UNIVERSITY OF TORONTO
Faculty of Arts and Science

DECEMBER 2008 EXAMINATIONS

CSC418H1F : Computer Graphics

Duration: 3 hours

No aids allowed

There are 15 pages total (including this page)

Given name(s): ________________________________

Family name: ________________________________

Student number: ________________________________

Question	Marks
1	________/15
2	________/15
3	________/30
4	________/25
5	________/30
6	________/15
7	________/15
8	________/25
9	________/10
Total	________/180

1 Line Scan Conversion (15 marks total)

Your task is to write a scan-conversion function that returns the coordinates of the k^{th} pixel on the line segment between any two pixels (x_1, y_1) and (x_2, y_2):

void **unrestricedSC**(int x_1, int y_1, int x_2, int y_2, int k, bool first, int& x, int& y)

where first=true indicates that k counts pixels starting from (x_1, y_1).

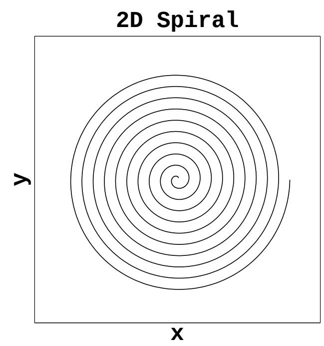
To help you in this task, you are given a restricted version of this function that works only for line segments whose slope is between 0 and 1:

void **restrictedSC**(int x_1, int y_1, int x_2, int y_2, int k, bool first, int& x, int& y)

Using this function as a subroutine, give below the pseudocode for function **unrestricedSC**(). Be as complete as possible but do not specify variable data types. You can assume that k is always between 0 and the total number of pixels on the line segment.

2 Curves and Surfaces (15 marks total)

(a) **[7 Marks]** Suggest a parametric representation for the following curve. You are free to make whatever assumptions you need to keep the parametric representation as simple as possible.



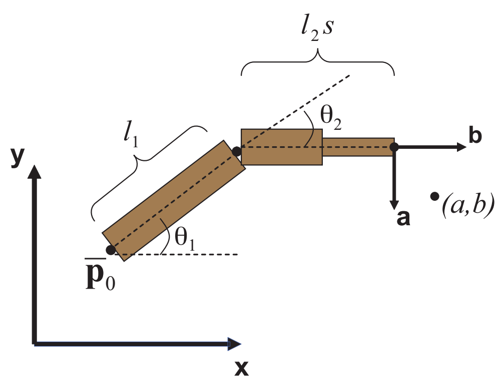
(b) **[8 Marks]** Give the parametric representation for the *torus*.

3 Transformations and Cameras (30 marks total)

(a) **Transformations [15 Marks]** Consider a two-link, two-dimensional robotic arm with the following properties:

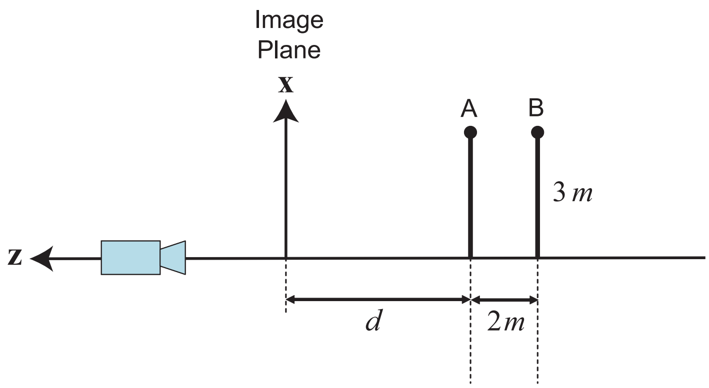
- its base is affixed to a point $\bar{\mathbf{p}}_o$;
- its first link has length l_1 and can rotate freely about $\bar{\mathbf{p}}_o$ by an angle θ_1 ;
- its second link has a *variable length* equal to $l_2 s$, with $0.5 \leq s \leq 1$;
- the second link can rotate freely about the common joint by an angle θ_2.

Suppose we attach a local coordinate system to the end of the second joint, as shown in the figure below, with its b-axis aligned with the joint and its origin at the joint's end:



Express the transformation that maps local coordinates (a, b) to world coordinates (x, y) as a product of elementary homogeneous transformation matrices. You should show the elements of each matrix, but you do not need to perform any matrix multiplications.

