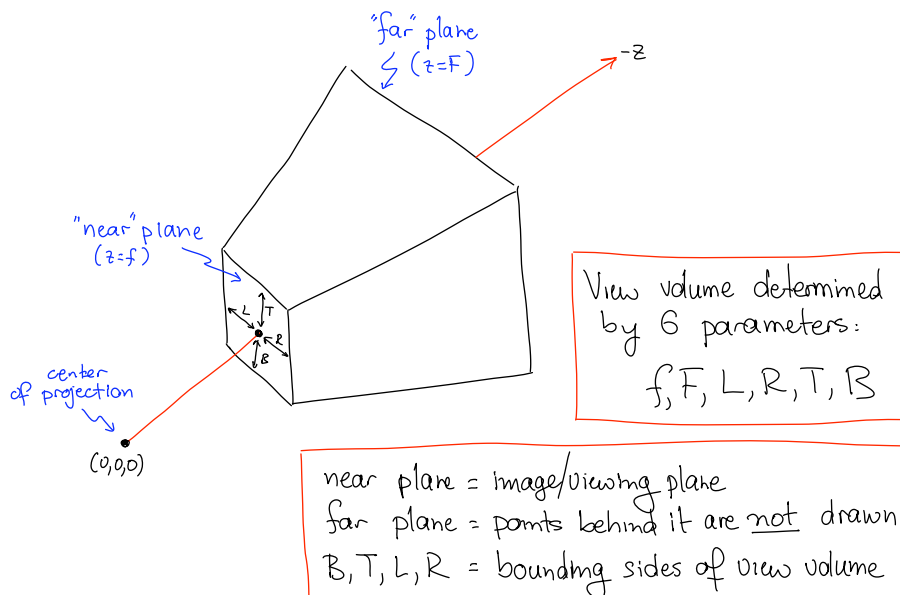


Polygon Clipping

Goal: Remove points and parts of objects outside the view volume

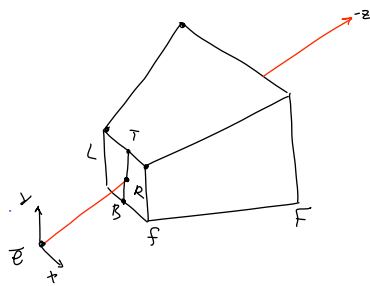
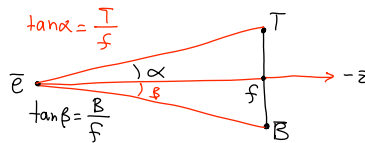
Clipping is especially important when objects/scenes contain large #'s of polygons (most of which are not in the field of view)

The View Volume



The Field of View

The parameters L, R, T, B control which 3D points are within the field of view (FOV)



- angular FOV = $\alpha + \beta$
- for fixed T, B :
 small $f \Rightarrow$ wide FOV
 large $f \Rightarrow$ small FOV

Transformation Chain for 3D Viewing

(complete)

Object-to-world transformation (M_{ow}) $\begin{bmatrix} x_w \\ y_w \\ z_w \\ 1 \end{bmatrix} = \begin{bmatrix} R_{ow} & e_{ow} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_o \\ y_o \\ z_o \\ 1 \end{bmatrix}$ a 4x4 matrix that maps object-centered coords to world-centered coords

World-to-camera transformation (M_{wc}) $\begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix} = \begin{bmatrix} R_{wc} & e_{wc} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_w \\ y_w \\ z_w \\ 1 \end{bmatrix}$ a 4x4 matrix that maps world-centered coords to camera-centered coords

Camera-to-canonical/view transformation (M_{cv}) $\begin{bmatrix} x_v \\ y_v \\ z_v \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & f \end{bmatrix} \begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix}$ a 4x4 matrix that maps camera-centered coords to canonical/view coords

Canonical view-to-image transformation (M_{vi}) (a.k.a. projection) $\begin{bmatrix} x_i \\ y_i \\ z_i \\ 1 \end{bmatrix} \approx \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_v \\ y_v \\ z_v \\ 1 \end{bmatrix}$ a 4x4 matrix that maps canonical/view 3D coords to 2D image coords

Orthographic projection matrix

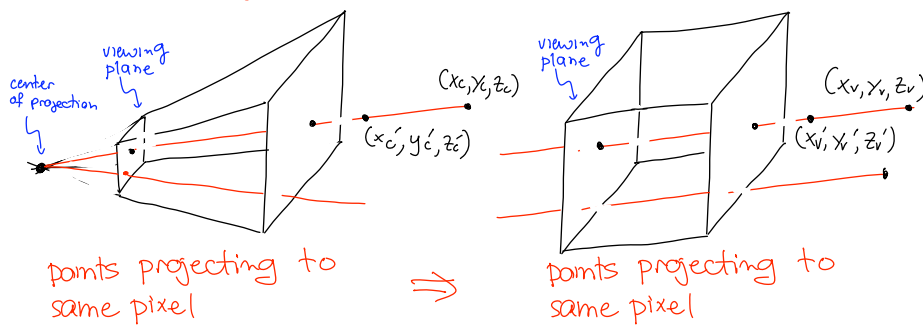
The Canonical View Volume Transform

$$\begin{bmatrix} x_v \\ y_v \\ z_v \\ 1 \end{bmatrix} \stackrel{M_{cv}}{\approx} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{1}{f} & 0 \end{bmatrix} \begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix}$$

rays converging to center of projection



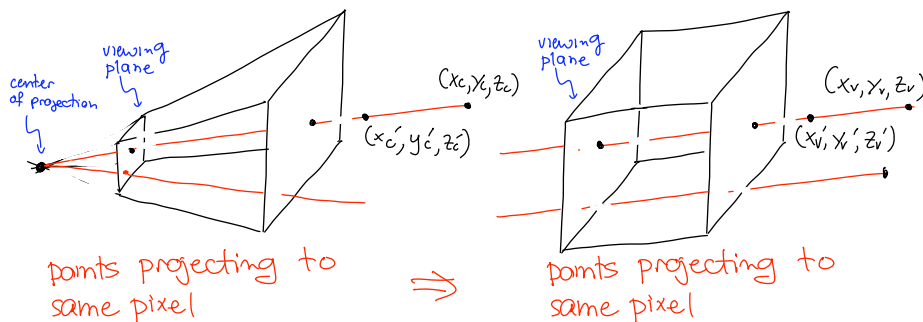
rays parallel to z-axis



The Canonical View Volume Transform

Why do we care about transforming the viewing "cone" of lines of sight into a "cube" where lines of sight are parallel?

