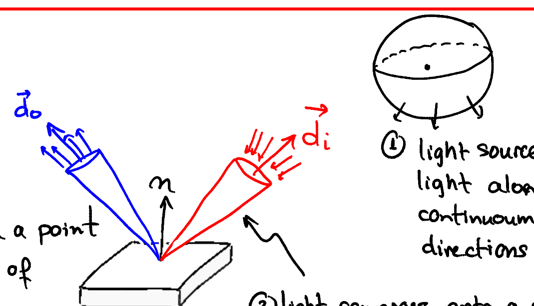
Topic 13:

Radiometry

- The big picture
- Measuring light coming from a light source
- Measuring light falling onto a patch: Irradiance
- Measuring light leaving a patch: Radiance
- The Light Transport Cycle
- The Bidirectional Reflectance Distribution Function

Five Issues We Have Ignored So Far...



① light sources emit light along a continuum of directions

② light converges onto a point from a continuous "cone" of directions

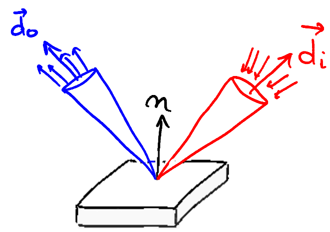
③ light is reflected from a point to a continuous "cone" of directions

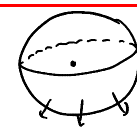
④ real objects are made of surfaces, not isolated points

$$\rho(\vec{d}_i, \vec{d}_o) = \frac{\text{emitted light in direction } \vec{d}_o}{\text{incident light in direction } \vec{d}_i}$$

⑤ must be expressed in terms of physical quantities

Radiometry: Getting the Physics Right





Radiometry: measurement of electromagnetic radiation

Physics:
Joules
Watts

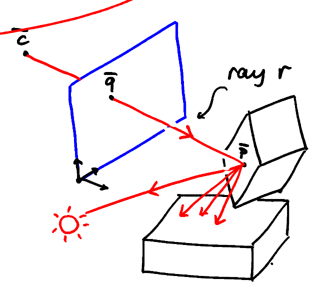
Geometry:
differential
- patches
- directions
- solid angles
- foreshortened area

Radiometry:
radiant energy
radiant flux
irradiance
radiance
radiant exitance
BRDF

"Radiometrically-Correct" Ray Tracing

Basic loop:
for each pixel $\bar{q}$
1. cast ray r through $\bar{q}$
2. find 1st intersection of $\bar{q}$ with scene (i.e. point $\bar{p}$)
3. estimate amount of light reaching $\bar{p}$
4. estimate amount of light travelling from $\bar{p}$ to $\bar{q}$ along ray r

Implemented by
- spawning a large set of rays at each step
- computing integrals of radiometric quantities



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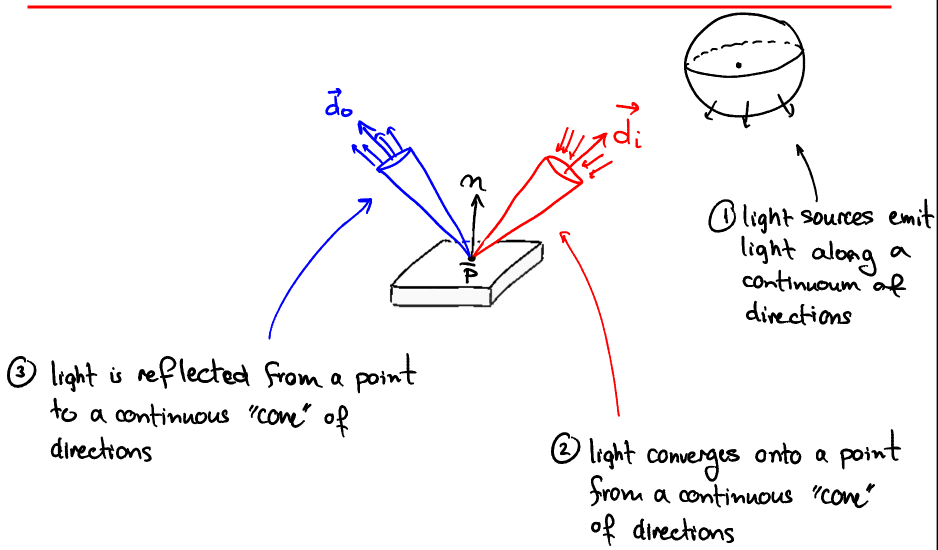
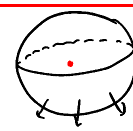
The Basic “Light Transport” Path



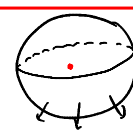
The Basic "Light Transport" Path



light sources emit light along a continuum of directions

Radiant Energy & Radiant Flux

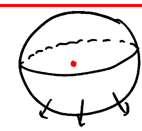
- Light is energy (i.e. photons)
  - ⇒ measured in Joules (J) (4×10^{-19} J per photon)
  - ⇒ energy emitted by a light source is called radiant energy
- We are interested in "steady-state" conditions (energy per unit time interval)
  - ⇒ energy per unit time (a.k.a power)
  - ⇒ measured in Watts (= J/sec)
  - ⇒ called radiant flux Φ



light source emits radiant flux Φ (in Watts)

Measuring Light Emitted from a (Point) Source

In rendering, light emitted by the light source falls onto an object surface.



Q: How do we measure emission along specific directions?

Q: How do we measure light received at a point on a surface?

Q: What units should we use?

Flux Through an Arc (for 2D, Uniform Source)

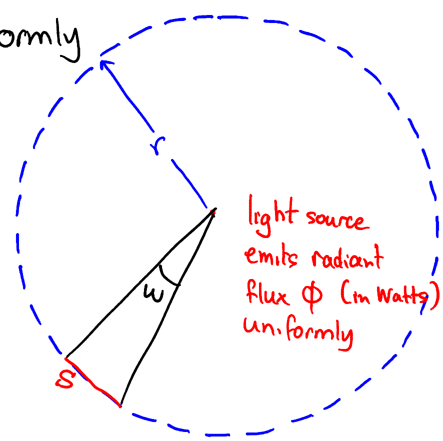
- Suppose source emits uniformly in all directions
- How much flux flows through arc S'?

$$\Phi \frac{\text{length of } S'}{\text{perimeter}} =$$

$$\Phi \frac{\omega \cancel{r}}{2\pi \cancel{r}} = \omega \frac{\Phi}{2\pi}$$

measured in radians (rad)

measured in Watts per radian (W/rad)



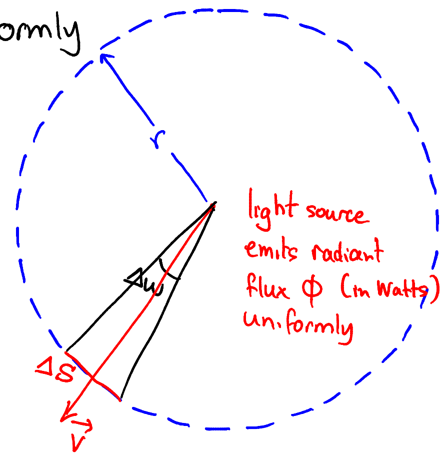
Flux Along a Direction (for 2D, Uniform Source)

- Suppose source emits uniformly in all directions
- How much flux flows along direction v?
  - Consider a small arc Δs centered at $\vec{v}$

    $$\Delta\phi = \phi \frac{\text{length } \Delta s}{\text{perimeter}} = \Delta\omega \frac{\phi}{2\pi}$$

  - Define flux along $\vec{v}$ to be the limit of $\Delta\phi$ as $\Delta\omega \to 0$:

    $$d\phi = d\omega \frac{\phi}{2\pi} \stackrel{\text{def}}{=} \lim_{\Delta\omega \to 0} \Delta\omega \frac{\phi}{2\pi}$$



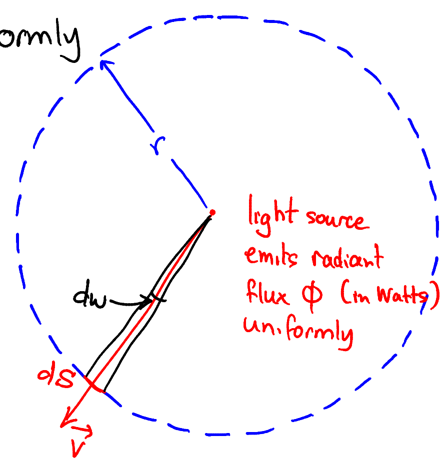
Flux Along a Direction (for 2D, Uniform Source)

- Suppose source emits uniformly in all directions
- How much flux flows along direction $\vec{v}$?

