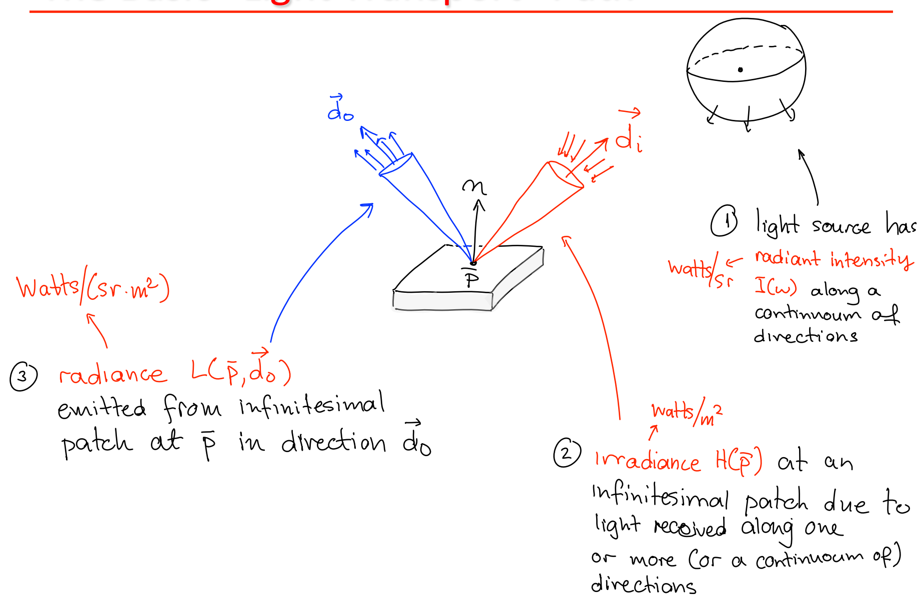
Topic 13:

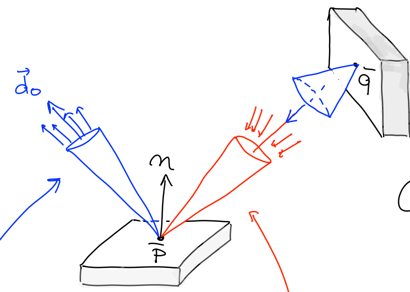
Radiometry

- The big picture
- Measuring light coming from a light source
- Measuring light falling onto a patch: Irradiance
- Measuring light leaving a patch: Radiance
- **The Light Transport Cycle**
- The Bidirectional Reflectance Distribution Function

The Basic "Light Transport" Path



Light Transport Between Patches



① radiance emitted at $\bar{q}$ is received as incident flux at $\bar{p}$

Watts/(sr·m²)

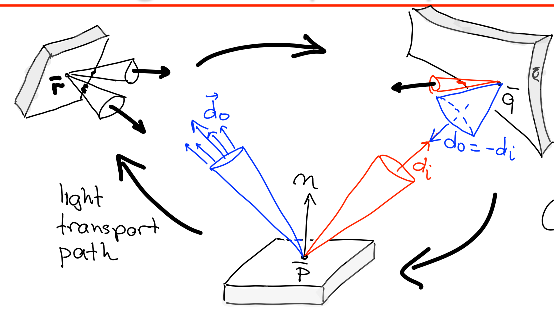
③ radiance $L(\bar{p}, \vec{d}_o)$ emitted from infinitesimal patch at $\bar{p}$ in direction $\vec{d}_o$

Watts/m²

② irradiance $H(\bar{p})$ at an infinitesimal patch due to light received along one or more (or a continuum of) directions

Now that we have defined radiance we can think of every surface point as a light source!

The General Light Transport Cycle



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One Step Along Path: Directional Integration

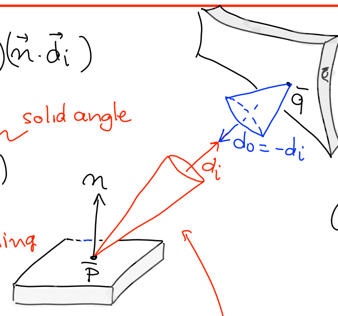
$$H(\bar{p}) = \int_{\text{all directions}} (\text{radiance along } -d_i)(\vec{n}\cdot\vec{d}_i)$$

$$= \int_{\vec{d}_i} L(\bar{q}, -d_i)(\vec{n}\cdot\vec{d}_i)\, d(d_i)$$

solid angle

radiance travelling to $\bar{p}$ from $\bar{q}$

foreshortening in case patch at $\bar{p}$ is slanted



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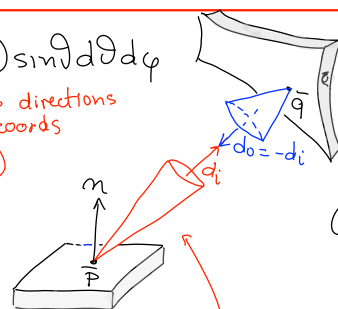
One Step Along Path: Directional Integration

$$H(\bar{p}) = \int_{\varphi}\int_{\vartheta} L(\bar{p}, -d_i)(\vec{n}\cdot\vec{d}_i)\sin\vartheta\, d\vartheta\, d\varphi$$

if we express directions in spherical coords

$(\cos\varphi\sin\vartheta, \sin\varphi\sin\vartheta, \cos\vartheta)$



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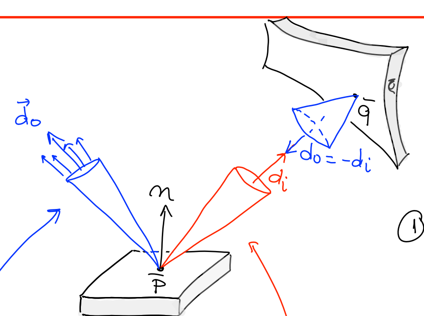
General Light Transport Cycle: Closing the Loop



① radiance emitted at $\bar{q}$ is received as incident flux at $\bar{p}$

Watts/(sr·m²)

③ radiance $L(\bar{p}, \vec{d}_o)$ emitted from infinitesimal patch at $\bar{p}$ in direction $\vec{d}_o$

Watts/m²

② irradiance $H(\bar{p})$ at an infinitesimal patch due to light received along one or more (or a continuum of) directions

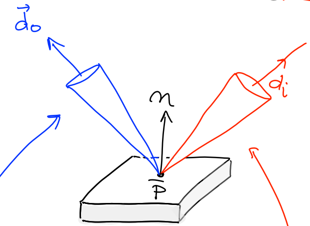
Q: How is incident light "transformed" to outgoing light?

Definition: The BRDF of a Point

BRDF: $\rho_{\bar{p}}(\vec{d}_i, \vec{d}_o) = \dfrac{\text{radiance}}{\text{irradiance}}$

in direction $\vec{d}_o$

due to flux arriving from an infinitesimal solid angle around $\vec{d}_i$



Watts/(sr·m²)

③ radiance $L(\bar{p}, \vec{d}_o)$
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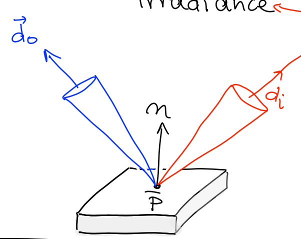


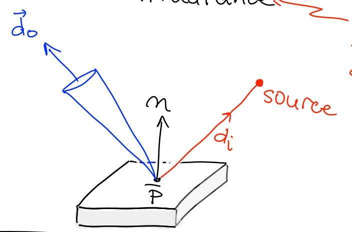
Intuition: The BRDF tells us how bright
$\bar{p}$ will appear if viewed along $\vec{d}_o$
when it receives light from a small
cone of directions along $\vec{d}_i$

Definition: The BRDF of a Point

$$\text{BRDF}: \rho_{\bar{p}}(\vec{d}_i, \vec{d}_o) = \frac{\text{radiance}}{\text{irradiance}}$$

radiance ← in direction $\vec{d}_o$

irradiance ← due to flux arriving from an infinitesimal solid angle around $\vec{d}_i$