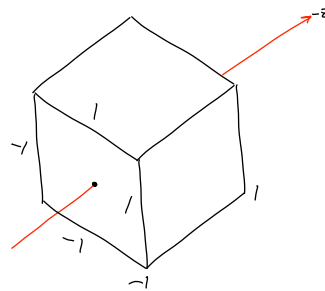
Ans: Clipping & visibility computation become much simpler



Clipping in the Canonical View Volume

Clipping rule very simple when done in a canonical view volume shaped like a CUBE whose faces are on planes $x = \pm 1$, $y = \pm 1$, $z = \pm 1$

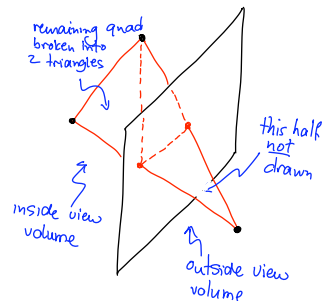
- Triangles whose edges lie outside the canonical view volume are not drawn
- Triangles whose edges cross the plane(s) $x = \pm 1$, $y = \pm 1$, $z = \pm 1$ must be clipped



Clipping in the Canonical View Volume

Clipping rule very simple when done in a canonical view volume shaped like a CUBE whose faces are on planes $x = \pm 1$, $y = \pm 1$, $z = \pm 1$

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Clipping in the Canonical View Volume

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- Triangles whose edges cross the plane(s) $x = \pm 1$, $y = \pm 1$, $z = \pm 1$ must be clipped

see Shirley
Ch. 12.1 - 12.2

Topic 8:

Visibility

- Elementary visibility computations:
 - Clipping
 - "Shaping" the canonical view volume
 - Backface culling
- Algorithms for visibility determination
 - Z-Buffering
 - Painter's algorithm
 - BSP Trees

"Shaping" the Canonical View Volume

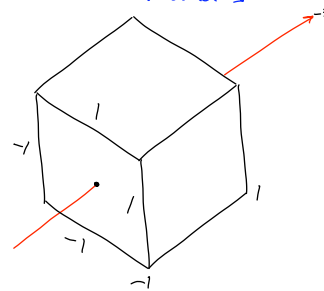
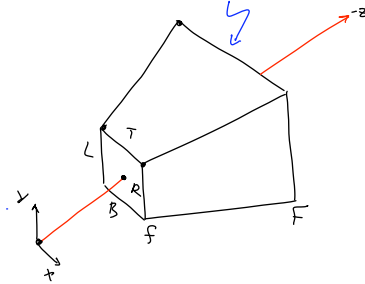
Q: What should the canonical view transform be to map the view volume onto a cube?

$$\begin{bmatrix} x_v \\ y_v \\ z_v \\ 1 \end{bmatrix} \approx \begin{bmatrix} ? & 0 & ? & 0 \\ 0 & ? & ? & 0 \\ 0 & 0 & ? & ? \\ 0 & 0 & 1/f & 0 \end{bmatrix} \begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix}$$

if point (x_c, y_c, z_c) lies inside this view volume



its (x, y, z) coordinates after the transform will be between -1 and 1



"Shaping" the Transformed z-Coordinates

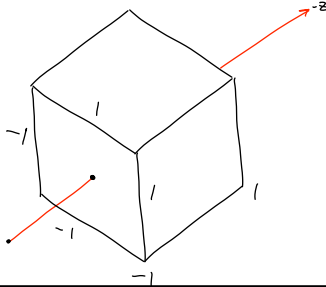
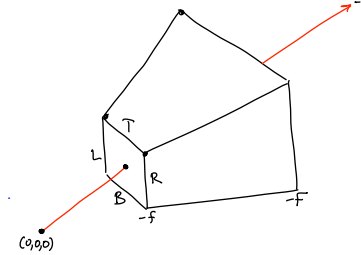
Q: What should the canonical view transform be to map the view volume onto a cube?

$$\begin{bmatrix} z_v \\ 1 \end{bmatrix} \approx \begin{bmatrix} az_c + b \\ z_c/f \end{bmatrix} = \begin{matrix} z_v \text{ is independent} \\ \text{of } x_c, y_c \\ \hline 0 & 0 & a & b \\ 0 & 0 & 1/f & 0 \end{matrix} \begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix}$$

we have two constraint eqs: $\left. \begin{matrix} b = af + 1 \\ F = afF - fb \end{matrix} \right\} \Rightarrow$
 $\left. \begin{matrix} -1 = af - b \\ 1 = \frac{f}{F}(af - b) \end{matrix} \right\} \begin{matrix} F = afF - af^2 - f \\ f + F = af(F - f) \end{matrix}$

$z_v = \frac{f}{z_c} (az_c + b)$

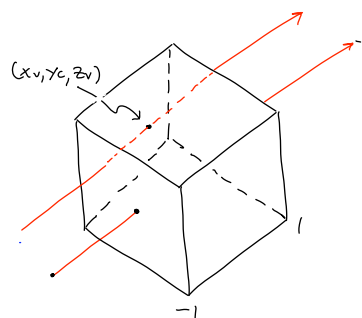
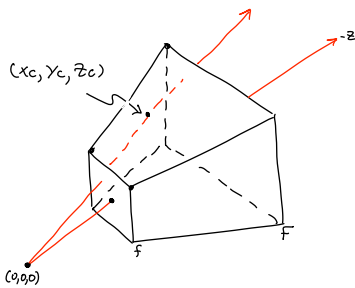
$a = \frac{(f+F)}{f(F-f)} \quad b = \frac{2F}{F-f}$



The Pseudo-Depth of a 3D Point

Q: What should the canonical view transform be to map the view volume onto a cube?

called the "pseudo-depth" \rightarrow
$$\begin{bmatrix} z_v \\ 1 \end{bmatrix} \approx \begin{bmatrix} 0 & 0 & \frac{f+F}{f(F-f)} & \frac{zF}{F-f} \\ 0 & 0 & \frac{1}{f} & 0 \end{bmatrix} \begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix}$$
 "true depth" of the point \leftarrow

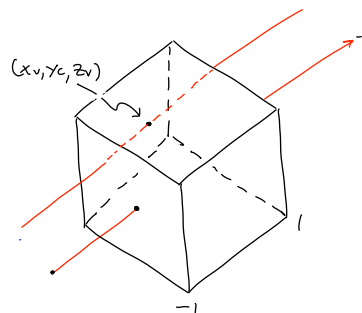
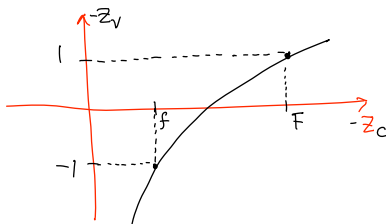


The Pseudo-Depth of a 3D Point

Q: What should the canonical view transform be to map the view volume onto a cube?

called the "pseudo-depth" \rightarrow
$$\begin{bmatrix} z_v \\ 1 \end{bmatrix} \approx \begin{bmatrix} 0 & 0 & \frac{f+F}{f(F-f)} & \frac{zF}{F-f} \\ 0 & 0 & \frac{1}{f} & 0 \end{bmatrix} \begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix}$$
 "true depth" of the point \leftarrow