Ans: A differential flux $d\phi$:

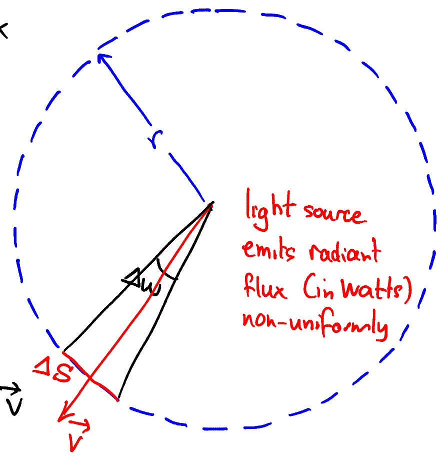
$$\underbrace{d\phi}_{\text{differential flux}} = \underbrace{d\omega}_{\text{differential angle}} \underbrace{\frac{\phi}{2\pi}}_{\text{measured in Watts per radian (W/rad)}}$$

these are both infinitesimal quantities!



Flux Along a Direction (2D, Non-Uniform Src)

- Suppose source emits flux non-uniformly.
- How do we quantify the source's emission "strength" in a given direction?
  - Consider a small arc ΔS centered at $\vec{v}$
  - To describe the light source's emission along $\vec{v}$ we need the fraction



$$\frac{\text{watts}}{\text{radian}} \left\{ \frac{\Delta\Phi}{\Delta\omega} \right.$$

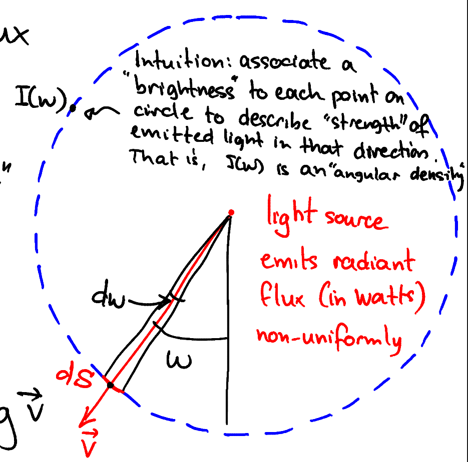
$\Delta\Phi$ ← flux "sent" through angle $\Delta\omega$

$\Delta\omega$ ← angle $\Delta\omega$

as $\Delta\omega \to 0$

Flux Along a Direction: Radiant Intensity

- Suppose source emits flux non-uniformly.
- How do we quantify the source's emission "strength" in a given direction?
  - Consider a small arc ΔS centered at $\vec{v}$
  - To describe the light source's emission along $\vec{v}$ we need the limit



$$\lim_{\Delta\omega \to 0} \frac{\Delta\Phi}{\Delta\omega} = \boxed{\frac{d\Phi}{d\omega} = I(\omega)}$$

Radiant intensity of source in direction ω $\left(\frac{\text{watts}}{\text{radian}}\right)$

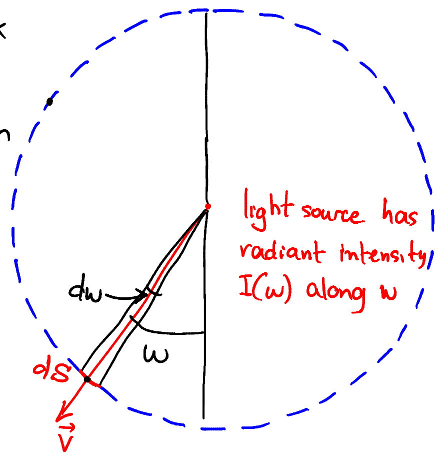
Flux Along a Direction: Radiant Intensity

- Suppose source emits flux non-uniformly.
- Its emission strength given by radiant intensity:

$$I(\omega) = \frac{d\Phi}{d\omega} \quad \left(\frac{\text{watts}}{\text{radian}}\right)$$

- Differential flux given by

$$d\Phi = d\omega \cdot I(\omega)$$



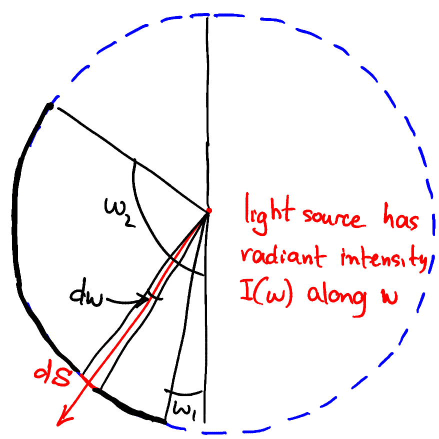
- Compare to a uniformly-emitting source of flux Φ

$$d\Phi = d\omega \cdot \frac{\Phi}{2\pi}$$

i.e. $I(\omega) = \frac{\Phi}{2\pi} = \text{const}$ for a uniform source

Flux Through a General Arc (2D, Non-Uniform)

- What is the total flux through the arc $[\omega_1, \omega_2]$?



- Differential flux given by

$$d\Phi = d\omega \cdot I(\omega)$$

$\Rightarrow$ Compute the integral over $[\omega_1, \omega_2]$

$$\boxed{\Phi_{\omega_1\omega_2} = \int_{\omega_1}^{\omega_2} d\Phi = \int_{\omega_1}^{\omega_2} I(\omega)\, d\omega}$$

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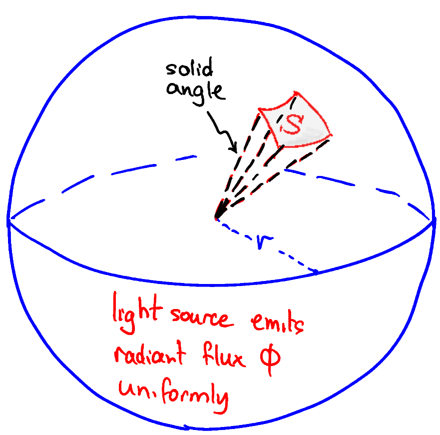
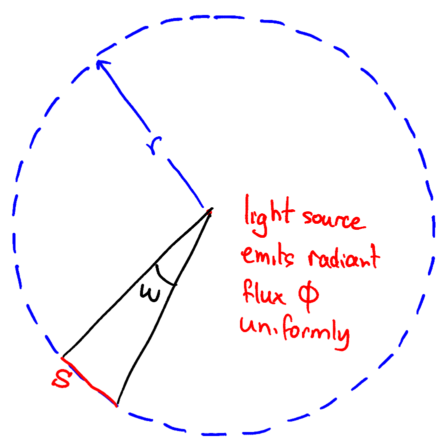
Flux Through an Arc (for 3D, Uniform Source)