Simpler: Suppose we only have a point light source

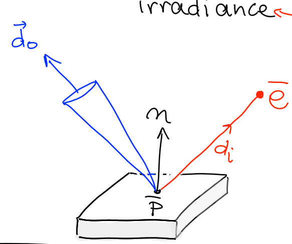
Intuition: The BRDF tells us how bright $\bar{p}$ will appear if viewed along $\vec{d}_o$ and the source is along $\vec{d}_i$

Radiance Due to a Point Light Source

$$\text{BRDF}: \rho_{\bar{p}}(\vec{d}_i, \vec{d}_o) = \frac{\text{radiance}}{\text{irradiance}}$$

radiance ← in direction $\vec{d}_o$

irradiance ← due to flux arriving from an infinitesimal solid angle around



Example #1: Source is at $\bar{e}$

Radiant intensity is $I(\bar{p}-\bar{e})$

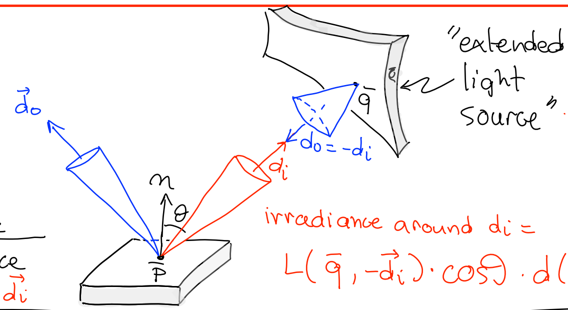
BRDF at $\bar{p}$ is $\rho_{\bar{p}}$

Reminder: $H(\bar{p}) = \frac{I(\omega)}{r^2}\cos\theta$

Q: What is the radiance along $\vec{d}_o$?

Ans: $L(\bar{p}, \vec{d}_o) = \rho(\vec{d}_i, \vec{d}_o) H(\bar{p}) = \rho(\vec{d}_i, \vec{d}_o) \frac{I(\bar{p}-\bar{e})}{\|\bar{p}-\bar{e}\|^2} \cos\theta$

Radiance Due to an Extended Source



$$\rho_{\bar p}(\vec d_i,\vec d_o) = \frac{\overset{\text{along } \vec d_o}{\text{radiance}}}{\underset{\text{around } \vec d_i}{\text{irradiance}}}$$

irradiance around d_i = $L(\bar q, -\vec d_i)\cdot\cos\theta\cdot d(\vec d_i)$

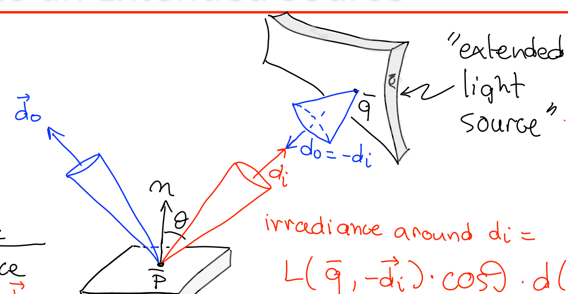
Example #2: Extended source with radiance $L(\bar q, \vec d_i)$

BRDF at $\bar p$ is $\rho_{\bar p}$

Q: What is the radiance along $\vec d_o$?

Ans: $$L(\bar p,\vec d_o) = \int_{\vec d_i} \rho_{\bar p}(\vec d_i,\vec d_o)\cdot\underbrace{(\text{irradiance around } \vec d_i)}_{\substack{\parallel \\ L(\bar q,-\vec d_i)(\vec n\cdot\vec d_i)}}\, d(\vec d_i)$$

Radiance Due to an Extended Source



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Example #2: Extended source with radiance $L(\bar q, \vec d_i)$

BRDF at $\bar p$ is $\rho_{\bar p}$

Q: What is the radiance along $\vec d_o$?

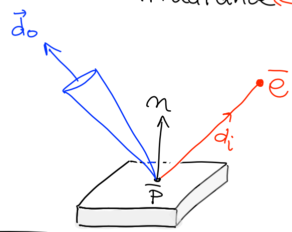
Using spherical coords (θ,φ) for $\vec d_i$:

Ans: $$L(\bar p,\vec d_o) = \int_\theta\int_\varphi \rho_{\bar p}(\vec d_i,\vec d_o)\cdot\underbrace{(\text{irradiance around } \vec d_i)}_{\substack{\parallel \\ L(\bar q,-\vec d_i)(\vec n\cdot\vec d_i)}}\sin\theta\, d\theta\, d\varphi$$

The BRDF of a Diffuse Point

BRDF: $\rho_{\bar{p}}(\vec{d_i},\vec{d_o}) = \dfrac{\text{radiance}}{\text{irradiance}}$ ← in direction $\vec{d_o}$

← due to flux arriving from an infinitesimal solid angle around $\vec{d_i}$



Example # 3: What is the BRDF of a diffuse surface point?

For diffuse points:

- brightness independent of $\vec{d_o}$
- brightness depends only on total incident flux (i.e. irradiance) not illumination dir

The BRDF of a Diffuse Point

BRDF: $\rho_{\bar{p}}(\vec{d_i},\vec{d_o}) = \dfrac{\text{radiance}}{\text{irradiance}}$ ← in direction $\vec{d_o}$

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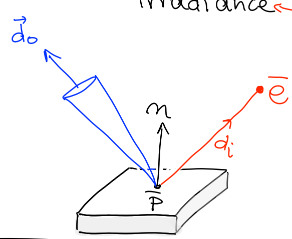


Example # 3: What is the BRDF of a diffuse surface point?

For diffuse points:

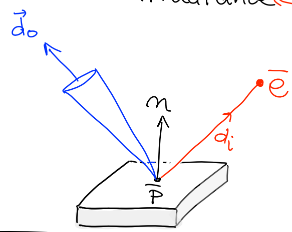
radiance = constant fraction of irradiance

$\Rightarrow \rho(\vec{d_i},\vec{d_o}) = \text{constant}$

↑ what is it equal to?

The BRDF of a Diffuse Point

BRDF : $\rho_{\bar{p}}(\vec{d}_i, \vec{d}_o) = \dfrac{\text{radiance}}{\text{irradiance}}$ ← in direction $\vec{d}_o$ ← due to flux arriving from an infinitesimal solid angle around



Example # 3: What is the BRDF of a diffuse surface point?

For diffuse points:

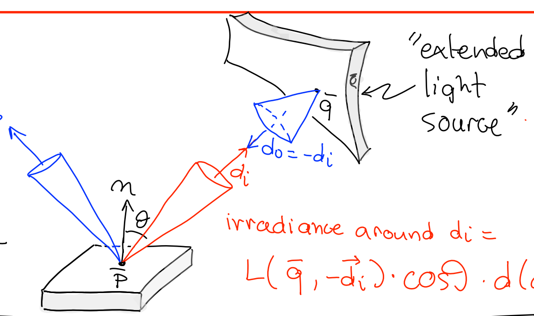
conservation of energy

total light coming in = total light going out
(irradiance) (radiant exitance)

can show that $\rho = \frac{1}{\pi}$ (see Leonid's slides)

Radiance of a Diffuse Point Due to Extended Src

$\rho_{\bar{p}}(\vec{d}_i, \vec{d}_o) = \dfrac{\text{radiance}}{\text{irradiance}}$ along $\vec{d}_o$; around $\vec{d}_i$



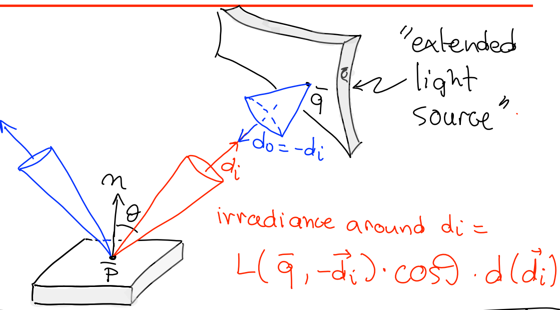
irradiance around $\vec{d}_i$ =
$L(\bar{q}, -\vec{d}_i) \cdot \cos\theta \cdot d(\vec{d}_i)$

Example # 4: Extended source with radiance $L(\bar{q}, \vec{d}_i)$
$\bar{p}$ is a diffuse point

Q: What is the radiance along $\vec{d}_o$?