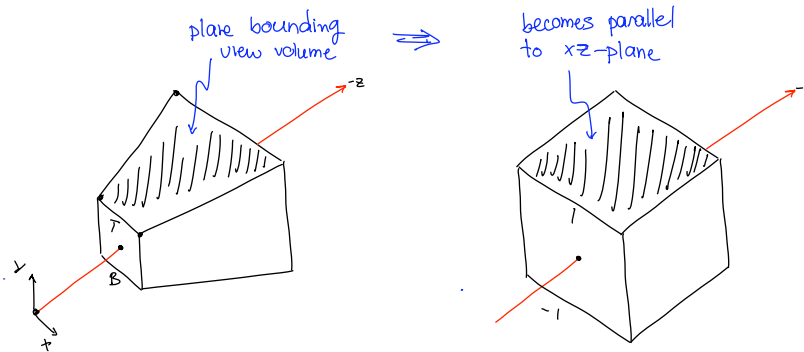
Key property of pseudo-depth:
it is monotonic with z_c



"Shaping" the Transformed y-Coordinates

Q: What should the canonical view transform be to map the view volume onto a cube?

$$\begin{bmatrix} y_v \\ z_v \\ 1 \end{bmatrix} \approx \begin{bmatrix} ay_c + bz_c \\ z_c/f \end{bmatrix} = \begin{bmatrix} 0 & a & b & 0 \\ 0 & 0 & f/f & z_c/f \\ 0 & 0 & f(f-f)/f & f-f \end{bmatrix} \begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix}$$



"Shaping" the Transformed y-Coordinates

Q: What should the canonical view transform be to map the view volume onto a cube?

$$\begin{bmatrix} y_v \\ z_v \\ 1 \end{bmatrix} \approx \begin{bmatrix} ay_c + bz_c \\ z_c/f \end{bmatrix} = \begin{bmatrix} 0 & a & b & 0 \\ 0 & 0 & f/f & z_c/f \\ 0 & 0 & f(f-f)/f & f-f \end{bmatrix} \begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix}$$

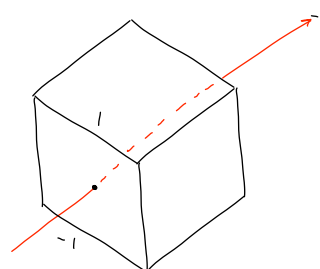
we have 2 constraints:
 $(aT + bz_c)f/z_c = 1$
 $(aB + bz_c)f/z_c = -1$

Consider the point $(0, T, -f)$:

$$\left. \begin{aligned} (aT - bf)(f)/f &= 1 \\ (aB - bf)(f)/f &= -1 \end{aligned} \right\} \Rightarrow$$

$$\left. \begin{aligned} aT - bf &= 1 \\ aB - bf &= -1 \end{aligned} \right\} \xrightarrow{\text{subtract}} \Rightarrow$$

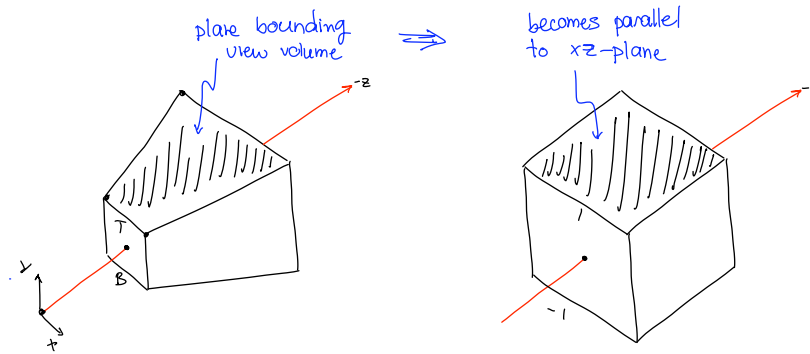
$$aT - aB = 2 \Rightarrow \boxed{a = \frac{2}{B-T}} \quad b = \frac{aT+1}{f} \Rightarrow \boxed{b = \frac{B+T}{f(B-T)}}$$



"Shaping" the Transformed y-Coordinates

Q: What should the canonical view transform be to map the view volume onto a cube?

$$\begin{bmatrix} y_v \\ z_v \\ 1 \end{bmatrix} \approx \begin{bmatrix} 0 & \frac{z}{B-T} & \frac{BFT}{f(B-T)} & 0 \\ 0 & 0 & \frac{f(F-f)}{f(F-f)} & \frac{zF}{F-f} \\ 0 & 0 & \frac{1}{f} & 0 \end{bmatrix} \begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix}$$

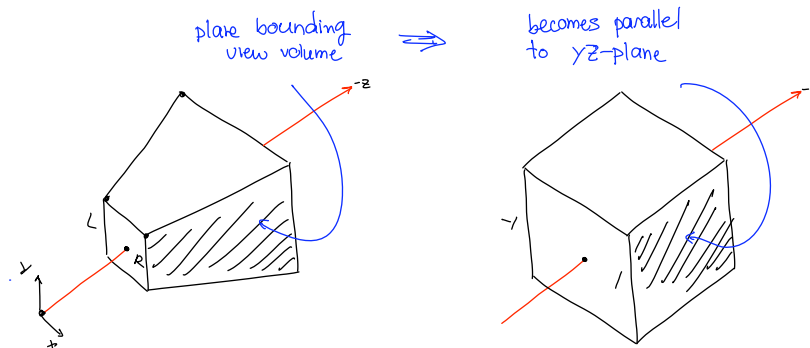


"Shaping" the Transformed x-Coordinates

Q: What should the canonical view transform be to map the view volume onto a cube?

$$\begin{bmatrix} x_v \\ y_v \\ z_v \\ 1 \end{bmatrix} \approx \begin{bmatrix} \frac{z}{R-L} & 0 & \frac{R+L}{f(R-L)} & 0 \\ 0 & \frac{z}{B-T} & \frac{BFT}{f(B-T)} & 0 \\ 0 & 0 & \frac{f(F-f)}{f(F-f)} & \frac{zF}{F-f} \\ 0 & 0 & \frac{1}{f} & 0 \end{bmatrix} \begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix}$$

1st row computed exactly the same way as for the scaling along y



The Full Canonical View Transformation

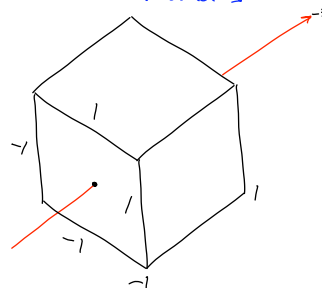
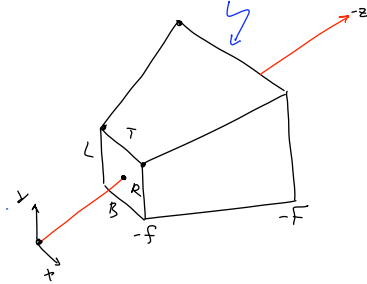
Q: What should the canonical view transform be to map the view volume onto a cube?

$$\begin{bmatrix} x_v \\ y_v \\ z_v \\ 1 \end{bmatrix} \approx \begin{bmatrix} \frac{z}{L-R} & \frac{L+F}{f(L-F)} & 0 \\ 0 & \frac{B-T}{f(B-T)} & 0 \\ 0 & \frac{f+F}{f(F-f)} & \frac{2F}{F-f} \\ 0 & 0 & \frac{1}{f} & 0 \end{bmatrix} \begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix} \xrightarrow{\text{mult by } \frac{f}{f}} \begin{bmatrix} \frac{zf}{L-R} & \frac{R+L}{L-R} & 0 \\ 0 & \frac{zf}{B-T} & \frac{B+T}{B-T} & 0 \\ 0 & 0 & \frac{zf+F}{F-f} & \frac{2zf}{F-f} \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix}$$

if point (x_c, y_c, z_c) lies
inside this view volume



its (x, y, z) coordinates
after the transform will be
between -1 and 1



Topic 8:

Visibility

- Elementary visibility computations:
 - Clipping
 - "Shaping" the canonical view volume
 - Backface culling
- Algorithms for visibility determination
 - Z-Buffering
 - Painter's algorithm
 - BSP Trees

Less is More...