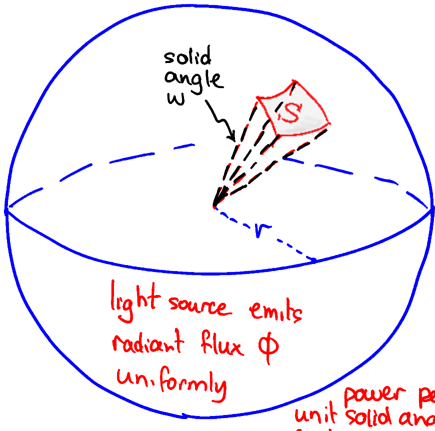
flux through S:

$$\Phi \frac{\text{area}(S)}{4\pi r^2} = \frac{\text{area}(S)}{r^2} \frac{\Phi}{4\pi}$$

flux through S:

$$\Phi \frac{\text{length}(S)}{\text{perimeter}} = \omega \cdot \frac{\Phi}{2\pi} \ (W/rad)$$

Arcs/Angles in 2D ⇔ Areas/Solid Angles in 3D



- Definition:

  Solid angle ω of a patch S' on a sphere of radius r:

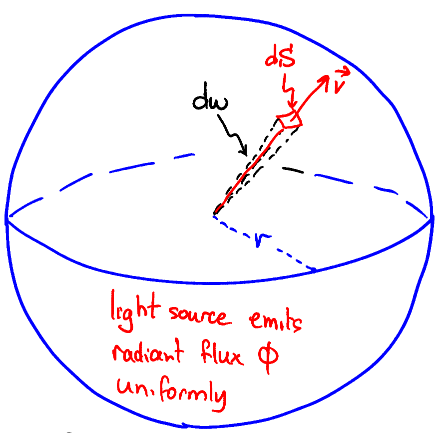
$$\boxed{\omega = \frac{\text{area}(S')}{r^2}}$$

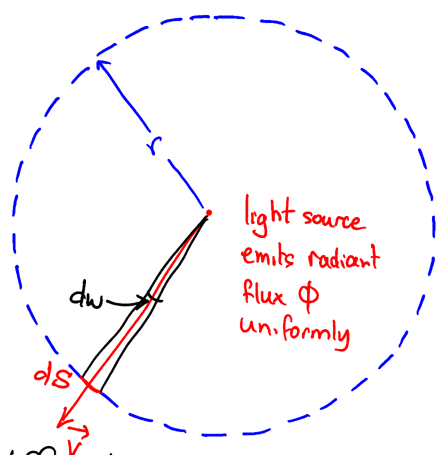
- Solid angles are measured in steradians (sr)
- The solid angle of a full sphere is 4π

flux through S':

$$\Phi \frac{\text{area}(S')}{4\pi r^2} = \underbrace{\omega}_{\text{solid angle (sr)}} \overbrace{\frac{\Phi}{4\pi}}^{\text{power per unit solid angle (W/sr)}}$$

Flux Along a Direction (for 3D, Uniform Source)





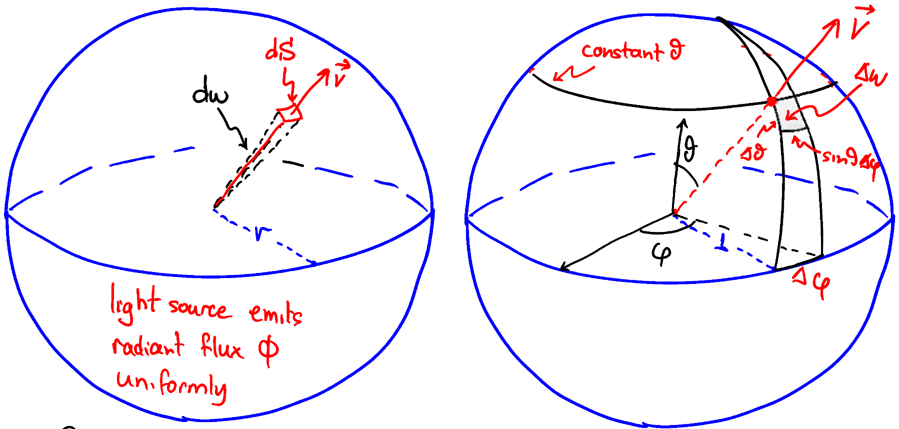
differential flux along direction $\vec{v}$:

$$d\Phi = \underbrace{d\omega}_{\text{differential solid angle}} \overbrace{\frac{\Phi}{4\pi}}^{\text{power per unit solid angle}}$$

differential flux along direction $\vec{v}$:

$$d\Phi = \underbrace{d\omega}_{\text{differential angle}} \frac{\Phi}{2\pi}$$

Differential Solid Angles ⟺ Spherical Coords



differential flux along direction $\vec{v}$:

$$d\Phi = d\omega \frac{\Phi}{4\pi}$$

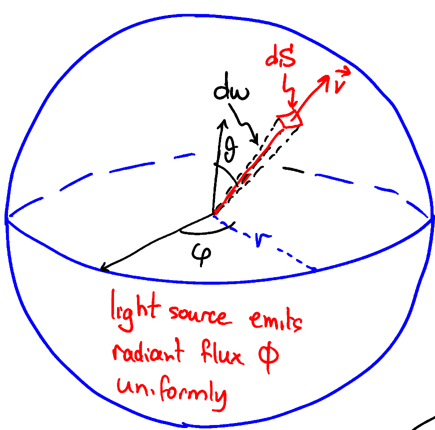
power per unit solid angle

if $v = (\sin\vartheta\cos\varphi, \sin\vartheta\sin\varphi, \cos\vartheta)$

$\text{area}(\Delta\omega) \simeq \Delta\vartheta \cdot \Delta\varphi \sin\vartheta$

when $\Delta\omega \to 0$, $\boxed{d\omega = d\vartheta\, d\varphi \sin\vartheta}$

Flux Along a Direction (for 3D, Uniform Source)



Differential flux along direction (ϑ, φ) for a source emitting uniformly in all directions:

$$\boxed{d\Phi = d\vartheta\, d\varphi \sin\vartheta \frac{\Phi}{4\pi}}$$

combining boxed expressions

differential flux along direction $\vec{v}$:

$$\boxed{d\Phi = d\omega \frac{\Phi}{4\pi}}$$

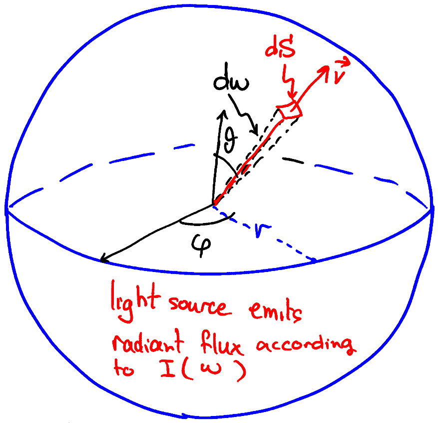
power per unit solid angle

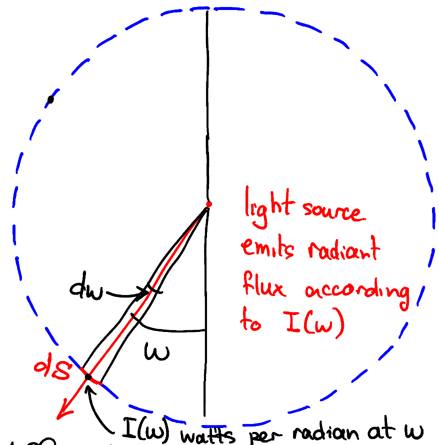
if $v = (\sin\vartheta\cos\varphi, \sin\vartheta\sin\varphi, \cos\vartheta)$

