UNIVERSITY OF TORONTO
Faculty of Arts and Science

APRIL / MAY 2004 EXAMINATIONS

CSC418H1S: Computer Graphics

Duration: 3 hours

No aids allowed

There are 14 pages total (including this page)

Family name: ________________________________

Given names: ________________________________

Student number: ________________________________

Question	Marks
1	___________/ 8
2	___________/11
3	___________/12
4	___________/16
5	___________/10
6	___________/12
7	___________/ 6
8	___________/10
9	___________/15
Total	__________/100

1 Euler Angles

(a) [3 marks] What is the homogeneous matrix $\mathbf{R}_x(\theta)$ for a rotation in three dimensions counterclockwise about the x axis by an angle of θ?

$\mathbf{R}_x(\theta) =$

What is the homogeneous matrix $\mathbf{R}_y(\phi)$ for a counterclockwise rotation about the y axis by an angle of ϕ?

$\mathbf{R}_y(\phi) =$

What is the homogeneous matrix $\mathbf{R}_z(\psi)$ for a counterclockwise rotation about the z axis by an angle of ψ?

$\mathbf{R}_z(\psi) =$

(b) [2 marks] The angles θ, ϕ, and ψ are known as *Euler angles*. How would you combine the above elementary rotation matrices $\mathbf{R}_x$, $\mathbf{R}_y$, and $\mathbf{R}_z$ to specify a matrix $\mathbf{R}(\theta, \phi, \psi)$ that rotates an object by arbitrary Euler angles?

$\mathbf{R}(\theta, \phi, \psi) =$

(c) [3 marks] To animate a 3D model, we often want to rotate about small "incremental" angles from frame to frame. Using the "small angle approximations" $\sin\psi \approx \psi$ and $\cos\psi \approx 1$, the so-called "incremental rotation matrix" about the z axis is

$$\begin{matrix} 1 & -\theta & 0 & 0 \\ \theta & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{matrix}$$

Does this matrix specify a true rotation transformation?

Answer "Yes" or "No": __________

If you answered "Yes", briefly explain why it is a rotation matrix; if you answered "No", briefly explain why it is not:

What is the drawback of trying to rotate objects by repeatedly applying this transformation many times?
(Hint: Consider points a unit distance from the origin.)

2 Sweep Surface

(a) [3 marks] Consider the line segment defined by the endpoints $\mathbf{p}_1 = [0, 0, 0]^T$ and $\mathbf{p}_2 = [1, 0, 0]^T$. Write the parametric equation $\mathbf{l}(\alpha)$, for $0 \leq \alpha \leq 1$, of this line segment in homogeneous form.

(b) [5 marks] Let us create a spiral-staircase-like surface, called a "line sweep surface", by translating the above line by 2 units along the z-axis, while rotating it counterclockwise about the z-axis through an angle of 2π radians. The surface generated can be written parametrically as $\mathbf{s}(\alpha, \beta)$. The homogeneous transformation matrix $\mathbf{M}(\beta)$, parameterized by $0 \leq \beta \leq 1$, that accomplishes this sweep can be written in terms of a homogeneous translation matrix $\mathbf{T}(\beta)$ and a homogeneous rotation matrix $\mathbf{R}(\beta)$. Specify $\mathbf{T}(\beta)$ and $\mathbf{R}(\beta)$, and write $\mathbf{M}(\beta)$ in terms of these matrices.

(c) [3 marks] What are the homogeneous coordinates of the point $\mathbf{s}(0.5, 0.5)$ on the line sweep surface? Show your work.

3 Reflectance

[12 marks] Using the Phong reflectance model, give the mathematical expression for the reflectance toward a camera eye (center of projection) at location $\mathbf{e}$ from a surface point $\mathbf{p}$ with unit normal $\mathbf{n}$, given a point light source at location $\mathbf{l}$. Define any other variables needed by the model, and express all directions in terms of $\mathbf{p}$, $\mathbf{n}$, $\mathbf{e}$, and $\mathbf{l}$.

4 Ray Intersections

Consider a hemispherical object; i.e., a sphere of unit radius that is cut by a plane passing through the centre of the sphere (like half an orange). The surface of the object comprises two parts, the quadratic hemisphere and the planar base.

(a) [6 marks] Express this object model mathematically in implicit form using constraints.

(b) [10 marks] Using this mathematical model, find the intersections (if any) of the object surface and a ray $\mathbf{r}(\lambda) = \mathbf{a} + \lambda\mathbf{b}$, assuming that the ray originates outside the object.

5 Ray Tracing: Motion Blur

(a) [2 marks] In one or two sentences, describe what causes motion blur.

(b) [2 marks] Suppose you have a function $f(p, t)$ that traces a ray in an animated scene, for pixel p at time t, and returns the intensity along that ray. Write a definite integral that describes the intensity of pixel p for a given image.

(c) [2 marks] In one or two sentences, describe one way that a photographer can reduce motion blur by changing the camera settings. How does this correspond to changing the above variables?