Clipping is a rudimentary form of the following principle:

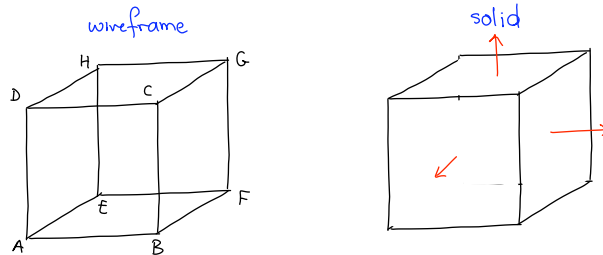
Don't spend cycles drawing what you don't have to
(= whatever will not contribute to final image)

Two other ways this is applied:

- . Backface culling
- . Visibility determination

Backface Culling

Goal: Remove surface patches that point away from the camera (i.e. the backfacing patches)

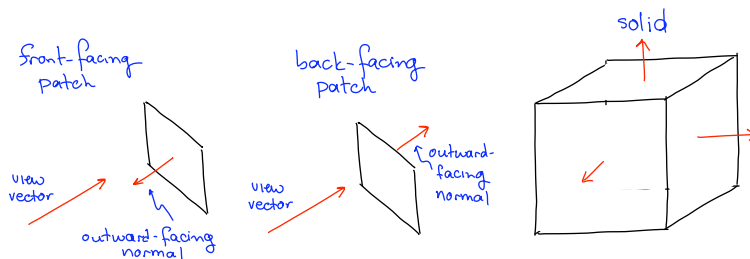


Backfacing faces on the wireframe:

ADHE, EHGF, AEFB
↑ will not be drawn

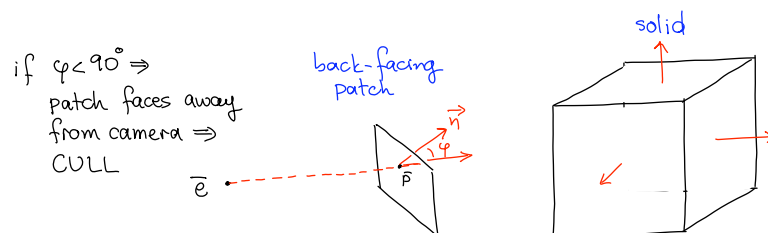
Front-facing vs. Back-facing Polygons

Goal: Remove surface patches that point away from the camera (i.e. the backfacing patches)



Backface Culling Criterion

Goal: Remove surface patches that point away from the camera (i.e. the backfacing patches)



Culling criterion:

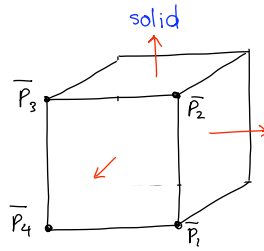
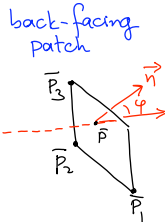
$$(\vec{p} - \vec{e}) \cdot \vec{n} > 0 \Rightarrow \text{CULL}$$

$$(\vec{p} - \vec{e}) \cdot \vec{n} \leq 0 \Rightarrow \text{DO NOT CULL (may be visible)}$$

Computing Outward-Facing Normals

Goal: Remove surface patches that point away from the camera (i.e. the backfacing patches)

if $\varphi < 90^\circ \Rightarrow$
 patch faces away
 from camera \Rightarrow
 CULL



Computing \vec{n} :

• if $\bar{P}_1, \bar{P}_2, \bar{P}_3$ are patch vertices
 in CCW order as seen from
 outside:

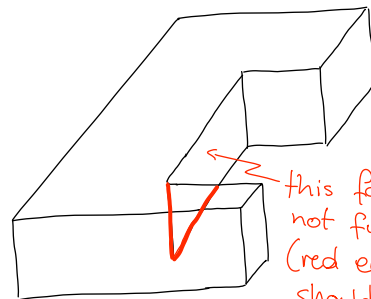
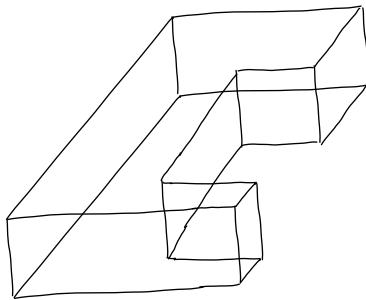
$$\vec{n} = (\bar{P}_2 - \bar{P}_1) \times (\bar{P}_3 - \bar{P}_1)$$

(uses right-hand rule)

• do computation in world
 coordinates (no point in
 transforming vertices/edges
 that will never be drawn.)

Backface Culling is not Enough ...

Which faces will not be drawn if backface culling is applied to this polyhedron?



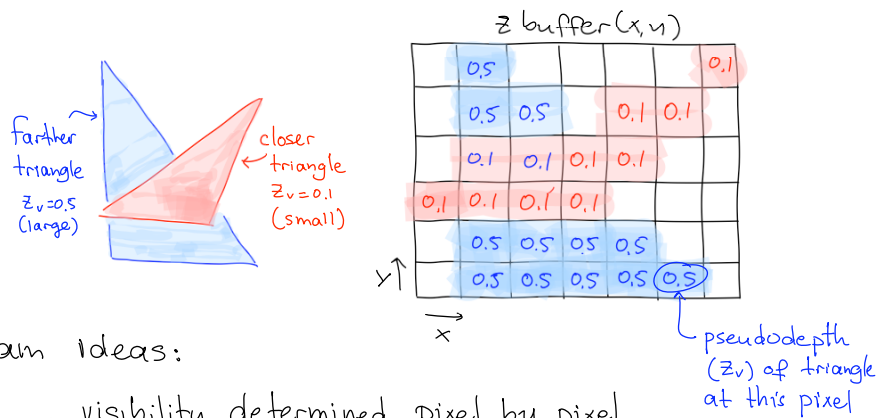
this face is
 not full visible!
 (red edges
 should not
 be drawn)

Topic 8:

Visibility

- Elementary visibility computations:
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 - Backface culling
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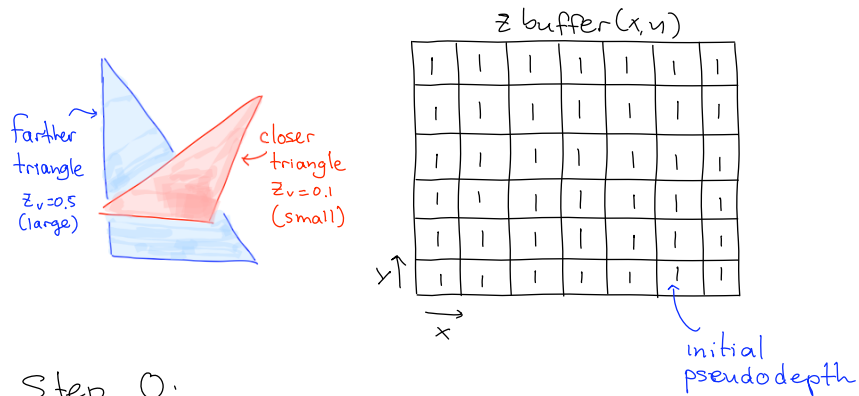
Z-Buffering



Main ideas:

- visibility determined pixel by pixel during polygon scan-conversion
- to draw an $M \times N$ -pixel image, we maintain an $M \times N$ buffer that holds closest z -value at each pixel of polygons drawn so far

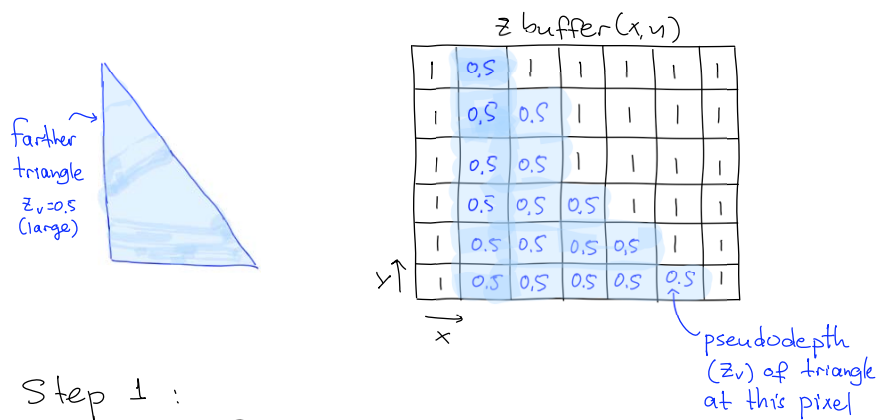
Z-Buffering & Scan Conversion



Step 0:

- Start with blank image
- Initialize z-buffer to z_{max} (always = 1)

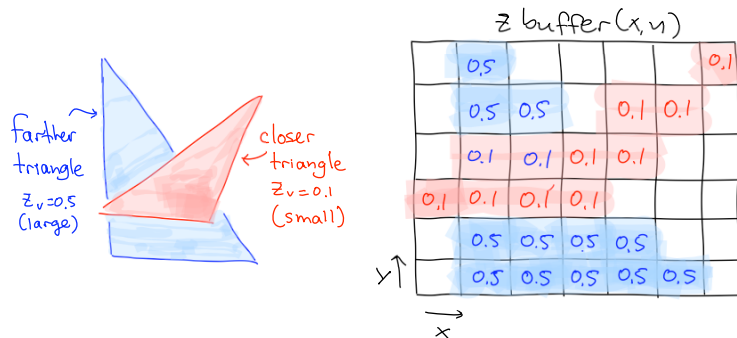
Z-Buffering & Scan Conversion



Step 1:

- Scan convert a polygon, copying the polygon's color to each pixel & updating the z-buffer

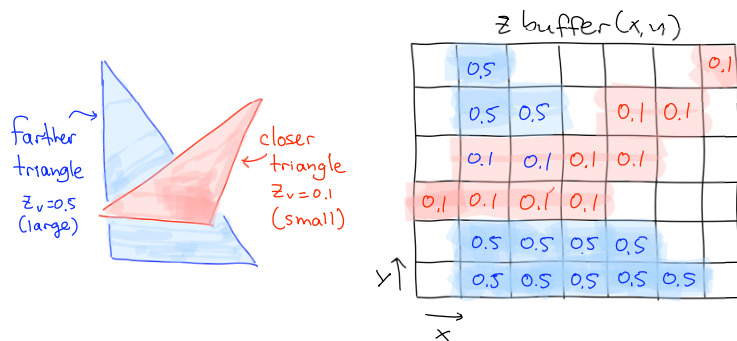
Z-Buffering & Scan Conversion



Step 2:

- Scan-convert next polygon
- To draw color c at pixel (x,y) with depth d :
 - if $d < zbuffer(x,y)$
 - putpixel (x,y,c)
 - end $zbuffer(x,y) = d$

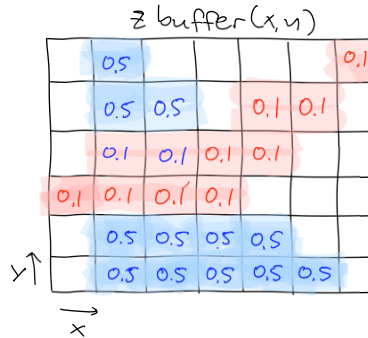
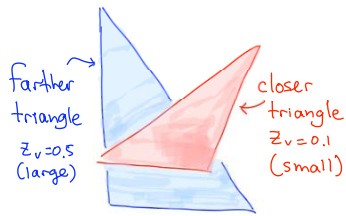
Rendering Order of Polygons



Q: What would be the result if we drew the closest triangle first?

Ans: The result would be the same.

Z-Buffering: Pros & Cons



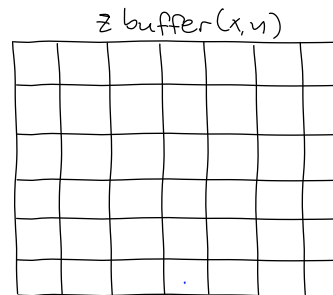
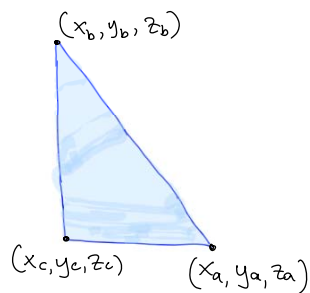
Advantages:

- . simple, accurate
- . independent of order polygons are drawn

Disadvantages:

- . memory for zbuffer (not a problem these days!)
- . wasted computation when over-writing distant points