$\text{area}(\Delta\omega) \simeq \Delta\vartheta \cdot \Delta\varphi \sin\vartheta$

when $\Delta\omega \to 0$, $\boxed{d\omega = d\vartheta\, d\varphi \sin\vartheta}$

Radiant Intensity (for 3D, Non-Uniform Src)





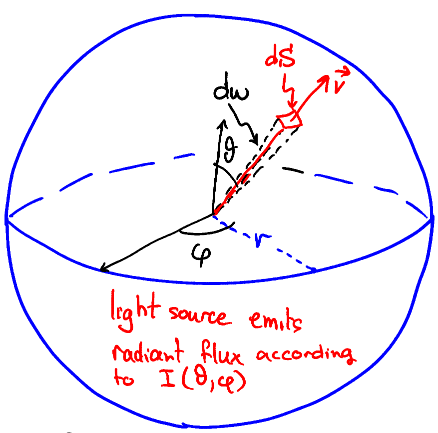
differential flux along direction $\vec{v}$:

$d\phi = d\omega \, I(\omega)$ Radiant intensity (watts/steradian)

differential flux along direction $\vec{v}$:

$d\phi = d\omega \cdot I(\omega)$

Radiant Intensity (for 3D, Non-Uniform Src)



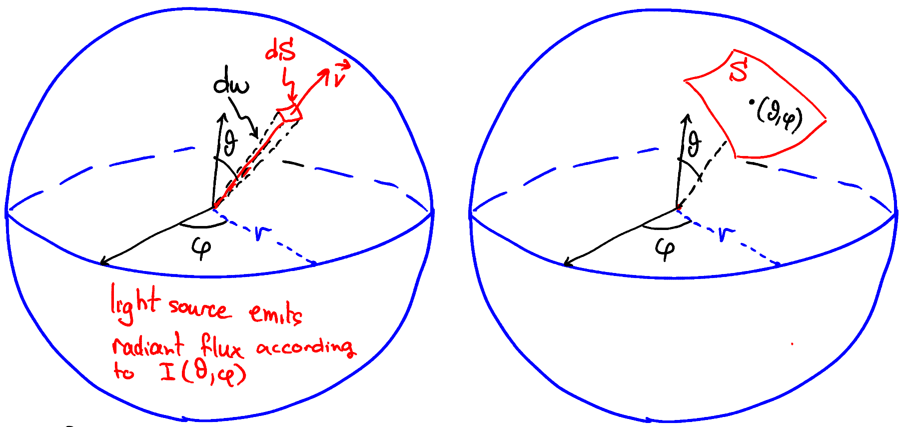
differential flux along direction $\vec{v}$:

$d\phi = d\omega \, I(\omega)$

- $I(\omega)$ is called radiant intensity (flux along specific direction)
- Measured using W/sr
- Can also be written as a function of θ, φ:
  $I(\theta, \varphi)$
- differential flux along (θ, φ)

$$\boxed{d\phi = d\theta \, d\varphi \sin\theta \, I(\theta, \varphi)}$$

Flux Through a General Patch (3D, Non-Uniform)



differential flux along direction $\vec{v}$:

$$d\Phi = d\vartheta \, d\varphi \sin\vartheta \, I(\vartheta,\varphi)$$

total flux through region S:

$$\Phi_S = \iint_{(\vartheta,\varphi)\in S} \sin\vartheta \, I(\vartheta,\varphi) \, d\vartheta \, d\varphi$$

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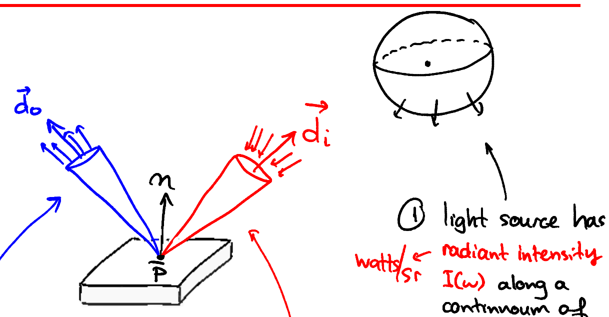
The Basic "Light Transport" Path



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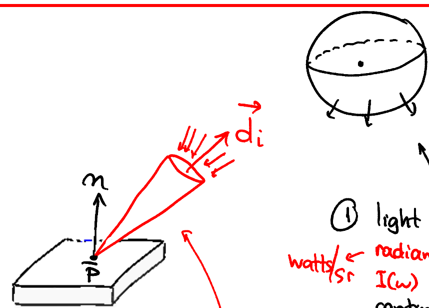
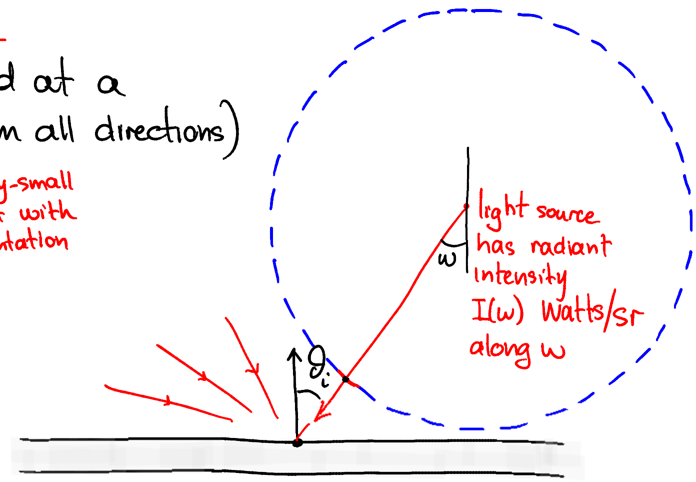


Measuring Incident Light: Irradiance

Irradiance H

Flux received at a ~~point~~ (from all directions)

= an infinitesimally-small surface element with a specific orientation



Definition of Irradiance (for "small patches")

Irradiance H

Flux received per unit area:

$$H = \frac{\text{flux received (all directions)}}{\text{area of patch}}$$

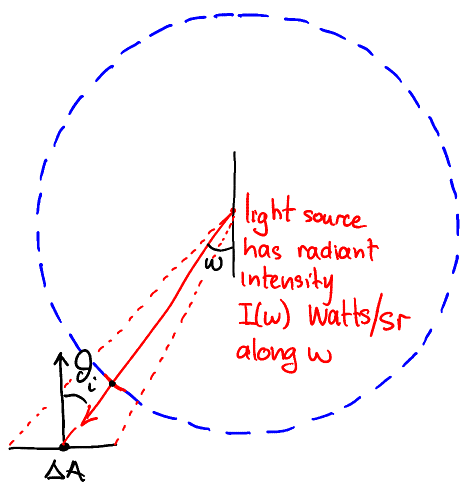
(measured in Watts/m²)

For small patches

$$H = \frac{\Delta \Phi}{\Delta A}$$

total flux received from all directions

area of patch



Definition of Irradiance (for differential areas)