Ans: $L(\bar{p}, \vec{d}_o) = \displaystyle\int_{\vec{d}_i} \rho_{\bar{p}}(\vec{d}_i, \vec{d}_o) \cdot \underbrace{(\text{irradiance around } \vec{d}_i)}_{= L(\bar{q}, -\vec{d}_i)(\vec{n}\cdot\vec{d}_i)} \, d(\vec{d}_i)$

Radiance of a Diffuse Point Due to Extended Src



along $\vec{d}_o$

$$\rho_{\bar{p}}(\vec{d}_i, \vec{d}_o) = \frac{\text{radiance}}{\text{irradiance}}$$

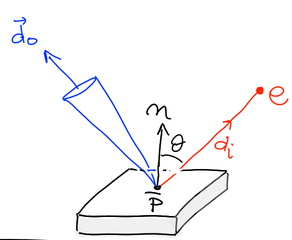
around $\vec{d}_i$

Example # 4: Extended source with radiance $L(\bar{q}, \vec{d}_i)$
$\bar{p}$ is a diffuse point

Q: What is the radiance along $\vec{d}_o$?

Ans: $$L(\bar{p}, \vec{d}_o) = \frac{1}{\pi}\int_{\vec{d}_i} L(\bar{q}, -\vec{d}_i)(\vec{n}\cdot\vec{d}_i)\, d(\vec{d}_i)$$

Radiance of Diffuse Point due to Point Light Src



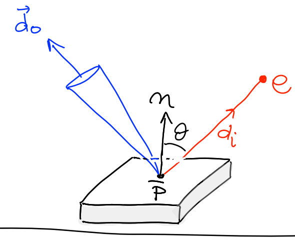
Example # 5: Point light source at distance r
$\bar{p}$ is a diffuse point

Q: What is the radiance along $\vec{d}_o$?

Ans: $$L(\bar{p}, \vec{d}_o) = \frac{1}{\pi}\cancel{\int_{\vec{d}_i}} \underbrace{L(\bar{q}, -\vec{d}_i)}_{\frac{I(\bar{p}-\bar{e})}{\|\bar{p}-\bar{e}\|^2}}(\vec{n}\cdot\vec{d}_i)\, \cancel{d(\vec{d}_i)}$$

only one direction

Radiance of Diffuse Point due to Point Light Src



Example # 5: Point light source at distance r
$\bar p$ is a diffuse point

Q: What is the radiance along $\vec d_o$?

Ans: $L(\bar p, \vec d_o) = \frac{1}{\pi} \cdot \frac{I(\bar p - \bar e)}{\cancel{\|\bar p - \bar e\|^2}} \cdot (\vec n \cdot \vec d_i) = \frac{1}{\pi} I(\bar p - \bar e)(\vec n \cdot \vec d_i)$

can be ignored if light very far away

"Radiometrically-Correct" Ray Tracing

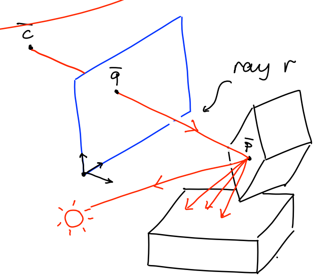
Basic loop:

for each pixel $\bar q$

1. cast ray r through $\bar q$
2. find 1st intersection of $\bar q$ with scene (i.e. point $\bar p$)
3. estimate amount of light reaching $\bar p$
4. estimate radiance $L(\bar q, \bar p - \bar q)$ from $\bar p$ to $\bar q$

Implemented by

- spawning a large set of rays at each step
- directional integration to compute $H(\bar p)$



Distribution Ray Tracing

- In **Whitted Ray Tracing** we computed lighting very crudely
  - Phong + specular global lighting

- In **Distributed Ray Tracing** we want to compute the lighting as accurately as possible
  - Use the formalism of Radiometry
  - **Compute irradiance at each pixel** (by integrating all the incoming light)
  - Since integrals are can not be done analytically, we will employ **numeric approximations**

Benefits of Distribution Ray Tracing

- Better global diffuse lighting
  - Color bleeding
  - Bouncing highlights
- Extended light sources
- Anti-aliasing
- Motion blur
- Depth of field
- Subsurface scattering

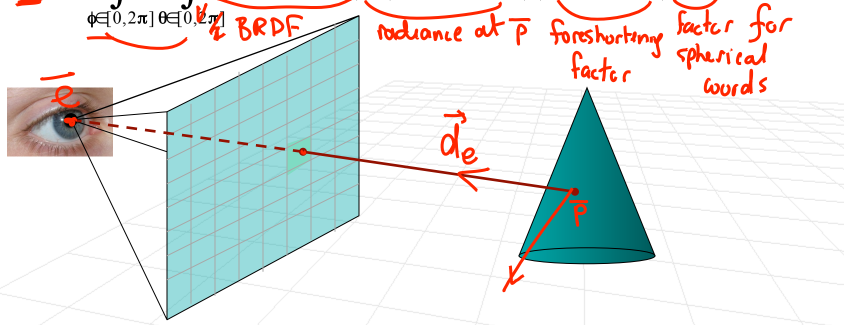
Radiance at a Point

- Recall that radiance (shading) at a surface point is given by

$$L(\bar{p},\vec{d}_e)=\int_{\Omega}\rho(\vec{d}_e,\vec{d}_i)L(\bar{p},-\vec{d}_i)(\vec{n}\cdot\vec{d}_i)\,d\omega$$

- If we parameterize directions in spherical coordinates and assume small differential solid angle, we get

$$L(\bar{p},\vec{d}_e)=\int_{\phi\in[0,2\pi]}\int_{\theta\in[0,2\pi]}\underbrace{\rho\left(\vec{d}_e,\vec{d}_i(\phi,\theta)\right)}_{\text{BRDF}}\underbrace{L\left(\bar{p},-\vec{d}_i(\phi,\theta)\right)}_{\text{radiance at }\bar{p}}\underbrace{\left(\vec{n}\cdot\vec{d}_i(\phi,\theta)\right)}_{\text{foreshortening factor}}\underbrace{\sin\theta}_{\text{factor for spherical coords}}\,d\theta\,d\phi$$



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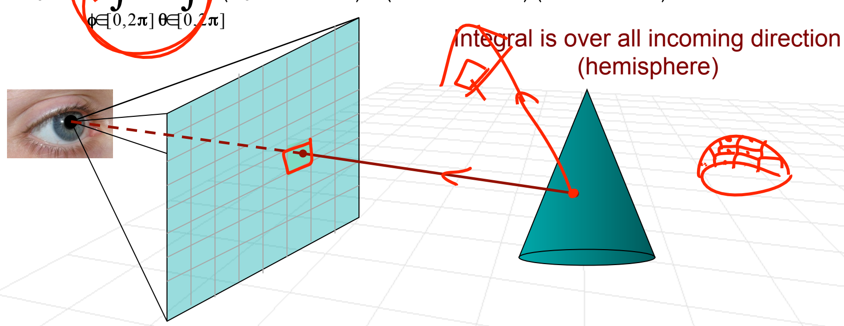
$$L(\bar{p},\vec{d}_e)=\int_{\phi\in[0,2\pi]}\int_{\theta\in[0,2\pi]}\rho\left(\vec{d}_e,\vec{d}_i(\phi,\theta)\right)L\left(\bar{p},-\vec{d}_i(\phi,\theta)\right)\left(\vec{n}\cdot\vec{d}_i(\phi,\theta)\right)\sin\theta\,d\theta\,d\phi$$



