UNIVERSITY OF TORONTO
Faculty of Arts and Science

DECEMBER 2009 EXAMINATIONS

CSC418H1F : Computer Graphics

Duration: 3 hours

No aids allowed

There are 13 pages total (including this page)

Given name(s): ________________________________

Family name: ________________________________

Student number: ________________________________

Question	Marks
1	__________/20
2	__________/20
3	__________/35
4	__________/30
5	__________/30
6	__________/20
7	__________/10
8	__________/15
Total	_________/180

1 Surfaces (20 marks total)

(a) **[10 Marks]** Consider the following parameterization:

$$\bar{f}(\phi,\theta) \;=\; (\;\;(0.5\cos\phi+4)\cos\theta,\;(0.5\cos\phi+4)\sin\theta,\;\;\sin\phi\;\;)$$

with

$$\phi \in [-\pi, 0] \quad \text{and} \quad \theta \in [0, 2\pi]\ .$$

What surface does it correspond to? Draw it and explain your reasoning.

(b) **[10 Marks]** Compute the unit surface normal for $\phi = \theta = 0$.

2 Cameras (20 marks total)

Consider two 3D lines passing through points

$$\bar{\mathbf{p}}_0 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \ \ \bar{\mathbf{p}}_1 = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$$

and

$$\bar{\mathbf{p}}_2 = \bar{\mathbf{p}}_0 + \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}, \ \ \bar{\mathbf{p}}_3 = \bar{\mathbf{p}}_1 + \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix},$$

respectively. All points are expressed in Euclidean, camera-centered coordinates.

(a) **[10 Marks]** Suppose the camera's projection matrix is

$$\mathbf{\Pi} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0.5 & 0 \end{bmatrix}.$$

Where do the lines' projections intersect on the image plane?

(b) **[6 Marks]** Now suppose the camera's projection matrix is

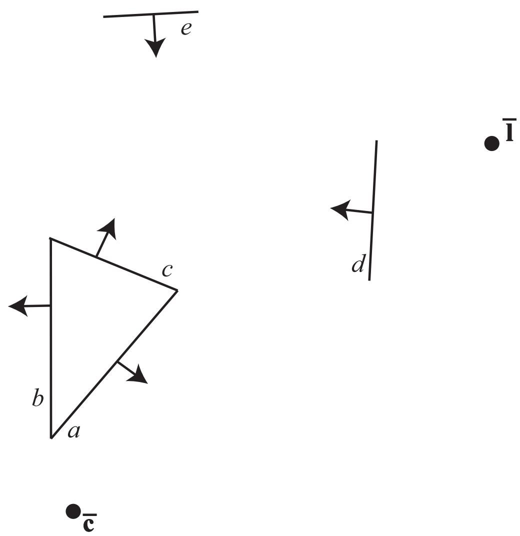
$$\mathbf{\Pi}' = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} .$$

Where do the lines' projections intersect in this case?

(c) **[4 Marks]** What camera focal length corresponds to the projection matrix in (a)?

3 Visibility & Shadows (35 marks total)

Consider the following 2D scene with a light source at point $\bar{\mathbf{l}}$, a camera at point $\bar{\mathbf{c}}$, and the outward normals of polygon segments as shown:



(a) **[10 Marks]** Draw the BSP tree for the scene by adding the segments in the labeled order (*i.e.*, from *a* to *e*).

(b) **[8 Marks]** Suppose you want to compute, for every point on a segment, whether that point will be in shadow. Explain how to do this efficiently using the BSP tree in (a).

(c) **[7 Marks]** Now suppose that segment d is a perfect mirror. Since segments may now be visible indirectly via their reflection in the mirror, your visibility computations must also take mirror reflection into account. Would this change the BSP tree computed in (a)? If your answer is yes, show the new BSP tree; if your answer is no, explain how you would use the existing tree.