Irradiance H

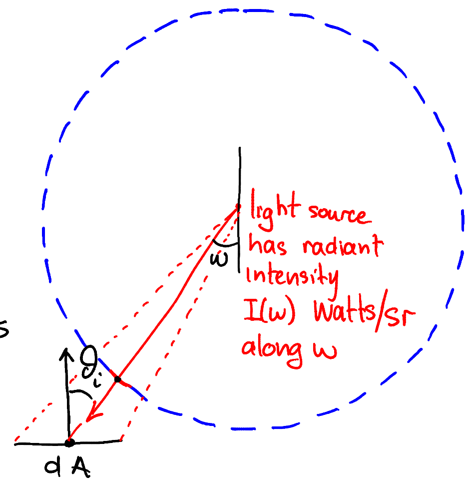
Flux received per unit area:

$$H = \frac{\text{flux received (all directions)}}{\text{area of patch}}$$

(measured in watts/m^2)

For infinitesimal patches

$$H = \lim_{\Delta A \to 0} \frac{\Delta \Phi}{\Delta A} = \frac{d\Phi}{dA}$$



Computing Irradiance: Normal Incidence

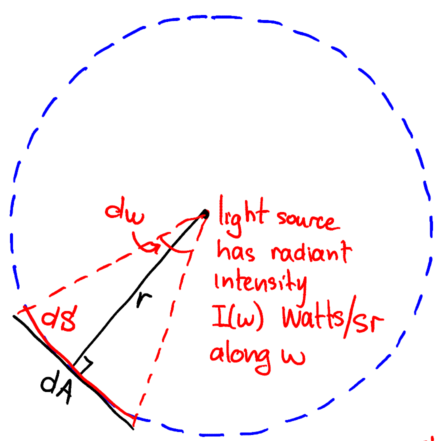
Irradiance H

Flux received per unit area:

$$H = \frac{\text{flux received (all directions)}}{\text{area of patch}}$$

(measured in watts/m^2)

Example 1: Calculate the irradiance at a planar patch dA that faces the source and is distance r away from it



$$\boxed{H = \frac{d\Phi}{dA} = \frac{I(\omega)}{r^2}}$$

- let $d\Phi$ be the flux through dA
- $d\Phi$ = flux through dS = $d\omega \cdot I(\omega) = \frac{dS}{r^2} \cdot I(\omega)$
- for infinitesimal patches $dA \simeq dS \Rightarrow d\Phi = \frac{dA}{r^2} I(\omega)$

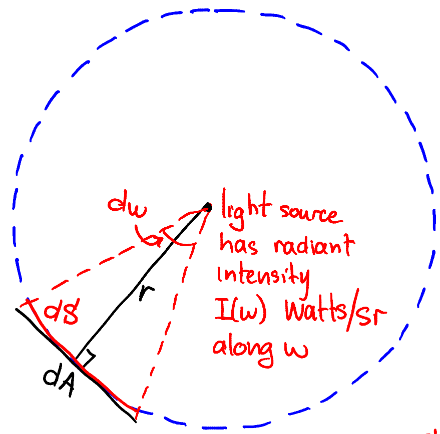
Computing Irradiance: Normal Incidence

- Irradiance H

Flux received per unit area:

$$H = \frac{\text{flux received (all directions)}}{\text{area of patch}}$$

(measured in watts/m^2)



Example 1: Calculate the irradiance at a planar patch dA that faces the source and is distance r away from it

$$\boxed{H = \frac{d\Phi}{dA} = \frac{I(\omega)}{r^2}}$$

⇒ Irradiance decreases quadratically with distance ("squared-distance fall-off")

⇒ the farther the patch is, the less light it gets

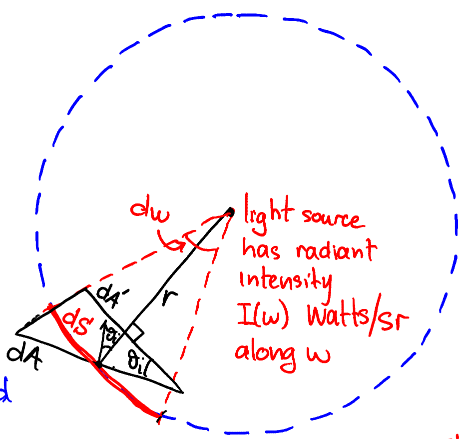
Computing Irradiance: Tilted Patches

- Irradiance H

Flux received per unit area:

$$H = \frac{\text{flux received (all directions)}}{\text{area of patch}}$$

(measured in watts/m^2)



Example 2: Calculate the irradiance at a planar patch dA at angle θ with source and distance r away from it

$$\boxed{H \neq \frac{d\Phi}{dA'} = \frac{I(\omega)}{r^2}}$$

- define dA' to be the patch at distance r that faces the light source
- for infinitesimal patches, $dA' \simeq dA\cos\theta$ ⇒ $\boxed{H = \frac{d\Phi}{dA} = \frac{I(\omega)\cos\theta_i}{r^2}}$

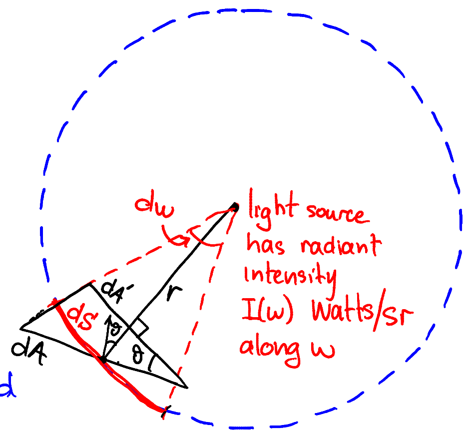
Computing Irradiance: Foreshortening

- Irradiance H

  Flux received per unit area:

  $$H = \frac{\text{flux received (all directions)}}{\text{area of patch}}$$

  (measured in Watts/m^2)



Example 2: Calculate the irradiance at a planar patch dA at angle θ with source and distance r away from it

The foreshortening effect:
patches tilted relative to the source receive less light per unit area

$$\boxed{H = \frac{d\Phi}{dA} = \frac{I(\omega)}{r^2}\cos\theta}$$

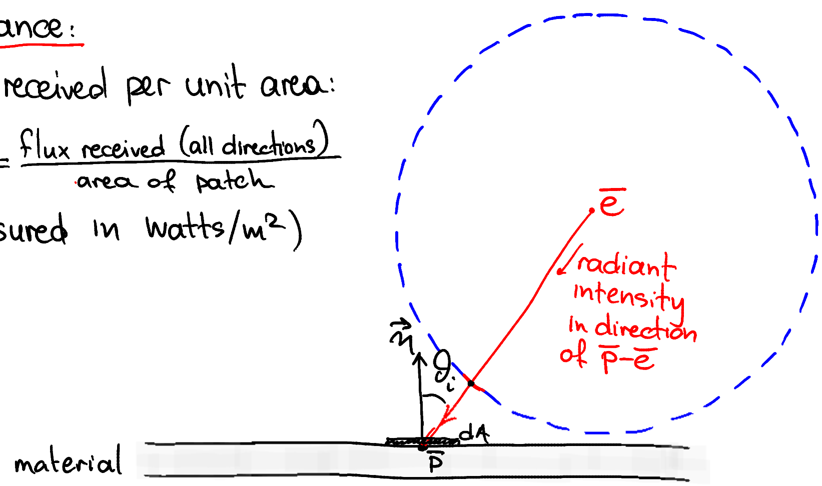
Example: Irradiance due to Point Light Source

Irradiance:

Flux received per unit area:

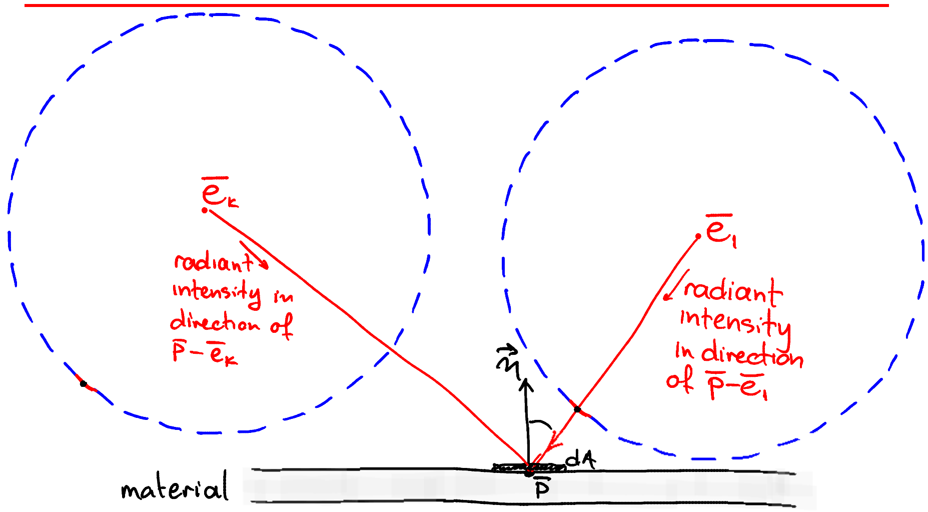
$$H = \frac{\text{flux received (all directions)}}{\text{area of patch}}$$

(measured in Watts/m^2)



Irradiance at $\bar{p}$ (one light): $H(\bar{p}) = \dfrac{I(\bar{p}-\bar{e})}{\|\bar{p}-\bar{e}\|^2}\, \vec{n}\cdot\dfrac{(\bar{p}-\bar{e})}{\|\bar{p}-\bar{e}\|}$

Example: Irradiance due to Multiple Sources



Irradiance at $\bar{p}$ (K lights): $H(\bar{p}) = \sum_{k=1}^{K} \frac{I(\bar{p}-\bar{e}_k)}{\|\bar{p}-\bar{e}_k\|^2} \, \vec{n} \cdot \frac{(\bar{p}-\bar{e}_k)}{\|\bar{p}-\bar{e}_k\|}$

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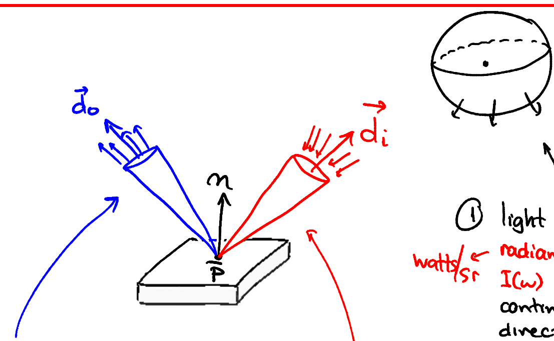
- **Measuring light leaving a patch: Radiance**
- The Light Transport Cycle
- The Bidirectional Reflectance Distribution Function

The Basic "Light Transport" Path



① light source has radiant intensity $I(\omega)$ along a continuum of directions

watts/sr

② irradiance $H(\bar{p})$ at an infinitesimal patch due to light received along one or more (or a continuum of) directions

watts/m^2

③ light is reflected from a point to a continuous "cone" of directions

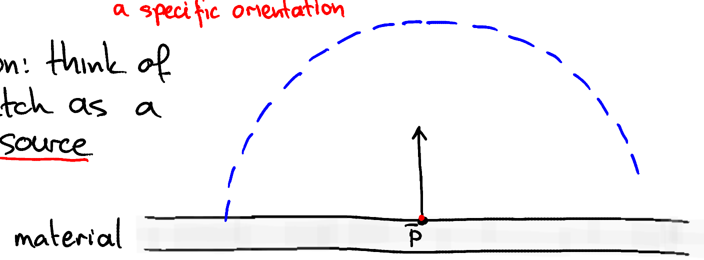
Measuring Outgoing Light: Radiance

Surface Radiance $L(\bar{p}, \vec{d}_o)$

Flux emitted in a particular direction by a ~~surface point~~

an infinitesimally-small surface element with a specific orientation

Intuition: think of the patch as a light source

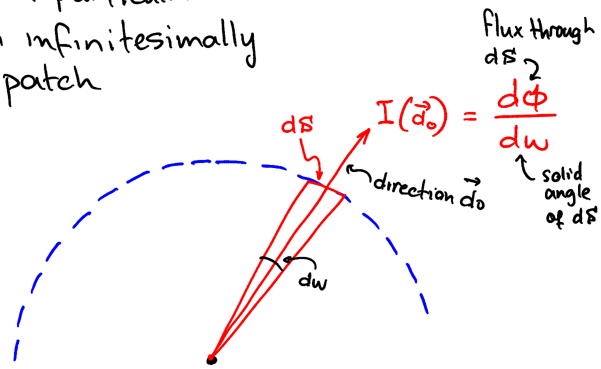


Defining Radiance: Basic Intuition

Surface Radiance $L(\bar{p}, \vec{d}_o)$

Flux emitted in a particular direction by an infinitesimally small surface patch

Intuition: think of the patch as a light source



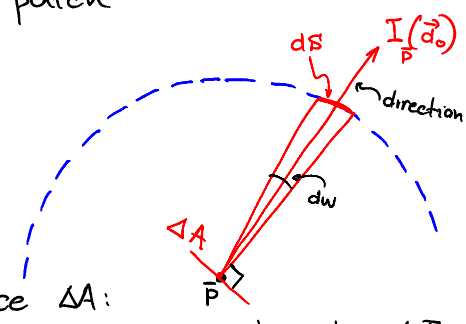
When the light source is a point its emission is quantified using its radiant intensity $I(\vec{d}_o)$

Defining Radiance: Basic Intuition

Surface Radiance $L(\bar{p}, \vec{d}_o)$

Flux emitted in a particular direction by an infinitesimally small surface patch

Intuition: think of the patch as a light source



For a patch source ΔA:

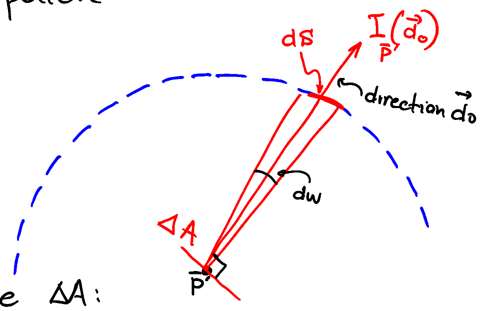
- Measure total radiant intensity ΔI through dS due to ΔA

Defining Radiance: Basic Intuition

Surface Radiance $L(\vec{p}, \vec{d}_o)$

Flux emitted in a particular direction by an infinitesimally small surface patch

Intuition: think of the patch as a light source



For a patch source ΔA:

- Measure total radiant intensity ΔI through dS due to ΔA

Defining Radiance: Basic Intuition

Surface Radiance $L(\vec{p}, \vec{d}_o)$

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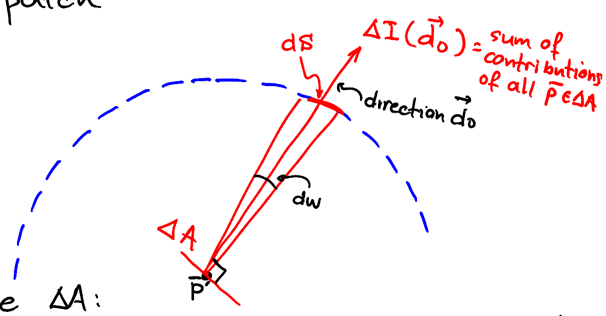