(b) **Cameras [15 Marks]** Consider a 2D scene consisting of two poles that are $3m$ high and $2m$ apart. Suppose that the scene is being viewed by a camera at a distance d away from the leftmost pole, as shown below:



(b1) **[5 Marks]** Suppose that the camera is orthographic. For each of the two poles, give the height of its projection on the image plane.

Projected height of Pole A =

Projected height of Pole B =

(b2) **[5 Marks]** Give the poles' projected heights for a perspective camera with focal length f.

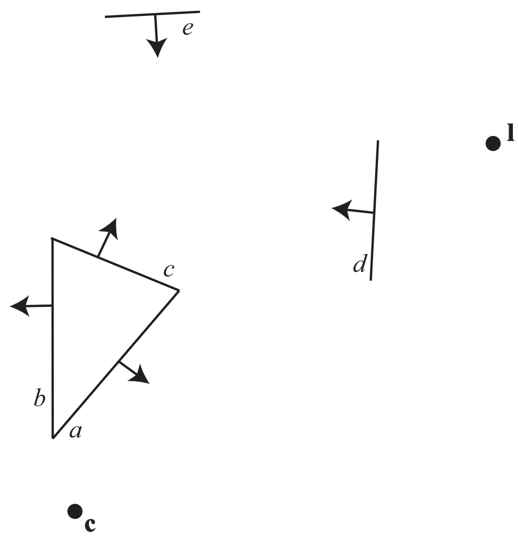
Projected height of Pole A =

Projected height of Pole B =

(b3) **[5 Marks]** How far should the camera in (b2) be from Pole A so that the projected height of Pole A is within 5% of the projected height of Pole B?

4 Visibility & Shadows (25 marks total)

Consider the following 2D scene with a light source at point **l**, a camera at point **c**, and the outward normals of polygon segments as shown:



(a) **[10 Marks]** Draw the BSP tree for the scene by adding the segments in the labeled order (*i.e.*, from *a* to *e*).

(b) **[8 Marks]** Suppose you want to compute, for every point on a segment, whether that point will be in shadow. Explain how to do this efficiently using the BSP tree in (a).

(c) **[7 Marks]** Now suppose that segment d is a perfect mirror. Since segments may now be visible indirectly via their reflection in the mirror, your visibility computations must also take mirror reflection into account. Would this change the BSP tree computed in (a)? If your answer is yes, show the new BSP tree; if your answer is no, explain how you would use the existing tree.

5 Lighting (30 marks total)

(a) **[10 Marks]** Give the expression for the Phong lighting model and briefly explain its parameters.

(b) **[10 Marks]** Suppose we replace the Phong lighting model with the following function of the surface normal $\vec{n}$, incidence direction $\vec{i}$, and outgoing direction $\vec{o}$:

$$I(\vec{n},\vec{i},\vec{o}) \ = \ e^{-(\frac{\theta}{m})^2}$$

where

$$\theta \ = \ \text{angle}\left(\vec{n},\ \frac{\vec{i}+\vec{o}}{2}\right) \quad \text{(in radians)}$$

and m is a constant. What appearance would function $I()$ simulate?

(c) **[10 Marks]** Suppose we want to compute the image of the diffusely-shaded ellipsoid $x^2 + (2y)^2 + z^2 = 1$ under orthographic projection along the z axis. Consider the following two procedures:

Procedure A:

1. *Squash* a tesselated unit sphere by applying the transformation $(x, y, z) \rightarrow (x, \frac{y}{2}, z)$ to all its polygons;
2. *render* the transformed polygons according to the Phong model (with an appropriate set of parameters).