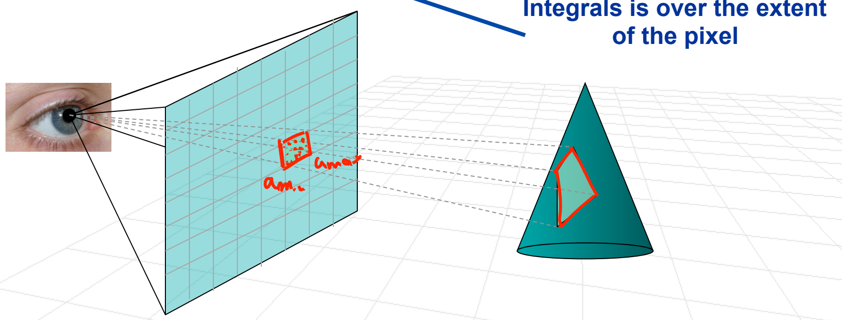
Irradiance at a Pixel

- To compute the color of the pixel, we need to compute **total light energy** (flux) **passing through the pixel** (rectangle) (i.e. we need to compute the total irradiance at a pixel)

$$\Phi_{i,j} = \int_{\alpha_{min} \le \alpha \le \alpha_{max}} \int_{\beta_{min} \le \beta \le \beta_{max}} H(\alpha,\beta)\, d\alpha\, d\beta$$

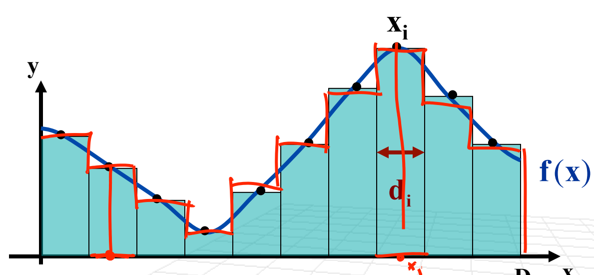
incident irradiance

Integrals is over the extent of the pixel



Numerical Integration (1D Case)

- **Remember:** integral is an area under the curve
- We can approximate any integral numerically as follows



area $= d_i \cdot f(x_i)$

$$\sum_{i=1}^{N} d_i f(x_i) \xrightarrow[N\to\infty]{} \int_0^D f(x)\,dx$$

Numerical Integration (1D Case)

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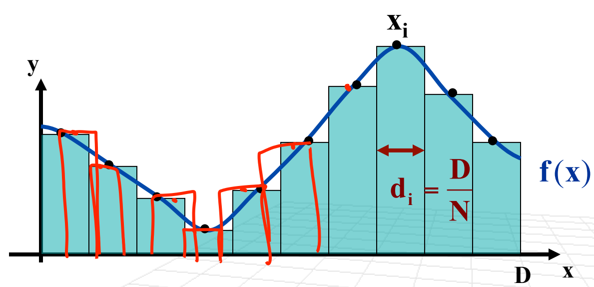
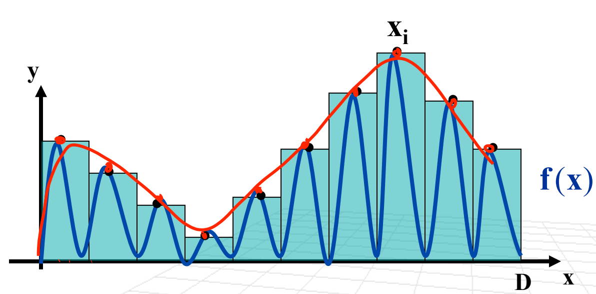


$$\int_0^D f(x)\,dx \approx \sum_{i=1}^{N} \frac{D}{N} f(x_i)$$

Numerical Integration (1D Case)

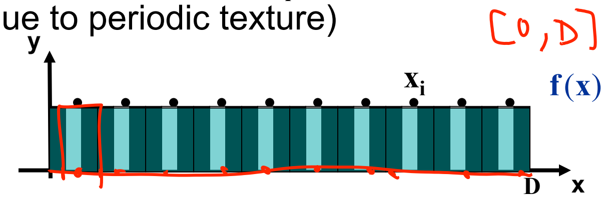
- **Problem:** what if we are really unlucky and our signal has the same structure as sampling?



$$\int_0^D f(x)\,dx \approx \sum_{i=1}^{N} \frac{D}{N} f(x_i)$$

Monte Carlo Integration

- **Idea:** randomize points $\mathbf{x_i}$ to avoid structured noise (e.g. due to periodic texture)



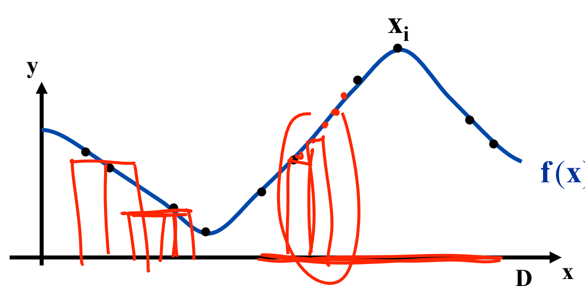
- Draw $\mathbf{N}$ random samples $\mathbf{x_i}$ independently from uniform distribution $\mathbf{Q(x)=U[0,D]}$ (i.e. $\mathbf{Q(x) = 1/D}$ is the uniform probability density function)
- Then approximation to the integral becomes

$$\frac{1}{N}\sum w_i f(x_i) \approx \int f(x)\,dx \quad \text{, for } w_i = \frac{1}{Q(x_i)} \quad \frac{1}{D}$$

- **We can also use other $\mathbf{Q}$'s for efficiency !!!** (a.k.a. importance sampling)

Monte Carlo Integration

$Q(x_i)$: probability of drawing x_i as a sample



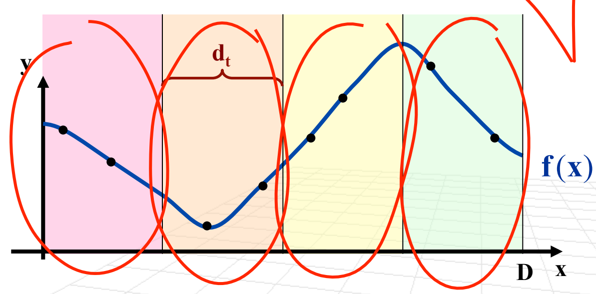
- Then approximation to the integral becomes

$$\frac{1}{N}\sum w_i f(x_i) \approx \int f(x)\,dx \quad \text{, for } w_i = \frac{1}{Q(x_i)}$$

- **We can also use other $\mathbf{Q}$'s for efficiency !!!** (a.k.a. importance sampling)

Stratified Sampling

- **Idea:** combination of uniform sampling plus random jitter
- Break domain into $\mathbf{T}$ intervals of widths $\mathbf{d_t}$ and $\mathbf{N_t}$ samples in interval $\mathbf{t}$

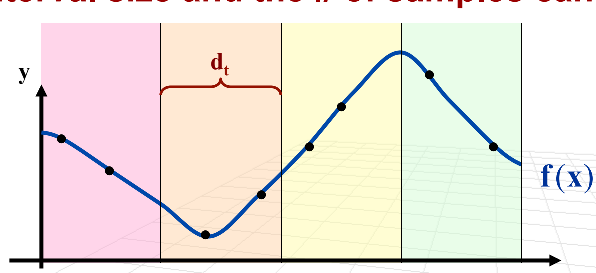


- Integral approximated using the following:

$$\sum_{t=1}^{T} \frac{1}{N_t} \sum_{j=1}^{N_t} d_t f(x_{t,j})$$

Stratified Sampling

- If intervals are uniform $\mathbf{d_t = D/T}$ and there are same number of samples in each interval $\mathbf{N_t = N/T}$ then this approximation reduces to: $$\sum_{t=1}^{T} \sum_{j=1}^{N_t} \frac{D}{N} f(x_{t,j})$$
- **The interval size and the # of samples can vary !!!**