For a patch source ΔA:

- Measure total radiant intensity ΔI through dS due to ΔA

- Divide by the area of the patch

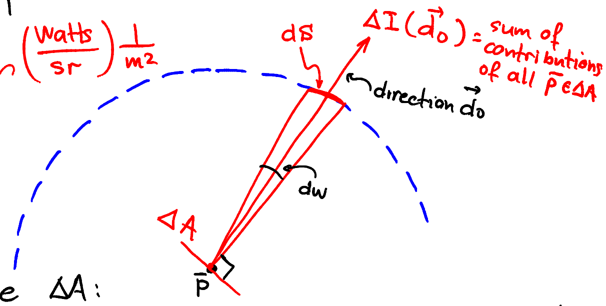
Defining Radiance: Basic Intuition

Surface Radiance $L(\bar{p}, \vec{d}_o)$

Flux emitted in a particular direction by an infinitesimally small surface patch

$$L(\bar{p}, \vec{d}_o) = \lim_{\Delta A \to 0} \frac{\Delta I(\vec{d}_o)}{\Delta A}$$

$\left(\frac{\text{Watts}}{\text{sr}}\right)\frac{1}{m^2}$



For a patch source ΔA:

- Measure total radiant intensity ΔI through dS due to ΔA
- Divide by the area of the patch (and take limit)

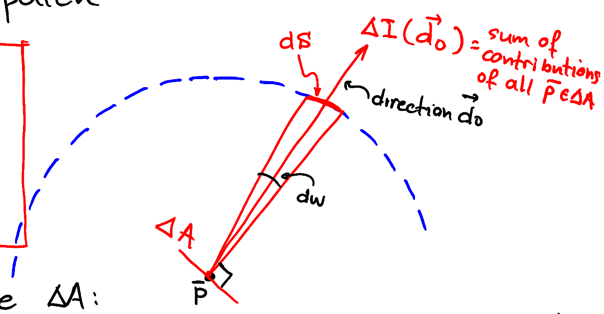
Definition of Radiance

Surface Radiance $L(\bar{p}, \vec{d}_o)$

Flux emitted in a particular direction by an infinitesimally small surface patch

$$\boxed{L(\bar{p}, \vec{d}_o) = \frac{dI}{dA} = \frac{d}{dA}\left(\frac{d\Phi}{d\omega}\right) = \frac{d^2\Phi}{dA\,d\omega}}$$

(in Watts/sr·m²)



For a patch source ΔA:

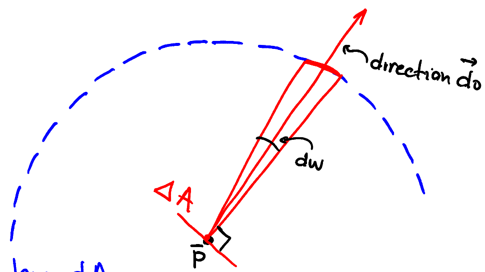
- Measure total radiant intensity ΔI through dS due to ΔA
- Divide by the area of the patch

Definition of Radiance: Normal Exitance

Surface Radiance $L(\bar{p}, \vec{d}_o)$

Flux emitted in a particular direction by an infinitesimally small surface patch

$$L(\bar{p}, \vec{d}_o) = \frac{dI}{dA} = \frac{d}{dA}\left(\frac{d\Phi}{d\omega}\right) = \frac{d^2\Phi}{dA\,d\omega}$$



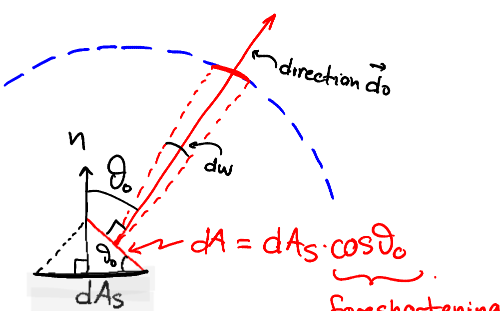
Attention: Division by dA assumes that patch is perpendicular to emission direction $\vec{d}_o$

Definition of Radiance for a Tilted Patch

Surface Radiance $L(\bar{p}, \vec{d}_o)$

Flux emitted in a particular direction by an infinitesimally small surface patch

$$L(\bar{p}, \vec{d}_o) = \frac{dI}{dA} = \frac{d^2\Phi}{dA\,d\omega}$$



dA: patch orthogonal to $\vec{d}_o$

dA_S: tilted surface patch

Definition of Radiance for a Tilted Patch

Surface Radiance for a tilted patch

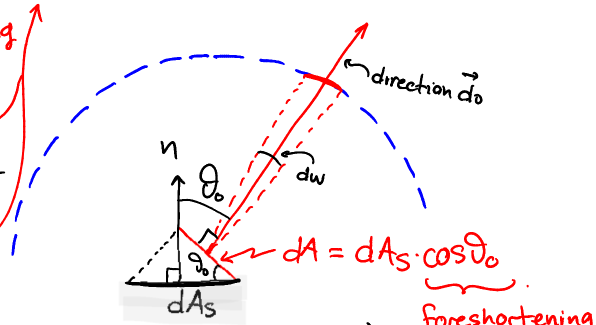
$$\frac{d^2\Phi}{dA_s d\omega}(\bar{p},\vec{d}_o) = \cos\vartheta_o \cdot L(\bar{p},\vec{d}_o)$$
$$= (\vec{n}\cdot\vec{d}_o)\, L(\bar{p},\vec{d}_o)$$

combining (*), (**)

So we have:

(*) $L(\bar{p},\vec{d}_o) = \dfrac{d^2\Phi}{dA\, d\omega}$

(**) $dA_s = \dfrac{1}{\cos\vartheta_o} \cdot dA$



dA: patch orthogonal to $\vec{d}_o$

dA_s: tilted surface patch

Measuring All Outgoing Light: Radiant Exitance

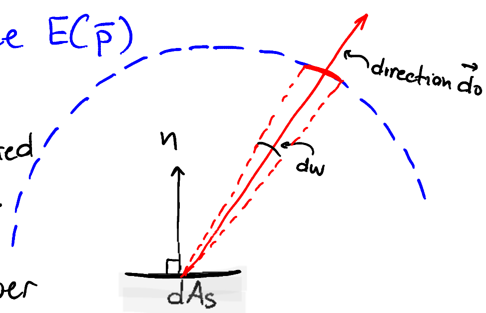
Surface Radiance for a tilted patch

$$\frac{d^2\Phi}{dA_s d\omega}(\bar{p},\vec{d}_o) = (\vec{n}\cdot\vec{d}_o)\, L(\bar{p},\vec{d}_o)$$

Radiant exitance $E(\bar{p})$ (aka Radiocity)

Total flux emitted from dA_s in all directions

$\Rightarrow$ an integral over directions:



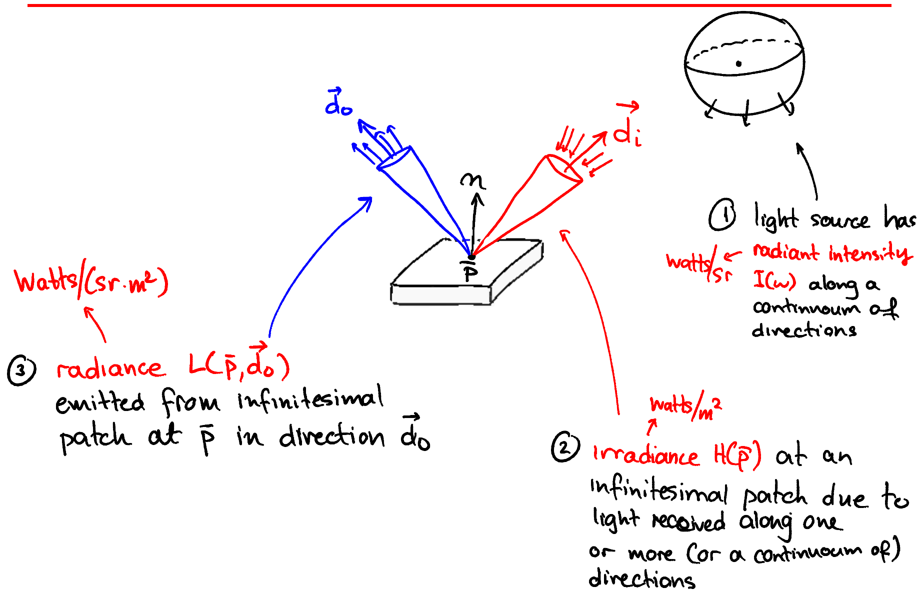
$$E(\bar{p}) = \int_{\vec{d}_o} (\vec{n}\cdot\vec{d}_o)\, L(\bar{p},\vec{d}_o)\, d(\vec{d}_o)$$

Topic 13:

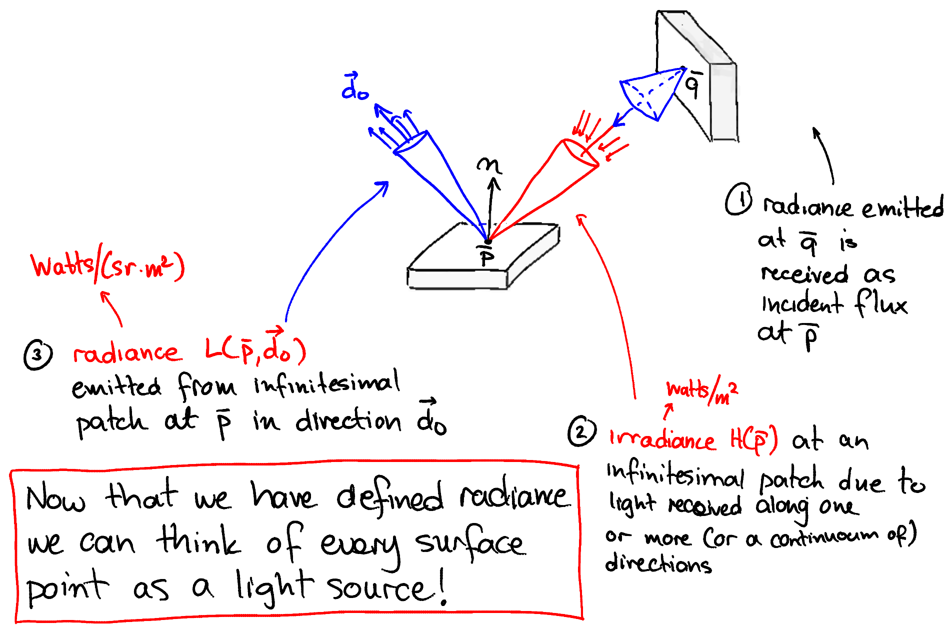
Radiometry

- The big picture
- Measuring light coming from a light source
- Measuring light falling onto a patch: Irradiance
- Measuring light leaving a patch: Radiance
- **The Light Transport Cycle**
- The Bidirectional Reflectance Distribution Function

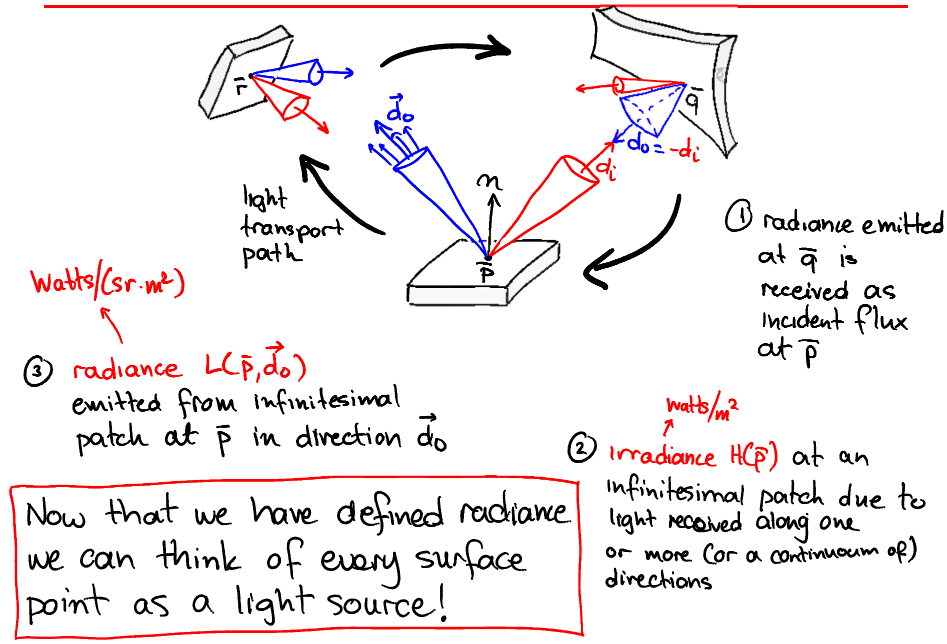
The Basic "Light Transport" Path



Light Transport Between Patches



The General Light Transport Cycle



One Step Along Path: Directional Integration

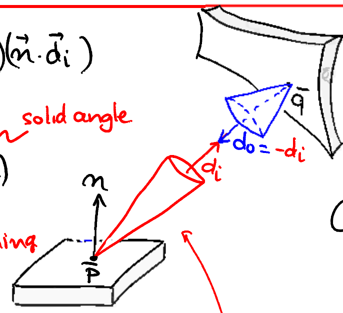
$$H(\bar{p}) = \int_{\text{all directions}} (\text{radiance along } -d_i)(\vec{n}\cdot\vec{d}_i)$$

$$= \int_{\vec{d}_i} L(\bar{q}, -d_i)\,(\vec{n}\cdot\vec{d}_i)\,d(d_i)$$

solid angle

radiance travelling to $\bar{p}$ from $\bar{q}$

foreshortening in case patch at $\bar{p}$ is slanted



① radiance emitted at $\bar{q}$ is received as incident flux at $\bar{p}$

② irradiance $H(\bar{p})$ at an infinitesimal patch due to light received along one or more (or a continuum of) directions

watts/m^2

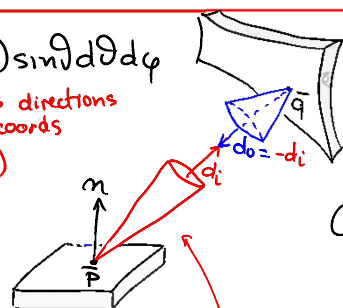
Now that we have defined radiance we can think of every surface point as a light source!

One Step Along Path: Directional Integration

$$H(\bar{p}) = \int_{\varphi}\int_{\vartheta} L(\bar{p}, -d_i)(\vec{n}\cdot\vec{d}_i)\sin\vartheta\, d\vartheta\, d\varphi$$

if we express directions in spherical coords

$(\cos\varphi\sin\vartheta, \sin\varphi\sin\vartheta, \cos\vartheta)$



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Now that we have defined radiance we can think of every surface point as a light source!