Triangle Scan Conversion with Z-Buffering



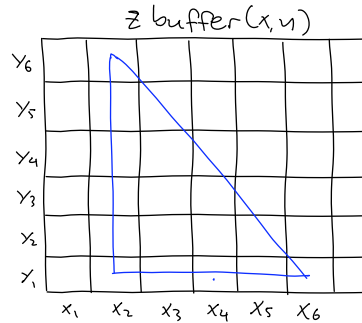
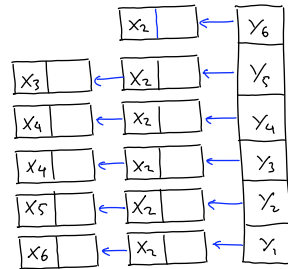
Step a: Build edge list

for each scanline,
store x-intersection &
pseudodepth of each
edge

Step b: Fill zbuffer &
image pixels

for each scanline,
interpolate pseudodepth
along scanline & compare
to z-buffer

Triangle Scan Conversion with Z-Buffering



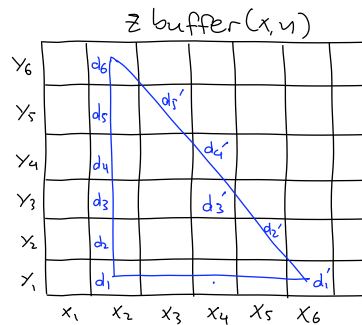
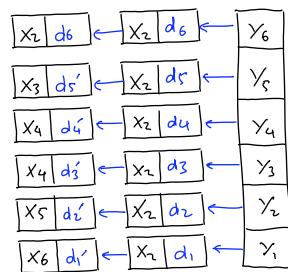
Step a: Build edge list

for each scanline,
store x-intersection &
pseudodepth of each
edge

Step b: Fill z-buffer &
image pixels

for each scanline,
interpolate pseudodepth
along scanline & compare
to z-buffer

Edge List Construction



Step a: Build edge list

for each scanline,
store x-intersection &
pseudodepth of each
edge

for each edge $[(x_u, y_u, d_u), (x_e, y_e, d_e)]$
with $y_u > y_e$:

$$x = x_e, d = d_e, \Delta x = \frac{x_u - x_e}{y_u - y_e}, \Delta d = \frac{d_u - d_e}{y_u - y_e}$$

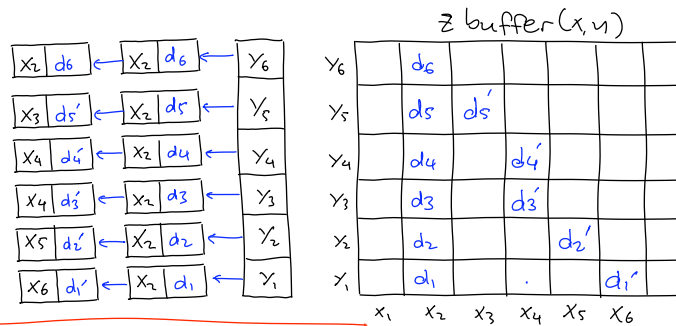
for $(y = y_e; y < y_u; y++)$

add (x, d) to list of scanline y

sort the list

$$x = x + \Delta x, d = d + \Delta d$$

Scanline Filling with Z-Buffering



```

y- = min(ya, yb, yc)  y+ = max(ya, yb, yc)
for (y=y-; y<=y+; y++)
  get (xe, de) and (xu, du) from
  edge list of y, with xe < xu
  Δd = (du - de) / (xu - xe)
  for (x=xe, d=de; x<=xu; x++)
    if d < zbuffer(x,y)
      putpixel(x,y,color), zbuffer(x,y)=d
      d = d + Δd
  
```

Step b1: Fill zbuffer & image pixels

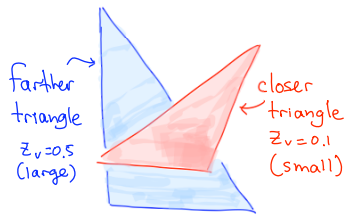
for each scanline,
interpolate pseudodepth
along scanline & compare
to z-buffer

Topic 8:

Visibility

- Elementary visibility computations:
 - Clipping
 - “Shaping” the canonical view volume
 - Backface culling
- Algorithms for visibility determination
 - Z-Buffering
 - Painter’s algorithm
 - BSP Trees

The Headless Painter's Algorithm



Main Idea:

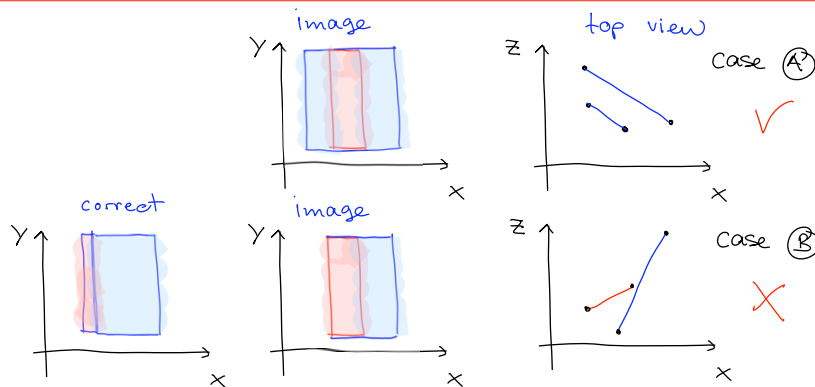
- Instead of deciding the depth order pixel by pixel, draw the polygons back to front
- Must sort polygons in decreasing z order

Question: How do we sort polygons that do not have a single z value. (i.e. not parallel to xy -plane)?

Ans: Sort according to depth of farthest vertex

Q: Does this always work? No!

The Headless Painter's Algorithm: Limitations

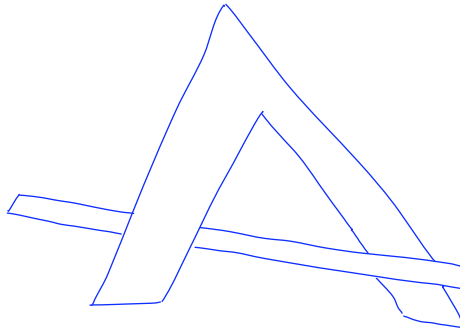


Ans: Sort according to depth of farthest vertex

Q: Does this always work? No!

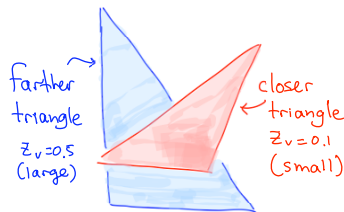
The Headless Painter's Algorithm: Limitations

Another failure case



- This example shows that in some cases there is no sort order that allow correct visibility handling
- What can we do in this case?
Ans: Break polygons into smaller (convex) parts

The Headless Painter's Algorithm: Limitations



Main issues/problems:

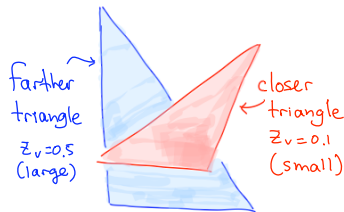
- Depth order depends on eye position (very expensive to recompute for 1000s of polygons at every frame)
- Correct visibilities may not be achievable w/out polygon splitting

Topic 8:

Visibility

- Elementary visibility computations:
 - Clipping
 - “Shaping” the canonical view volume
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 - BSP Trees