- Integral approximated using the following:

$$\sum_{t=1}^{T} \frac{1}{N_t} \sum_{j=1}^{N_t} d_t f(x_{t,j})$$

Back to Distribution Ray Tracing

- Based on one of the approximate integration approaches we need to compute
  - Let's try uniform sampling

$$L(\bar{p},\vec{d}_e) = \int_{\phi\in[0,2\pi]} \int_{\theta\in[0,\pi/2]} \rho\left(\vec{d}_e,\vec{d}_i(\phi,\theta)\right) L\left(\bar{p},-\vec{d}_i(\phi,\theta)\right)\left(\vec{n}\cdot\vec{d}_i(\phi,\theta)\right)\sin\theta\, d\theta\, d\phi$$

$$\approx \sum_{m=1}^{M}\sum_{n=1}^{N} \rho\left(\vec{\mathbf{d}}_e,\vec{\mathbf{d}}_i(\phi_m,\theta_n)\right)\mathbf{L}\left(\bar{\mathbf{p}},-\vec{\mathbf{d}}_i(\phi_m,\theta_n)\right)\left(\vec{\mathbf{n}}\cdot\vec{\mathbf{d}}_i(\phi_m,\theta_n)\right)\sin\theta\,\Delta\theta\,\Delta\phi$$

where

$$\theta_n = \left(n - \frac{1}{2}\right)\Delta\theta \qquad \Delta\theta = \frac{\pi/2}{M}$$

$$\phi_m = \left(m - \frac{1}{2}\right)\Delta\phi \qquad \Delta\phi = \frac{2\pi}{N}$$

midpoint of the interval (sample point) **Interval width**

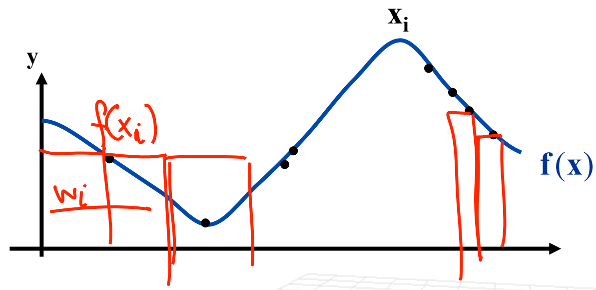
Importance Sampling in Distribution Ray Tracing

- **Problem:** Uniform sampling is too expensive (e.g. **100** samples/hemisphere with depth of ray recursion of **4** => $100^4 = 10^8$ samples per pixel … with 10^5 pixels => 10^{15} **samples**)

- **Solution:** Sample more densely (using **importance sampling**) where we know that effects will be most significant
  - Direction toward point or extended light source are significant
  - Specular and off-axis specular are significant
  - Texture/lightness gradients are significant
  - Sample less with greater depth of recursion

Importance Sampling

- **Idea:** randomize points $\mathbf{x_i}$ to avoid structured noise (e.g. due to periodic texture)



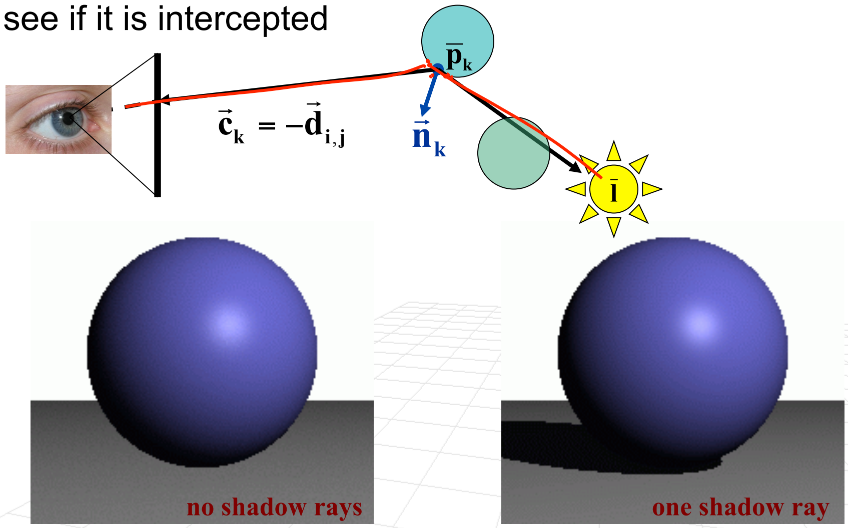
$$\frac{1}{N}\sum \mathbf{w_i f(x_i)} \approx \int \mathbf{f(x)dx} \quad \text{, for } \mathbf{w_i} = \frac{1}{\mathbf{Q(x_i)}}$$

Benefits of Distribution Ray Tracing

- Better global diffuse lighting
  - Color bleeding
  - Bouncing highlights
- Extended light sources
- Anti-aliasing
- Motion blur
- Depth of field
- Subsurface scattering

Shadows in Ray Tracing

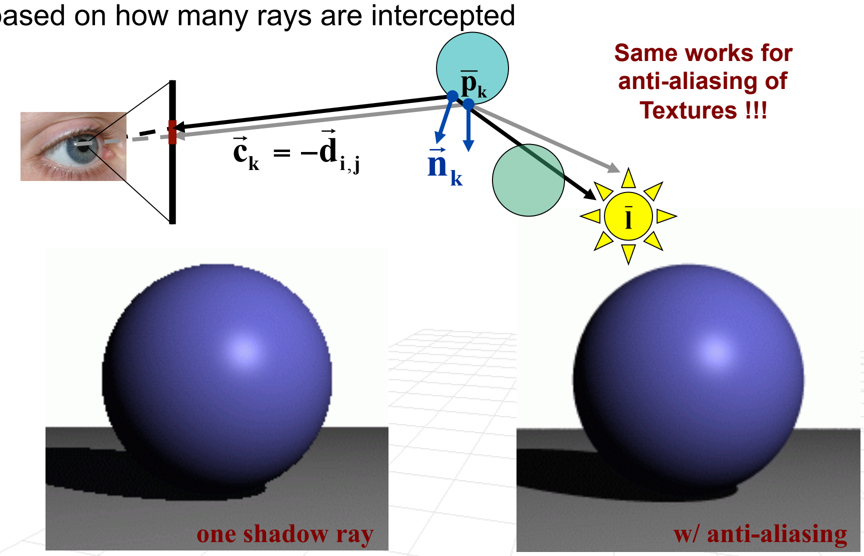
- Recall, we shoot a ray towards a light source and see if it is intercepted



Images from the slides by Durand and Cutler

Anti-aliasing in Distribution Ray Tracer

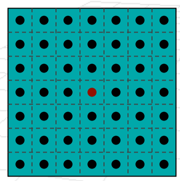
- Lets shoot multiple rays from the same point and attenuate the color based on how many rays are intercepted



Images from the slides by Durand and Cutler

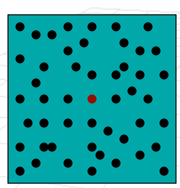
Anti-aliasing by Deterministic Integration

- **Idea:** Use multiple rays for every pixel
- **Algorithm**
  - Subdivide pixel ***(i,j)*** into squares
  - Cast ray through square centers
  - Average the obtained light
- Susceptible to structured noise, repeating textures



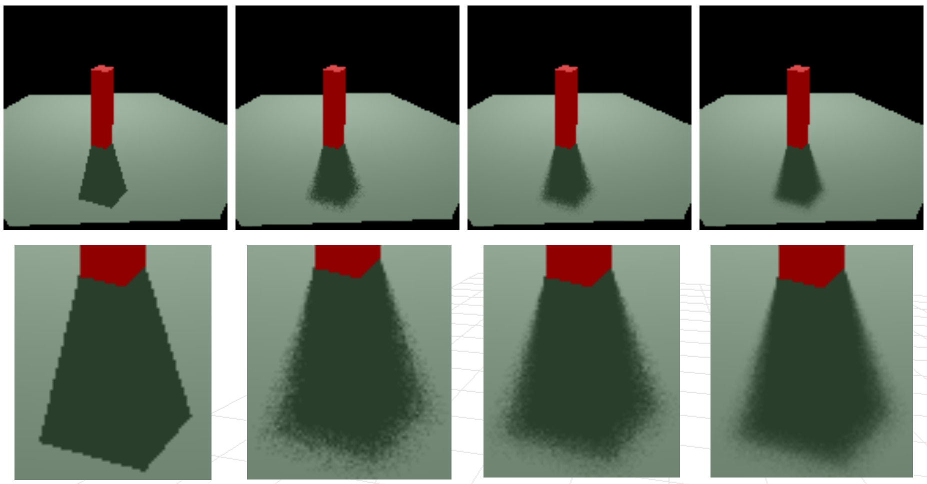
Anti-aliasing by Monte Carlo Integration

- **Idea:** Use multiple rays for every pixel
- **Algorithm**
  - Randomly sample point inside the pixel ***(i,j)***
  - Cast ray through point
  - Average the obtained light
- **Does not** suffer from structured noise, repeating textures



How many rays do you need?

