\}$	1	e_3, e_1, e_2
$e_4 = cd$	-	$\{e_4\}$	1	e_3, e_4, e_1, e_2
$e_5 = ce$	-	$\{e_5\}$	1	e_3, e_4, e_5, e_1, e_2
$e_6 = ef$	bd	$\{e_2, e_6\}$	3	$e_3, e_4, e_5, e_1, e_2, e_6$

Future Work

- Colouring Algorithm via VOCs.
- Certifying algorithms.
- Stepping outside perfection: AT-free graphs are closed under $L^2(\bullet)$ as well.

References

KÖhler & Mouatadid, *Linear Time MWIS on Cocomparability Graphs. IPL*

Brandstädt & Hoàng, *Maximum induced matchings for chordal graphs in linear time.* Algorithmica

Cameron, *Induced matchings in intersection graphs.* Discrete Math

Habib & Mouatadid*: Efficient MIM Algorithms via VOCs.* Submitted