Binary Space-Partitioning (BSP) Trees



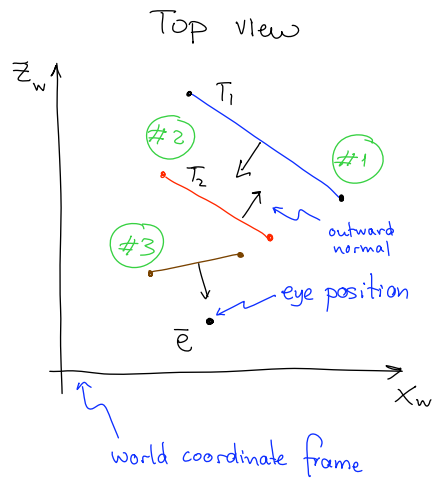
Main Idea:

- Maintain a data structure that allows fast computation of depth order for every-eye position
- Have mechanism to split polygons if necessary

Main issues/problems:

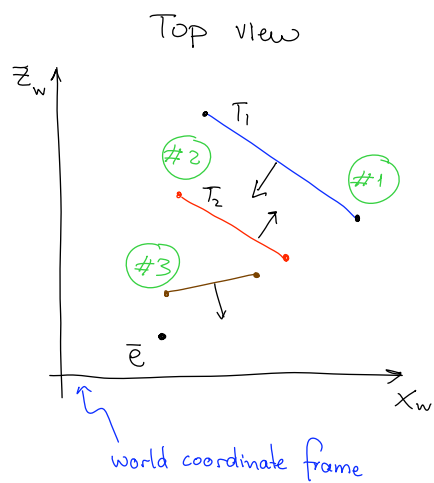
- Depth order depends on eye position (very expensive to recompute for 1000s of polygons at every frame)
- Correct visibilities may not be achievable w/out polygon splitting

Eye Position & Correct Drawing Order



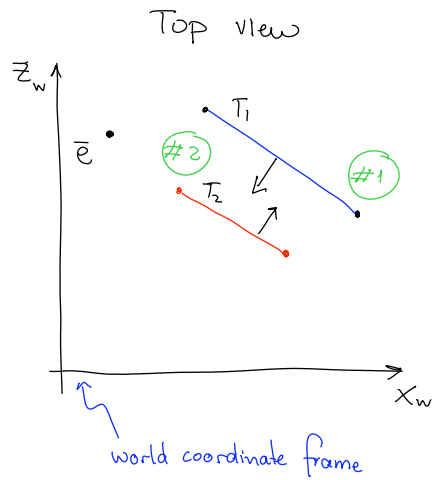
Q: What should the drawing order be when camera is at \bar{e} ?

Eye Position & Correct Drawing Order



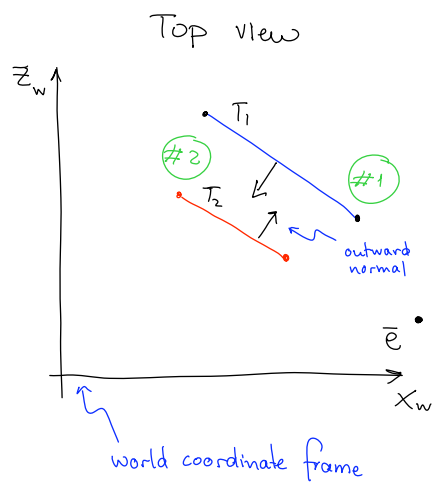
Q: What should the drawing order be when camera is at \bar{e} ?

Eye Position & Correct Drawing Order



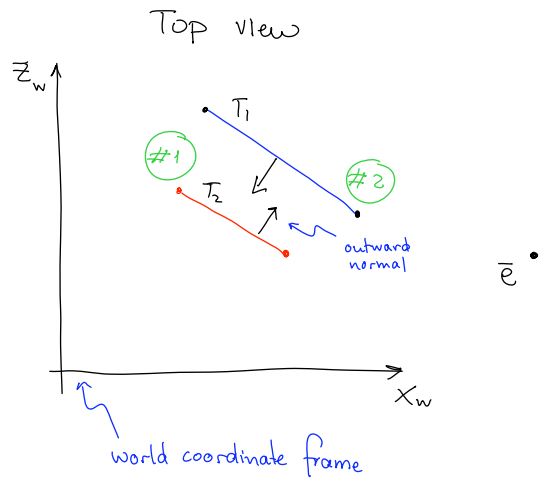
Q: What should the drawing order be when camera is at \bar{e} ?

Eye Position & Correct Drawing Order



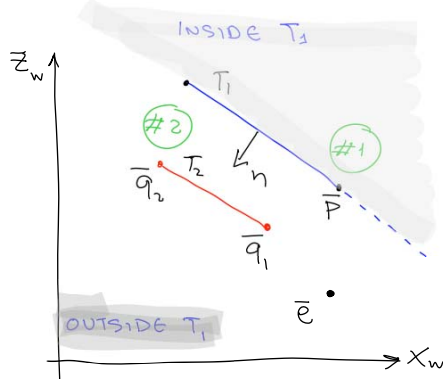
Q: What should the drawing order be when camera is at \bar{e} ?

Eye Position & Correct Drawing Order



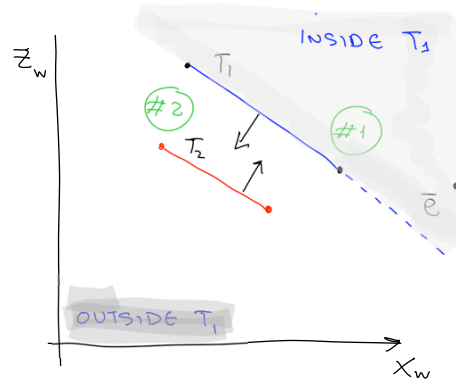
Q: What should the drawing order be when camera is at \bar{e} ?

Eye Position & Drawing Order: Basic Idea



if \bar{e}, T_2 on same side of T_1

\Rightarrow draw T_1 ,
then draw T_2



if \bar{e}, T_2 on opposite sides

\Rightarrow draw T_2 ,
then draw T_1

Eye Position & Drawing Order: Basic Idea

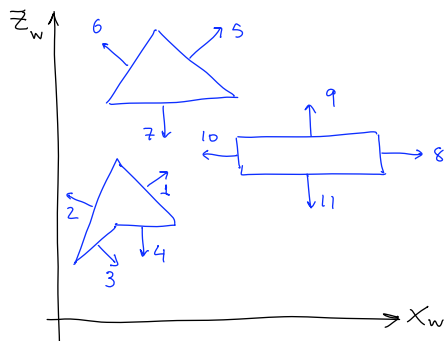
implicit plane eq: $f(\vec{q}) = (\vec{q} - \vec{p}) \cdot \vec{n}$

if $f(\vec{q}_1) \cdot f(\vec{e}) > 0$
 \Rightarrow draw T_1 ,
 then draw T_2

if $f(\vec{q}_1) \cdot f(\vec{e}) < 0$
 \Rightarrow draw T_2 ,
 then draw T_1

The BSP Tree

The BSP tree is an efficient data structure for quickly determining the inside/outside relation between polygons & the camera position