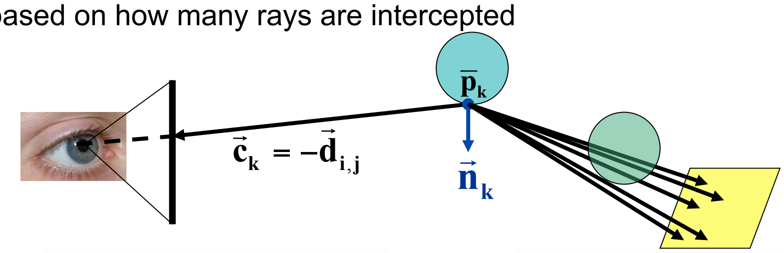
1 ray/light | 10 ray/light | 20 ray/light | 50 ray/light

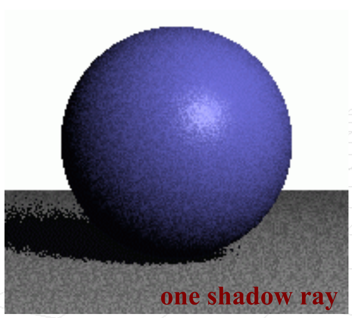


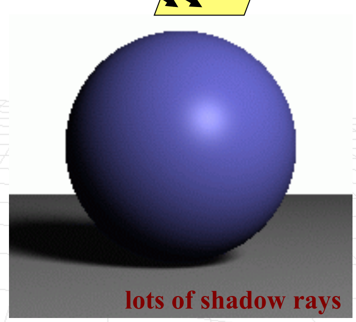
Images taken from http://web.cs.wpi.edu/~matt/courses/cs563/talks/dist_ray/dist.html

Soft Shadows with Distribution Ray Tracing

- Lets shoot multiple rays from the same point and attenuate the color based on how many rays are intercepted

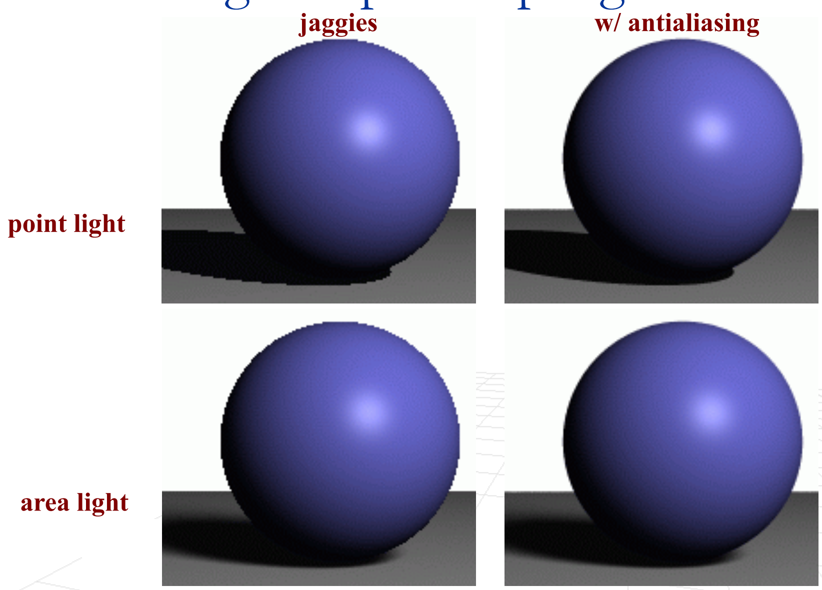






Images from the slides by Durand and Cutler

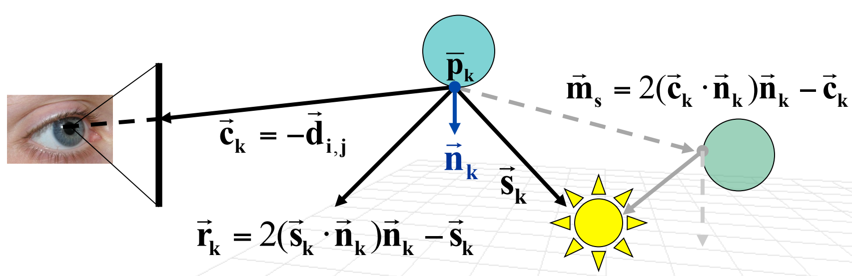
Antialiasing – Supersampling



Images from the slides by Durand and Cutler

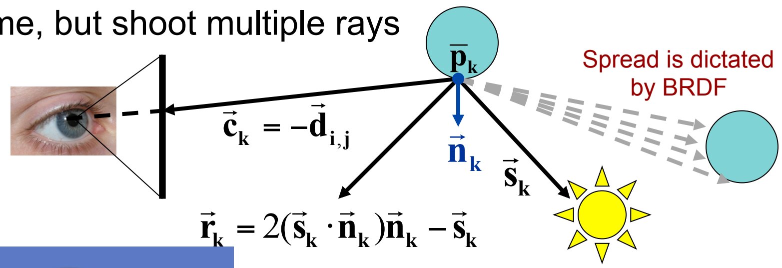
Specular Reflections

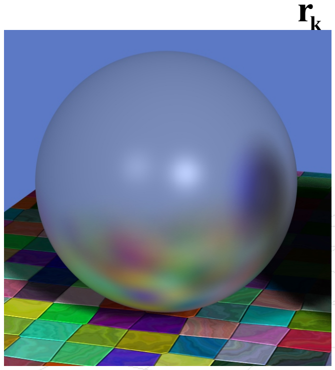
- Recall, we had to shoot a ray in a perfect specular reflection direction (with respect to the camera) and get the radiance at the resulting hit point



Specular Reflections with DRT

- Same, but shoot multiple rays





Justin Legakis

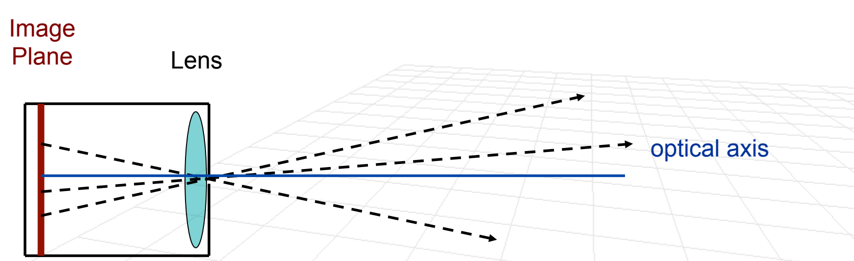
Perfect Reflections (Metal)

Perfect Reflections (glossy polished surface)

[Jensen]

Depth of Field

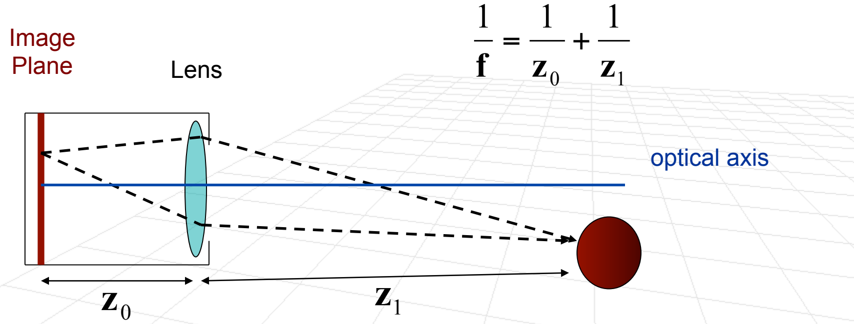
- So far with our Ray Tracers we only considered **pinhole camera model** (no lens)
  - or alternatively, lens, but tiny aperture



Depth of Field

- So far with our Ray Tracers we only considered pinhole camera model (no lens)
  - or alternatively, lens, but tiny aperture
- What happens if we put a lens into our “camera”
  - or increase the aperture
- Remember the thin lens equation?

