CSC418/2504: Computer Graphics
Term Test 1 October 14th, 2009 19:10-20:00

Student Number:______________________

Last Name:______________________ First Name:______________________

This exam consists of 3 questions on 7 single-sided pages (including cover page).

Aids allowed: None.

Total Marks: 50
Minutes: 50

Question	Marks
1a	______________/5
1b	______________/10
1c	______________/5
2a	______________/8
2b	______________/8
2c	______________/4
3	______________/5
Total	_____________/50

1. **Super-Ellipses & Implicit Equations [20 Marks]**

   In addition to their parametric representation discussed in class, super-ellipses can be represented implicitly with the following general equation:

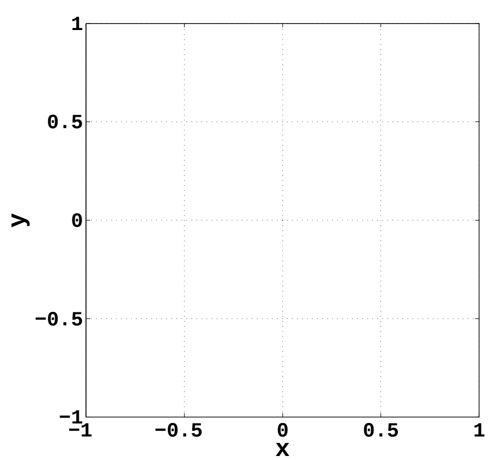
$$\left|\frac{x}{a}\right|^n + \left|\frac{y}{b}\right|^n - 1 = 0 \ , \tag{1}$$

   where (x, y) is a point on the super-ellipse; $|\ \ |$ denotes absolute value; and a, b, n are constants that control the curve's 2D shape.

   (a) [5 Marks] Consider the super-ellipse with $a = b = 1$ and $n = \frac{1}{2}$. Calculate the intersection point(s) of this curve with the line $x = \frac{1}{4}$.

(b) [10 Marks] Calculate the curve's unit normal at the intersection point(s) in (a).

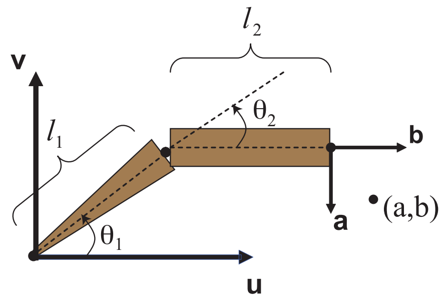
(c) [5 Marks] Draw the curve on the grid below. Briefly explain the reasoning behind your drawing (no marks will be awarded without an explanation). *Hint 1: the coordinates of all points on the curve satisfy* $-1 \leq x, y \leq 1$. *Hint 2: use your calculations from (a) and (b).*



2. **Coordinate Transformations [20 Marks]**

(a) [8 Marks] Consider a two-link, two-dimensional robotic arm with the following properties:

- its base is affixed to the origin of the world coordinate system;
- its first link has length l_1 and can rotate freely about the origin by an angle θ_1 ;
- its second link has length l_2 and can rotate freely by an angle θ_2 about the links' common joint.

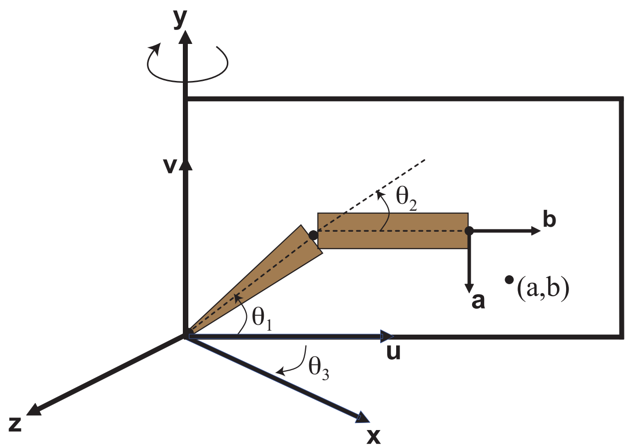


Suppose we attach a local coordinate system to the end of the second joint, as shown in the figure above, with its b-axis aligned with the joint and its origin at the joint's end.

Express the transformation $\mathbf{T}_1$ that maps local coordinates (a, b) to 2D world coordinates (u, v) as a product of elementary homogeneous transformation matrices. You should show the elements of each matrix and indicate the transformation the matrix corresponds to. You do not need to perform any matrix multiplications.

$\mathbf{T}_1 \ =$

(b) [8 Marks] Now suppose that the planar arm in (a) actually lives in a 3D world and has an extra degree of freedom: in addition to rotating its links by angles θ_1 and θ_2, the arm's uv-plane can rotate about the y-axis of the world coordinate system by an angle θ_3:

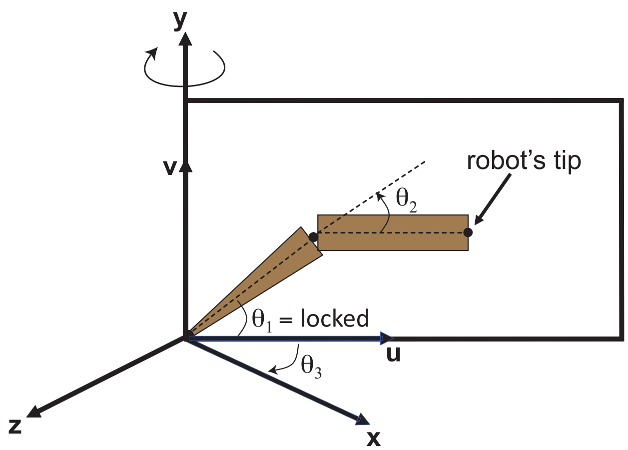


As shown in the figure, the arm's base is at the origin of the 3D world coordinate system and the v-axis of arm's plane is aligned with the y-axis.

Express the transformation $\mathbf{T}_2$ that maps local coordinates (a, b) to 3D world coordinates (x, y, z) as a product of elementary homogeneous transformation matrices.

$\mathbf{T}_2 \ =$

(c) [4 Marks] Finally, suppose that we "lock" angle θ_1 of the robot arm to some fixed value. This leaves us with just two degrees of freedom—angles θ_2 and θ_3.



Imagine moving the robot arm to every possible combination of angles $\theta_2, \theta_3 \in [0, 2\pi)$. This will cause the robot's tip to move to a set of points in 3D space. What is this set? Explain briefly (you do not have to provide formulas/equations).

3. **Lenses [5 Marks]**

State the thin-lens law and explain its various terms (your explanations can include a figure).