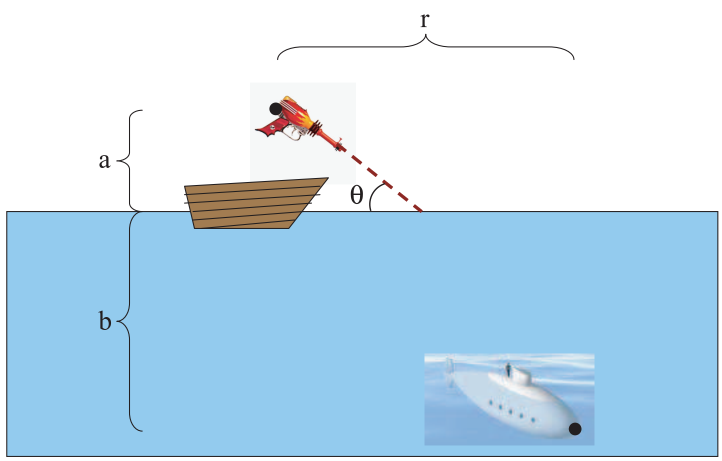
Procedure B:

1. *Render* the tesselated unit sphere with the same Phong parameters and the same rendering algorithm as in Procedure A, to obtain a temporary image of a diffusely-shaded sphere;
2. *squash* the temporary image by applying the transformation $(x, y) \rightarrow (x, \frac{y}{2})$.

Would both procedures give accurate results? Explain why or why not. You should assume that you have available very high-quality algorithms for rendering 3D scenes and for squashing images.

6 Ray Tracing (15 marks total)

Let's say you are in a boat with a ray gun, trying to hit an infinitesimally-small submarine that is b meters below the surface and r meters to the right of you. Assuming you are standing upright so that the gun is a meters above the water, at what angle θ should you aim the gun to hit the submarine?



7 Radiometry (15 marks total)

(a) **[5 Marks]** State the definition of *irradiance*. Be as specific as possible, and specify the quantity's units of measurement.

(b) **[10 Marks]** Indicate the truth or falsehood of the following statements about two infinitesimal surface patches A and B that reflect light onto each other. No explanation is necessary. You will receive 2 marks for each correct answer and -2 marks for each incorrect one (if this incurs a decifit, your marks in (a) will be reduced accordingly, to a minimum of zero).

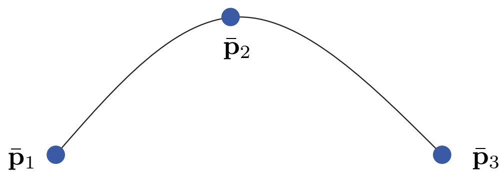
- The irradiance at A due to B depends on A's surface normal.
- The irradiance at A due to B is equal to the radiance at B in the direction of A.
- The irradiance at A due to B depends on the BRDF at B.
- The radiance from A to B depends on the BRDF at A.
- The radiance from A to B depends on the distance between A and B.

8 Interpolation (25 marks total)

You are given three 2D control points $\bar{\mathbf{p}}_1, \bar{\mathbf{p}}_2, \bar{\mathbf{p}}_3$ and two vectors $\vec{\mathbf{a}}, \vec{\mathbf{v}}$. Your goal is to find a polynomial interpolant, $\bar{\mathbf{c}}(t)$, that satisfies the constraints shown below:

$$\bar{\mathbf{c}}(0.5) = \bar{\mathbf{p}}_2$$

$$\frac{d\bar{\mathbf{c}}}{dt}(0.5) = \vec{\mathbf{v}} \qquad \frac{d^2\bar{\mathbf{c}}}{dt^2}(0.5) = \vec{\mathbf{a}}$$



$$\bar{\mathbf{c}}(0) = \bar{\mathbf{p}}_1 \qquad \bar{\mathbf{c}}(1) = \bar{\mathbf{p}}_3$$

(a) **[5 Marks]** What polynomial degree should you use in order to satisfy perfectly all the above constraints? Explain briefly.

(b) **[15 Marks]** Using matrix notation, show the linear system that must be solved in order to compute the coefficients of the interpolating polynomial. You only need to show this for the x coordinates.

(c) **[5 Marks]** Assuming that you have the computed all polynomial coefficients, give the expression for the tangent at $t = 0.9$.

$$\frac{d\bar{\mathbf{c}}}{dt}(0.9) =$$

9 Animation (10 marks total)

(a) **[5 Marks]** Describe in 2-3 sentences the difference between forward and inverse kinematics.

(b) **[5 Marks]** Give one example of an interpolation scheme that exhibits local control. Why is interpolation with local control preferable for computer animation?

END OF EXAM