UNIVERSITY OF TORONTO

Faculty of Arts and Sciences

APRIL / MAY 2006 EXAMINATIONS

CSC418H1S / CSC2504HS: Computer Graphics

Duration: 3 hours

Aids allowed: None

There are 11 pages total (including this page)

The eleventh page will be handed out as an addendum

Please answer questions on the exam pages in the space provided. There are 9 main questions, most of which have multiple parts. Read questions carefully and answer as neatly, clearly and concisely as possible. Should a question be unclear or ambiguous, make a *reasonable* interpretation and state what you have assumed before answering.

Family name: ______________________________

Given names: ______________________________

Student number: ______________________________

Question	Marks
1	________/ 12
2	________/ 16
3	________/ 8
4	________/ 6
5	________/ 10
6	________/ 10
7	________/ 12
8	________/ 16
9	________/ 10
Total	________/ 100

1 Short Answer Questions [12 marks]

(a) *[4 marks]* In traditional animation, what is anticipation? What role does it perform in animation?

(b) *[4 marks]* List two rendering effects that can be achieved by distribution ray tracing but not basic ray tracing. Briefly explain why each one cannot be achieved by basic ray tracing.

(c) *[4 marks]* The graphics pipeline performs clipping to the view volume in homogeneous coordinates. Briefly explain how this can be done without explicitly converting to Euclidean coordinates.

2 Surfaces and normals [16 marks]

An ellipsoid in 3D can be defined by the points (x, y, z) satisfying:

$$f(x,y,z) = \left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 + \left(\frac{z}{c}\right)^2 - 1 = 0 \tag{1}$$

It can also be defined in spherical coordinates as:

$$\begin{aligned} \bar{p}(\theta,\phi) &= (x(\theta,\phi), y(\theta,\phi), z(\theta,\phi)) && (2)\\ x(\theta,\phi) &= a\cos\theta\sin\phi && (3)\\ y(\theta,\phi) &= b\sin\theta\sin\phi && (4)\\ z(\theta,\phi) &= c\cos\phi && (5) \end{aligned}$$

(a) *[4 marks]* To show that these expressions define the same surface, show that every surface point produced by the parametric form (i.e., for all points $\bar{p}(\theta,\phi)$) satisfies the implict equation of the surface.

(b) *[2 marks]* Derive an expression for the surface normal at a point $\bar{q} = (x, y, z)$, using the implicit form.

(c) *[4 marks]* Derive two distinct tangent vectors for points on the surface as a function of (θ, ϕ), using the parametric form.

(d) *[6 marks]* Show that the tangent planes derived from the parameteric form and the implicit form are equivalent, assuming $\bar{q} = \bar{p}(\theta, \phi)$. *Hint:* what is the relationship between a normal vector and the tangent vectors at a point?

3 Bézier Curves [8 marks]

(a) *[4 marks]* Give the equation for the Bézier curve $\bar{p}(t)$ defined by the control points $\bar{p}_0$, $\bar{p}_1$, $\bar{p}_2$, and $\bar{p}_3$.

(b) *[4 marks]* Derive the tangent to the curve for an arbitrary point t.

4 Materials [6 marks]

The figure shows a photograph of a sphere illuminated by a point light source:

The sphere's reflectance includes diffuse and specular components. Describe in geometric terms the location of the highlight (bright spot) on the sphere.

5 Texture Mapping [10 marks]

(a) *[4 marks]* Let $\bar{q}(\alpha, \beta)$ be a triangle with vertices $\bar{p}_0, \bar{p}_1$, and $\bar{p}_2$. Give a parametric equation for $\bar{q}(\alpha, \beta)$ in terms of the vertices and the parameters (α, β), and specify bounds on α and β.

(b) *[6 marks]* Let $T(u, v)$ be a texture image parameterized by texture coordinates (u, v). Suppose we specify texture coordinates for each of the vertices: $(u_0, v_0), (u_1, v_1)$, and (u_2, v_2), such that all of these points lie within the bounds of the texture. What is the value of the texture that would be mapped to a point $\bar{q}(\alpha, \beta)$ on the triangle?

