CSC418/2504: Computer Graphics

Term Test 1 October 15th, 2008 19:10-20:00

Student Number:______________________

Last Name:______________________ First Name:______________________

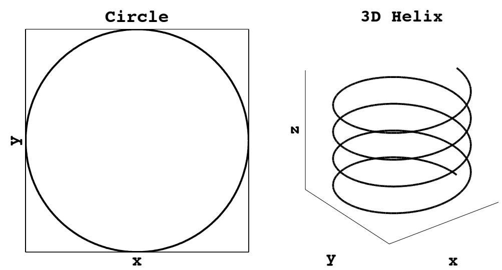
This exam consists of 4 questions on 6 single-sided pages (including cover page).
Aids allowed: None.

Total Marks: 50
Minutes: 50

Question	Marks
1a	______________/3
1b	______________/5
2a	______________/4
2b	______________/10
3a	______________/7
3b	______________/7
3c	______________/4
4	______________/10
Total	______________/50

1. **Parametric Curves [8 Marks]**

   Suggest a parametric representation for each of the two curves below:



   You are free to make whatever assumptions you need to keep the parametric representation simple (e.g., for the circle you may assume it is centered at (0,0) and has radius of 1).

   (a) [3 Marks] (Parametric representation of a circle)

   $c(s) \quad =$

   (b) [5 Marks] (Parametric representation of a 3D helix)

   Be sure to explain your reasoning.

   $c(s) \quad =$

2. **Implicit 3D Surfaces [14 Marks]**

The *conoid* is a surface whose implicit equation, $F(x, y, z) = 0$, uses the function

$$F(x, y, z) \;=\; ax^2 + by^2 - zx^2 - zy^2 \;, \tag{1}$$

where $a, b \neq 0$ are two constants.

(a) [4 Marks] (Point-Surface inclusion)

Show that point $\bar{p} = \left(1 \;, 1 \;, \frac{a+b}{2}\right)$ lies on the conoid.

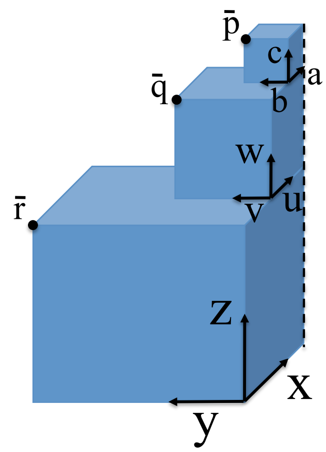
(b) [10 Marks] (Normal of the Conoid)

Compute the conoid's unit surface normal at point $\bar{p}$ in (a).

$\vec{n}(\bar{p}) \quad =$

3. **3D Coordinate Transformations [18 Marks]**

Consider the three-part object below, consisting of three stacked cubes whose edges have length 6, 3 and 1, respectively. The cubes are stacked with their dashed edges aligned and their faces parallel to each other:



Each part has its own object-centered coordinate system, as indicated. Within its own coordinate system, each part is just a unit cube. In particular, when expressed in the coordinate system of the cube on which they lie, vertices $\bar{p}, \bar{q}, \bar{r}$ all have coordinates $(0, 1, 1)$.

(a) [7 Marks] What are coordinates of vertex $\bar{p}$ in the coordinate system of the middle-sized cube?

$\bar{p} =$

(b) [7 Marks] Give the 4×4 homogeneous transformation matrix $\mathbf{T}_{s \to m}$ that maps the (a, b, c) coordinates of points on the small cube to the (u, v, w) coordinates of the middle-sized cube.

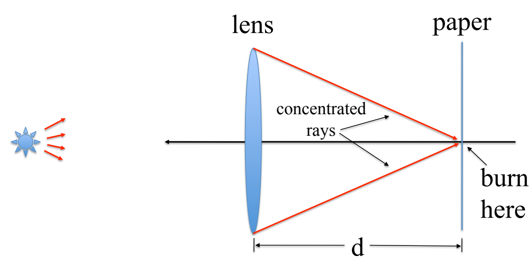
$$\mathbf{T}_{s \to m} =$$

(c) [4 Marks] Let $\mathbf{T}_{m \to l}$ be the 4×4 homogeneous transformation that maps the (u, v, w) coordinates of the middle-sized cube to the (x, y, z) coordinates of the large cube. Express the transformation from (a, b, c) coordinates of the small cube to (x, y, z) coordinates of the large cube in terms of $\mathbf{T}_{m \to l}$ and the matrix $\mathbf{T}_{s \to m}$ in (b).

$$\mathbf{T}_{s \to l} =$$

4. **Lenses & Projection [10 Marks]**

Legend has it that Euclid defeated his approaching enemies with an optical instrument that concentrated the sun's rays onto their ships. Suppose you want to do the same with the lens from your camera, in order to burn a small hole on a piece of paper: you break the camera open, remove the lens, point it to the sun, and put the sheet of paper behind it, as shown below.



Suppose that your lens behaves according to the thin-lens model and has a focal length f. How far behind the lens should you place the paper (i.e., distance d in the figure) for maximum burning effect? Give a brief explanation, and try to be as concrete as possible. No points will be awarded without an explanation.