$$\frac{1}{\mathbf{f}} = \frac{1}{\mathbf{z}_0} + \frac{1}{\mathbf{z}_1}$$



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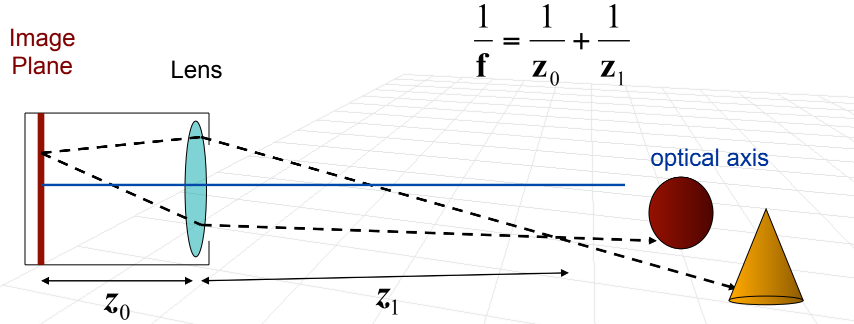
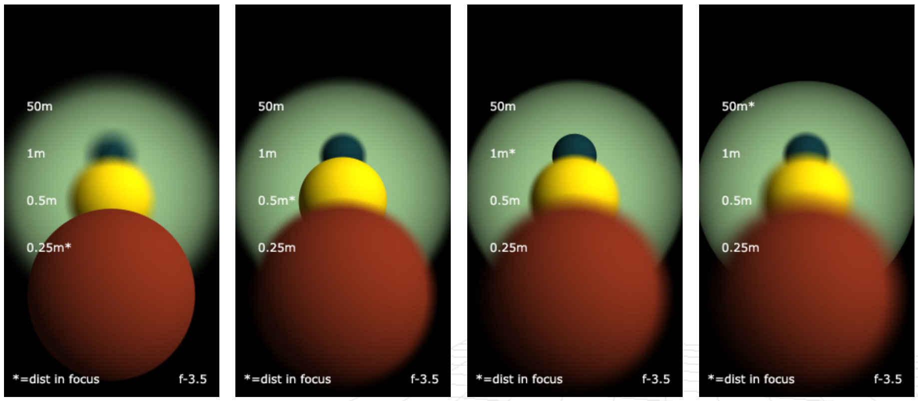
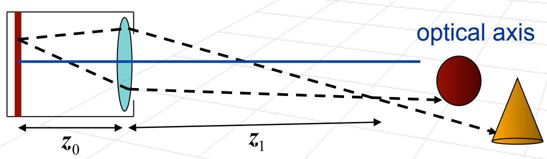


Changing the focal-length in DRT

increasing focal length →

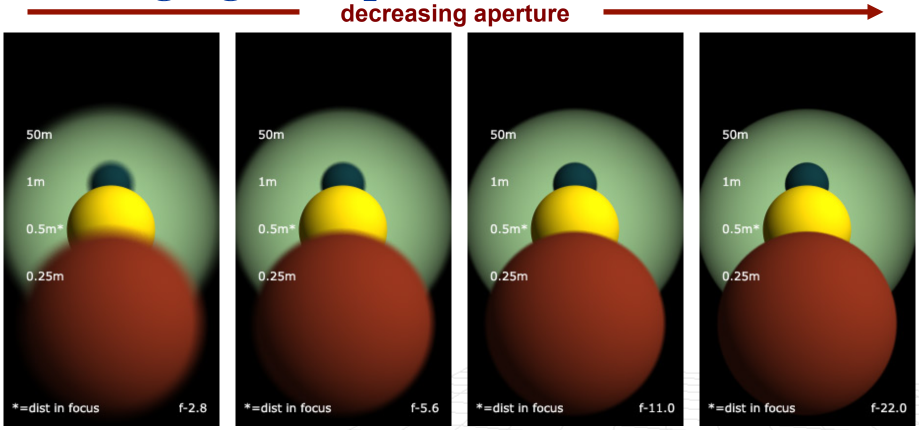


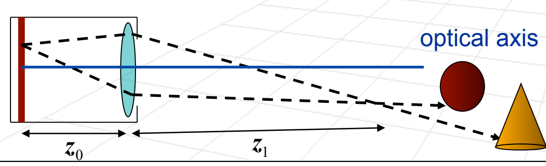


220x400 pixels
144 samples per pixel
~4.5 minutes to render

Changing the aperture in DRT

decreasing aperture →





220x400 pixels
144 samples per pixel
~4.5 minutes to render

Depth of Field

P. Haeberli

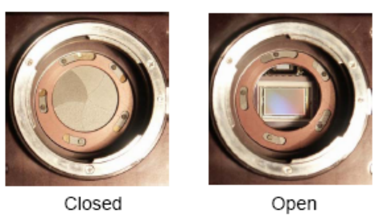
Depth of Field

Depth of Field

Depth of Field

Depth of Field

Camera Shutter



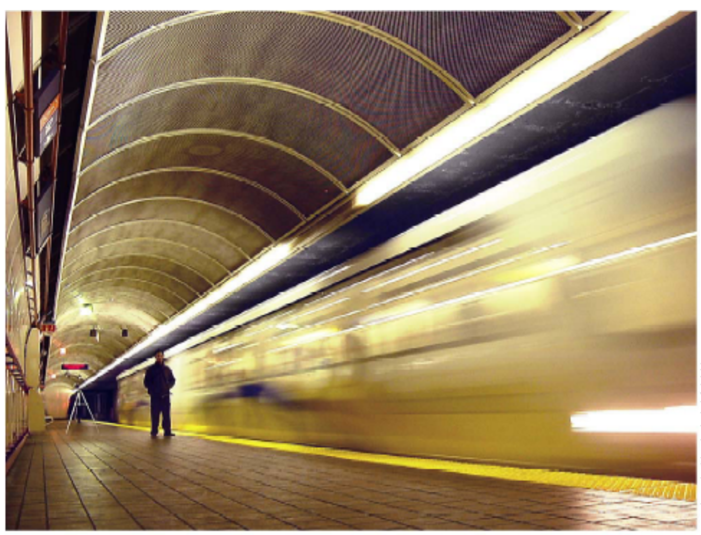
- We ignored the fact that **it takes time to form the image**
  - We ignored this for radiometry
- During that time the shutter is open and light is collected
  - We need to **integrate temporally**, not only spatially

$$\int_t \int_\alpha \int_\beta H(\alpha,\beta,t)\, d\alpha\, d\beta\, dt$$

Motion Blur

Cook, Porter & Carpenter

Motion Blur



Long Exposure Photography

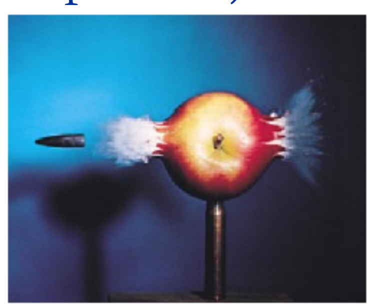
Motion Blur (long exposures)

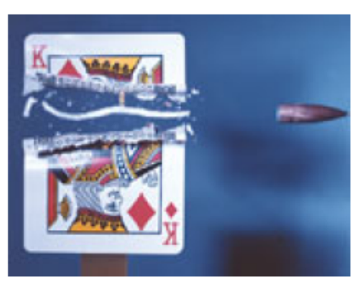
Golden Gate Bridge
30 sec. exposure @ f4

Bodie State Park
30 min. exposure @ f4

Motion Blur (short exposures)

Doc Edgerton, 1936





Sub-surface Scattering

radiance
irradian.