(d) **[10 Marks]** Prove or disprove the following statement:

If the BSP rendering order of all polygon segments is the same for two camera positions $\bar{\mathbf{c}}_1$ *and* $\bar{\mathbf{c}}_2$*, it will be the same for any position* $\bar{\mathbf{c}}$ *between them (i.e., for* $\bar{\mathbf{c}} = t\bar{\mathbf{c}}_1 + (1-t)\bar{\mathbf{c}}_2$, $0 \leq t \leq 1$*).*

4 Shading & Lighting (30 marks total)

(a) **[20 Marks]** Let $\bar{\mathbf{p}}_0, \bar{\mathbf{p}}_1$ and $\bar{\mathbf{p}}_2$ be the vertices of a triangle in a triangular mesh and let $\vec{\mathbf{n}}_0, \vec{\mathbf{n}}_1$ and $\vec{\mathbf{n}}_2$ be their associated unit normals. Suppose we are given a function $C(\bar{\mathbf{p}}, \vec{\mathbf{n}})$ that calculates the color at a point $\bar{\mathbf{p}}$ with normal $\vec{\mathbf{n}}$ using the Phong reflection model.

Now let

$$\bar{\mathbf{p}}(a, b) \;=\; a\bar{\mathbf{p}}_0 + b\bar{\mathbf{p}}_1 + (1 - a - b)\bar{\mathbf{p}}_2 \;.$$

(a1) **[5 Marks]** What are the valid ranges of the values $a, b, a + b$ for point $\bar{\mathbf{p}}(a, b)$ to lie in the triangle?

(a2) **[5 Marks]** What is the color at $\bar{\mathbf{p}}(a, b)$ under the Gouraud shading model? Your expression should be in terms of function $C(\bar{\mathbf{p}}, \vec{\mathbf{n}})$; you do not need to define or explain this function.

(a3) **[5 Marks]** What is the color at $\bar{\mathbf{p}}(a, b)$ under the Phong shading model? Again, your expression should be in terms of function $C(\bar{\mathbf{p}}, \vec{\mathbf{n}})$.

(a4) **[5 Marks]** Briefly explain the difference between the two shading methods in terms of accuracy and efficiency of shading computations.

(b) **[10 Marks]** Suppose we replace the Phong reflection model with the following function of the surface normal $\vec{\mathbf{n}}$, incidence direction $\vec{\mathbf{i}}$, and outgoing direction $\vec{\mathbf{o}}$:

$$I(\vec{\mathbf{n}}, \vec{\mathbf{i}}, \vec{\mathbf{o}}) \;=\; e^{-(10\theta/\pi)^2}$$

where

$$\theta \;=\; \text{angle}(\vec{\mathbf{i}},\, \vec{\mathbf{o}}) \quad \text{(in radians)}$$

What appearance would function $I()$ simulate?

5 Basic Ray Tracing (30 marks total)

(a) **[20 Marks]** Briefly, give the main steps of the Whitted Ray Tracing algorithm.

(b) **[10 Marks]** Describe two effects that cannot be rendered with Whitted ray tracing and briefly explain why.

6 Radiometry (20 marks total)

For each of the expressions below, draw a figure and explain all terms:

(a) **[10 Marks]** Give an expression for the irradiance at a surface point $\bar{\mathbf{p}}$ with normal $\vec{\mathbf{n}}$ due to an anisotropic point light source located at point $\bar{\mathbf{l}}$.

(b) **[10 Marks]** Give an expression for the irradiance at a surface point $\bar{\mathbf{p}}$ with normal $\vec{\mathbf{n}}$ due to an area light source defined by a polygon $\mathcal{P}$.

7 Distribution Ray Tracing (10 marks total)

Let $\vec{d}$ be a ray entering the image plane and let $R(\vec{d}, t)$ be the light traveling along this ray (*i.e.*, radiance) at time t from an animated scene. If the camera shutter remains open from t_0 to t_1, the total light received along this ray is:

$$\int_{t_0 \le t \le t_1} R(\vec{d}, t) dt \ .$$

Give an algorithm for approximating this integral by numerical integration. You should assume that you have a function $R(\vec{d}, t)$ for computing the radiance along $\vec{d}$ at time t; you do not need to explain what this function does or how it is implemented.

8 Short-Answer Questions (15 marks total)

(a) **[5 Marks]** What is sub-surface scattering? Explain briefly why it cannot be represented as a BRDF.

(b) **[5 Marks]** How would you compute the normal vector of a vertex on a triangulated 3D polygonal mesh, assuming that the mesh is supposed to approximate a smooth surface?

(c) **[5 Marks]** Give one advantage of inverse kinematics over forward kinematics for character animation.

END OF EXAM (180 marks)