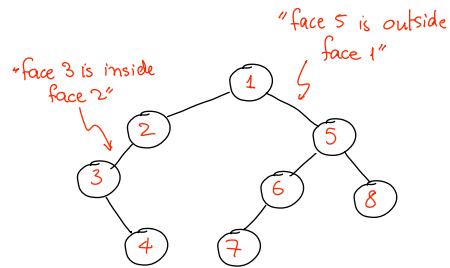
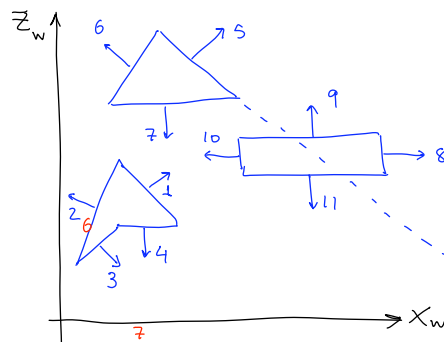
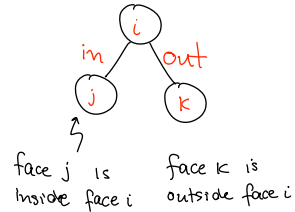
Two phases:

- Preprocessing phase (Tree Construction) done once per scene
- Rendering phase (Tree Traversal) done whenever eye position changes

BSP Tree Construction: Basic Idea

Idea:

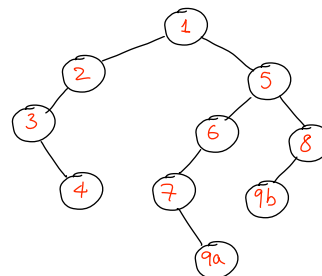
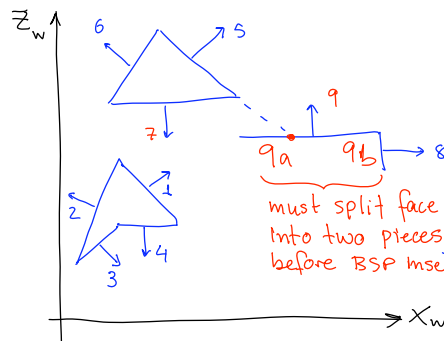
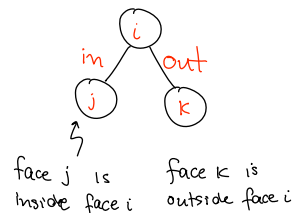
- order faces in some way
- associate a node to each face
- face #1 becomes root node
- for face j , traverse tree to a leaf node i & add it as "in" or "out" child



BSP Tree Construction: Splitting Faces

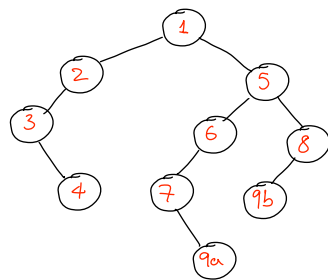
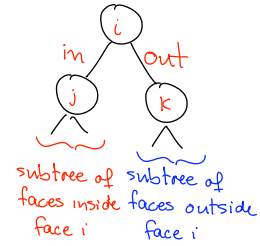
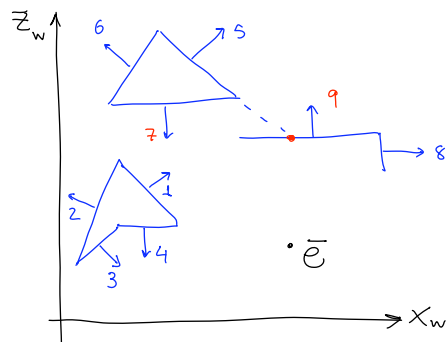
Idea:

- order faces in some way
- associate a node to each face
- face #1 becomes root node
- for face j , traverse tree to a leaf node i & add it as "in" or "out" child



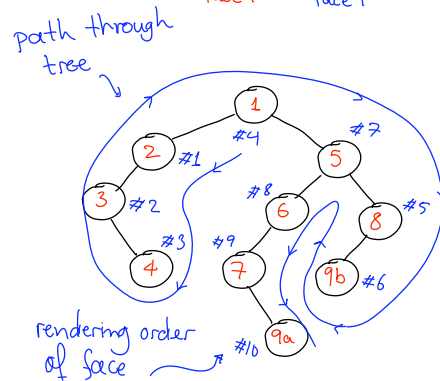
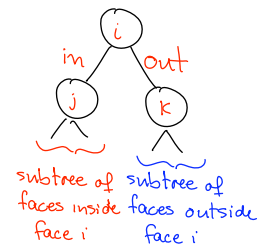
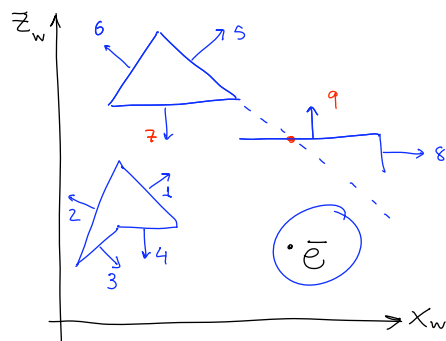
Rendering with BSP Trees: Main Idea

If \bar{e} is outside face i (eg. 1)
 nothing inside i can occlude i
 \Rightarrow can be drawn before i
 after drawing the inside faces,
 draw i & then the outside faces



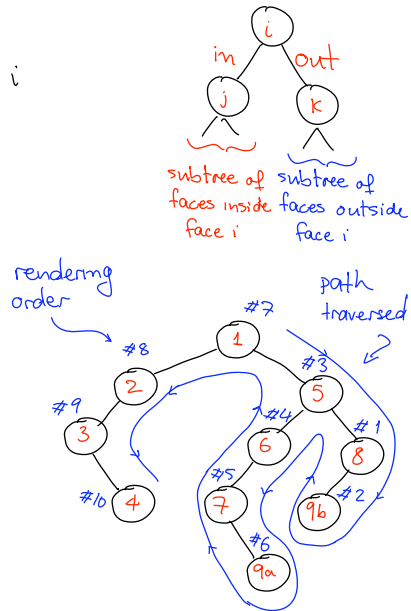
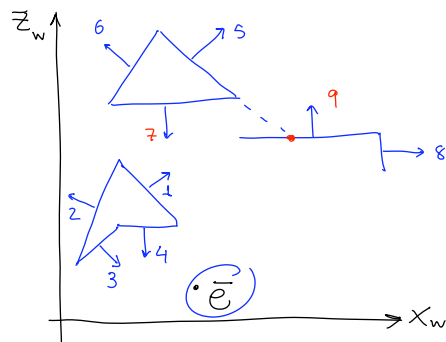
Rendering with BSP Trees: Algorithm

- If \bar{e} outside face i
 - draw everything inside i ← recursive calls to the same routine
 - draw i
 - draw everything outside i ←



Rendering with BSP Trees: Algorithm

- If \bar{e} inside face i
 - draw everything outside i
 - draw i
 - draw everything inside i



BSP Trees & Backface Culling

- If \bar{e} "inside" i then i is back-facing
- Everything here shown in 2D but construction is identical in 3D
- Only complication is triangle/polygon splitting (see Ch. 8.1 in book)