8

BSSRDF

H. W. Jensen

Sub-surface Scattering

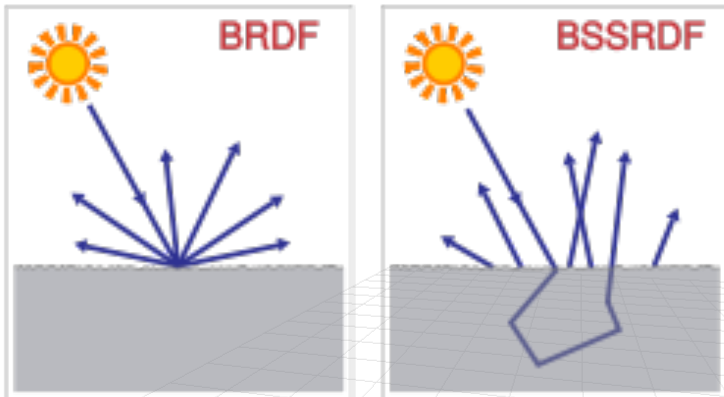
Bidirectional Surface Scattering Reflectance Distribution Function

Rendering with BRDF

Rendering with BSSRDF

H. W. Jensen

Bidirectional Surface Scattering Reflectance Distribution Function



[Images taken from Wikipedia]

Semi-Transparencies

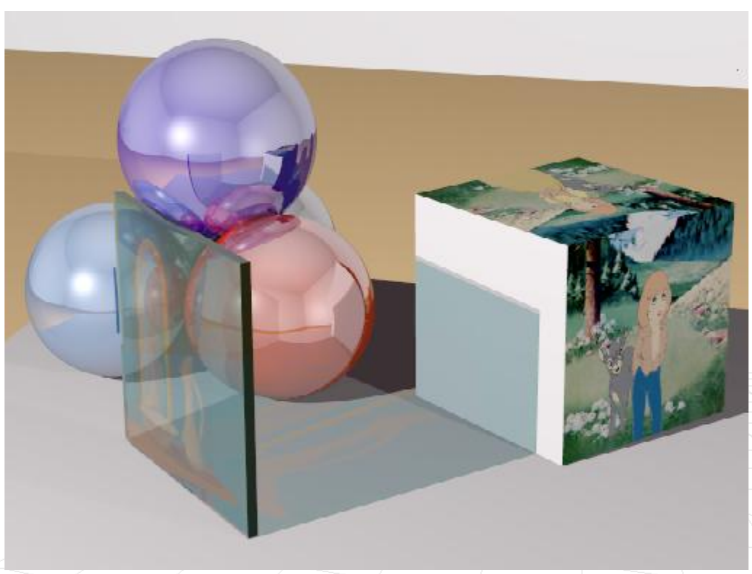


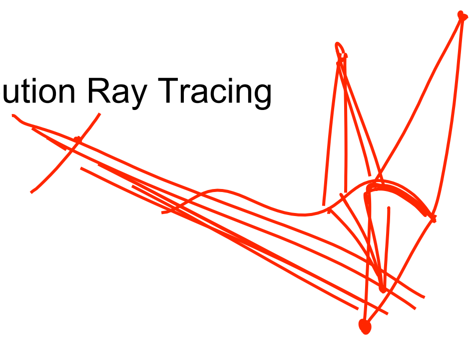
Image form http://www.graphics.cornell.edu/online/tutorial/raytrace/

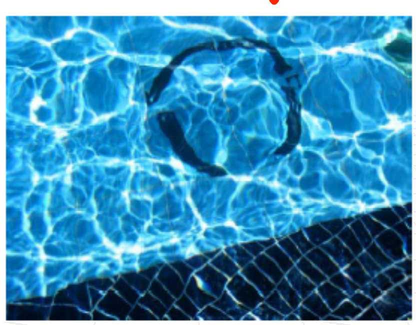
Texture-mapping and Bump-mapping in Ray Tracer

Image form http://www.graphics.cornell.edu/online/tutorial/raytrace/

Caustics

- Hard to do in Distribution Ray Tracing
  - **Why?**

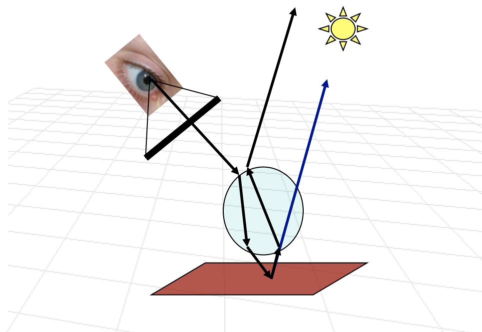




Caustics

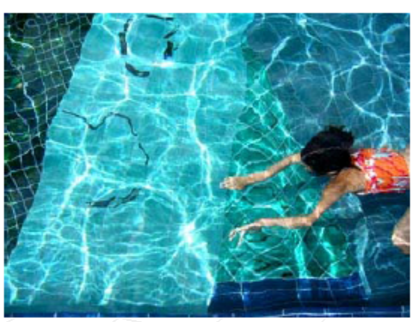
- Hard to do in Distribution Ray Tracing
  - **Why?**

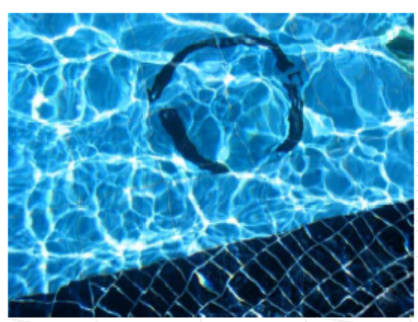
Hard to come up with a good importance function for sampling,
Hence, VERY VERY slow



Caustics

- Often done using bi-directional ray tracing (a.k.a. **photon mapping**)
  - Shoot light rays from light sources
  - Accumulate the amount of light (radiance) at each surface
  - Shoot rays through image plane pixels to “look-up” the radiance (and integrate irradiance over the area of the pixel)





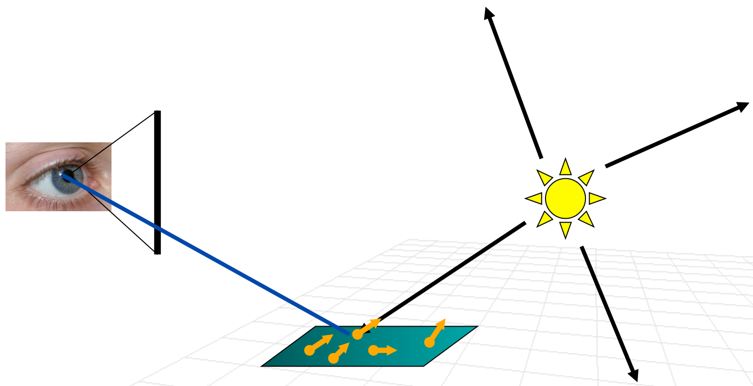
Photon Mapping

- Simulates **individual photons**
  - In DRT we were simulating radiance (flux)
- Photons are emitted from light sources
- Photons bounce off of specular surfaces
- Photons are **deposited on diffuse surfaces**
  - Held in a 3-D spatial data structure
  - Surfaces need not be parameterized
- Photons **collected by ray tracing** from eye

Photons

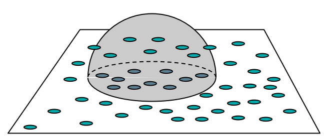
- A **photon** is a particle of light that carries flux, which is encoded as follows
  - magnitude (in Watts) and color of the flux it carries, stored as an RGB triple
  - location of the photon (on a diffuse surface)
  - the incident direction (used to compute irradiance)
- **Example** (point light source, photons emitted uniformly)
  - Power of source (in Watts) distributed evenly among photons
  - Flux of each photon equal to source power divided by total # of photons
  - 60W light bulb would sending 100 photons, will result in 0.6 W per photon

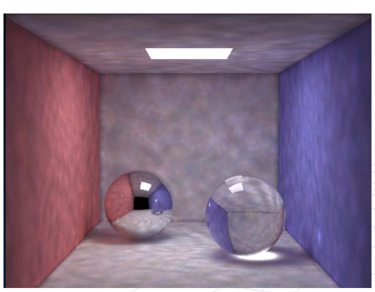
How does this actually work?



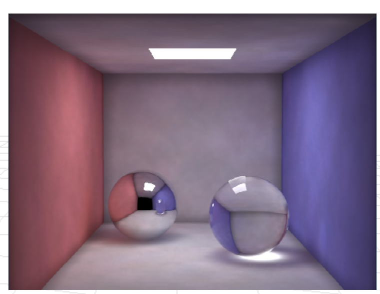
Special data structures are required to do fast look-up (KD-trees)

Photon Mapping Results





Radiance estimate using 50 photons



Radiance estimate using 500 photons