(d) [4 marks] Describe, briefly in words and/or pseudocode, how you could simulate motion blur using an OpenGL-like renderer (i.e., not a raytracer). The system does not need to run in real-time.

6 Ray Tracing: Shadows, Advanced Phenomena

(a) [1 marks] Under what condition will shadows become "soft"; i.e., exhibit umbral and penumbral regions?

(b) [5 marks] List 5 factors that determine the shape of these regions in the final rendered image.

(c) [6 marks] Apart from motion blur and soft shadows, name and describe each in one or two sentences, 3 other phenomena that are NOT captured by basic ray tracing (i.e., Whitted ray tracing in which rays only reflect specularly, are refracted, and are directed at point light sources).

7 Splines

(a) [6 marks] It is possible to define Bézier curves to have different degrees of continuity and to be closed. For a Bézier curve on five control points p_0, p_1, p_2, p_3, and p_4, answer the following (with figures to help illustrate your answers):

1. Specify a necessary condition on the control points for the curve to exhibit no continuity at its end points (i.e., not to be a closed curve):

2. Specify a necessary condition on the control points for the curve to be C^0 continuous everywhere (i.e., to be a closed curve):

3. Specify a necessary condition on the control points for the curve to be C^1 continuous everywhere (i.e., to be a smooth, closed curve):

8 Spherical Patches

The parametric representation of a unit sphere is

$$\begin{aligned} x &= \cos\theta\sin\phi \\ y &= \sin\theta\sin\phi \\ z &= \cos\phi \end{aligned}$$

We can define a quadrilateral spherical surface patch by restricting the parametric coordinates to $\theta_1 \leq \theta \leq \theta_2$ and $\phi_1 \leq \phi \leq \phi_2$.

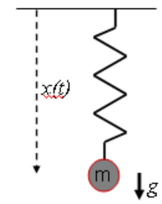
(a) [4 marks] What are the tangent vectors in the direction of each of the parametric coordinates at any point on this spherical patch (i.e., as a function of θ and ϕ)?

(b) [4 marks] What is the normal vector at any point on the patch?

(c) [2 marks] The "isoparametric curves" are the curves of constant θ (latitude) and the curves of constant ϕ (longitude) on the patch. Show that these curves cross orthogonally to one another. (Hint: Consider the tangent vector directions.)

9 Mass-Spring Systems

Consider the simple mass-spring system in the diagram. Assume that the particle can only move in the vertical direction; hence, vectors are unnecessary in what follows. Let t denote time. The particle's position $x(t)$ is measured as shown (positive x is in the downward direction). We know from Newton's second law of motion that the acceleration $a(t)$ of the particle is related to the total force $f(t)$ that acts on it, according to the formula $f = ma$, where m is the mass of the particle. We also know that $a = \frac{dv}{dt}$, where $v(t)$ is the particle's velocity, and that $v = \frac{dx}{dt}$.



(a) [3 marks] The natural length of the spring is 0 (zero) and its elasticity constant (spring stiffness) is c. What is the spring force $s(t)$ that acts on the particle?

$$s(t) = \tag{1}$$

The particle is retarded by a velocity-dependent damping force, with associated damping constant γ. What is the damping force $d(t)$ that acts on the particle?

$$d(t) = \tag{2}$$

The force of gravity, which acts downward, is given by the constant g. What then is the total force $f(t)$ that acts on the particle?

$$f(t) = \tag{3}$$

(b) [6 marks] Let us discretize the position, velocity, and acceleration of the particle in time as follows:

$$x_i = x(i\Delta t), \qquad v_i = v(i\Delta t), \qquad a_i = a(i\Delta t), \qquad \text{for } i = 0, 1, 2, \ldots,$$

where Δt is a small time-step. Replacing continuous time-derivatives with finite differences, write the *forward* finite difference approximation of the acceleration a_i in terms of the velocity variables v_i:

$$\frac{dv(t)}{dt} \approx a_i = \tag{4}$$

Write the *backward* finite difference approximation of v_i in terms of the position variables x_i:

$$\frac{dx(t)}{dt} \approx v_i = \tag{5}$$

Combining equations (4) and (5), write the *central* finite difference approximation of a_i in terms of x_{i+1}, x_i, and x_{i-1}:

$$a_i = \tag{6}$$

(c) [5 marks] Discretizing the force expression in equation (3), we define $f_i = f(i\Delta t)$. Use the central finite difference approximation of a_i obtained in equation (6), and use equation (5) as necessary, to write the discrete equation of motion $a_i = f_i/m$ of the particle in terms of x_{i+1}, x_i, and x_{i-1}:
(Hint: The equation will also involve the constants c, g, γ, Δt and, of course, m.)

(d) [1 marks] Rearranging this discrete equation of motion yields a *second-order* explicit Euler method for numerically integrating the equation through time starting from some initial position $x(0) = x_0$ and velocity $v(0) = v_0$ (i.e., $x_1 = x_0 + v_0\Delta t$) for the particle. Write this iterative integration formula, which gives the next position x_{i+1} of the particle in terms of its current position x_i and its previous position x_{i-1}:

$$x_{i+1} =$$

END OF EXAM