6 Solid Angle [10 marks]

(a) *[2 marks]* In words, why is solid angle important for determining how much light hits an object from a point light source?

(b) *[4 marks]* In words, what is foreshortening, and how does it affect solid angle? How does it affect the amount of light that hits a surface?

(c) *[4 marks]* Give the equation for radiance reflected from a surface point $\bar{p}$ with normal $\vec{n}$, in terms of the BRDF $\rho(\vec{d}_e, \vec{d}_i)$ and incoming radiance $L(\vec{d}_i, \bar{p})$. You may parameterize directions in terms of spherical coordinates, e.g., $\vec{d}(\theta, \phi)$.

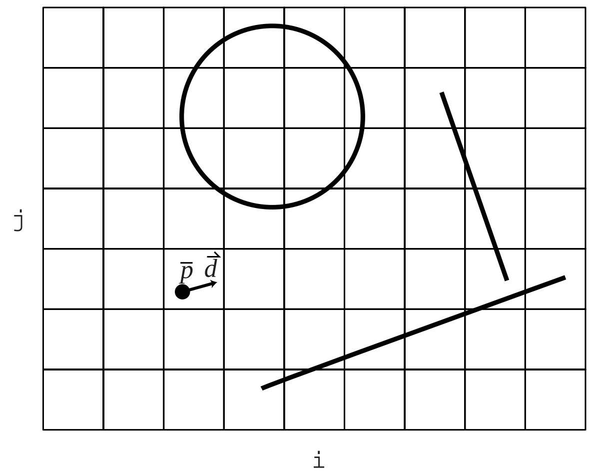
7 Radiometry [12 marks]

(a) *[6 marks]* Let $\bar{l} = (3, 3, 3)$ be the location of a point light source with radiant intensity 100 W/sr, and $\bar{p} = (0, 0, 0)$ be a surface point with normal $\vec{n} = (0, 1, 0)$. What is the irradiance $H(\bar{p})$ at $\bar{p}$ from the light source?

(b) *[6 marks]* Give an expression for the total flux from the light source onto the rectangle lying in the xz-plane, contained in the range $x \in [-2, 2]$ and $z \in [-2, 2]$. The expression should be in terms of the variables specified in the question; be as specific as possible. However, you do not need to evaluate the integral.

8 Visibility [16 marks]

One way to accelerate visibility computations is by placing all objects in a grid. In this problem, we will define a data structure called a **grid** to be used for 2D visibility computations. As illustrated in the figure, the $x - y$ plane has been discretized into an $M \times N$ array of grid cells.



Your job is to define a procedure **intersect**$(\bar{p}, \vec{d})$ that intersects the ray $\bar{r}(\lambda) = \bar{p} + \lambda\vec{d}, \lambda \geq 0$ with the scene. Your code should be efficient, i.e., do not iterate over the entire grid. To make things simpler, you may assume that the ray direction points to the right, and the slope of the ray in 2D is in the range $[-\pi/2, \pi/2]$. You may assume that the start of the ray lies in the grid and that there is at least one intersection point. Each grid cell has dimensions 1×1 in world space. You may also make use of the following functions; you do not need to explain how to implement them:

- The function **cell**(i, j) returns a list of all objects that intersect grid cell (i, j).
- The function **pointToCell**$(\bar{p})$ returns the (i, j) index of the cell containing a point $\bar{p}$.

You may describe the algorithm in words and/or pseudocode, but give sufficient formulas to define how to iterate over the grid.

9 Rendering [10 marks]

(a) *[6 marks]* What is the name of the material/rendering effect that distinguishes these glasses of liquid from each other? Explain the effect. Can it be rendered with basic ray tracing? Why or why not?

(b) *[4 marks]* Name another effect in the scene that cannot be rendered with basic ray tracing. Briefly explain why it cannot be rendered with basic ray tracing.

END OF EXAM