CSC320: Visual Computing
Term Test 1 March 3rd, 2006 9:10-10:00

Student Number:______________________

Last Name:______________________ First Name:______________________

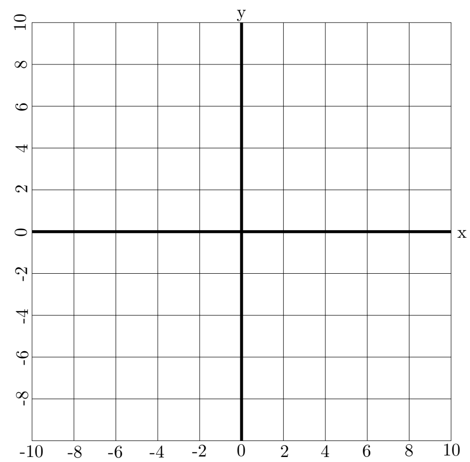
This exam consists of 3 questions on 7 single-sided pages (including cover page).
Aids allowed: None.

Total Marks: 50
Minutes: 50

1. **2D Curves [15 Marks]**

Consider the 2D curve $\gamma(\theta) = (2 + 6\cos\theta,\ 10\sin\theta)$ with $\theta \in [0, 2\pi)$.

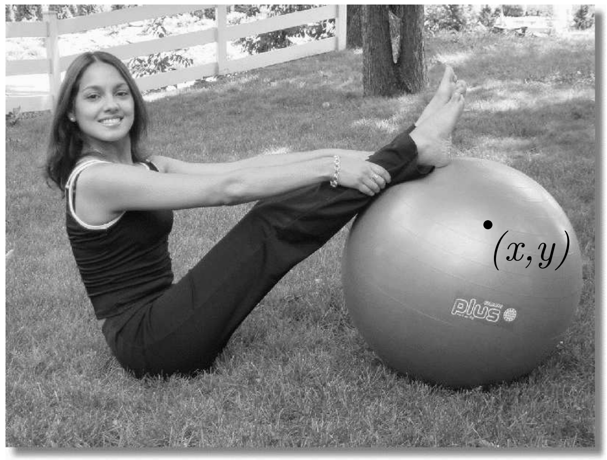
(a) [5 Marks] Draw the curve in the grid provided below. Be as precise as possible.



(b) [10 Marks] Derive the expression for the unit normal, $\mathbf{n}(\theta)$, at point $\gamma(\theta)$ along the curve.

2. **Isophotes & Image Gradients [15 Marks]**

Consider the image I shown below.



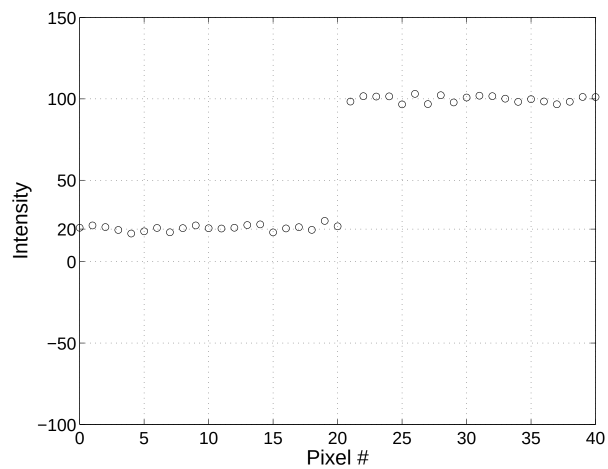
(a) [5 Marks] Give the definition of the *isophote* through pixel (x, y), and draw it on the image above.

(b) [5 Marks] Draw on the image a vector that begins at (x, y) and is in the direction of $\nabla I(x, y)$. Explain in a sentence its relationship to the isophote through (x, y).

(c) [5 Marks] Give the definition of the gradient magnitude using standard calculus notation.

3. **Weighted Least Squares Estimation [20 Marks]**

Consider the 1D image shown below, whose 41 pixels have intensities $I_0, \ldots, I_{40}$, respectively. We want to estimate the image intensity, $I(x)$, and its first derivative, $\frac{d}{dx}I(x)$, at pixel x using the sliding window algorithm with a first-order, weighted least squares fit. Assume the window has size 2*2+1 pixels and the weights are given by a function $\Omega(q)$, with $q \in [-2, 2]$.

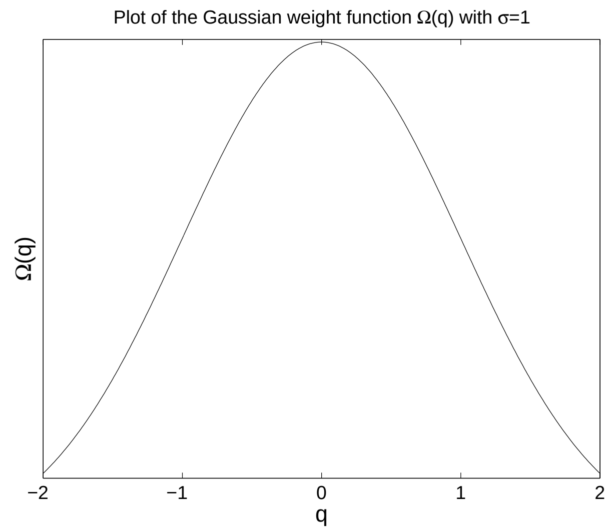


(a) [5 Marks] Using matrix notation, show the linear system that must be solved to compute the fit for pixel $x = 20$. Be sure to indicate the dimensions and contents of each matrix.

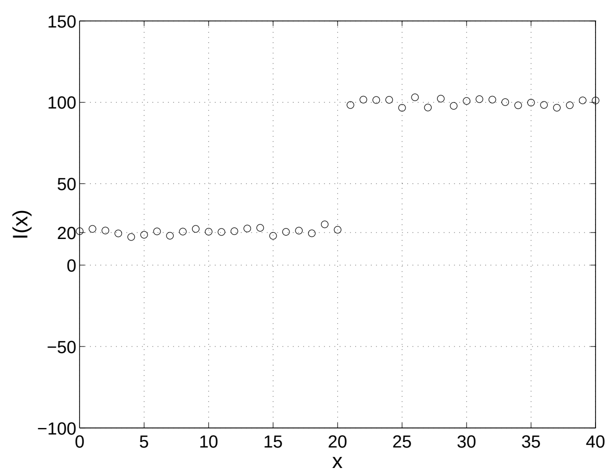
(b) [10 Marks] Now suppose that the weight function is a Gaussian

$$\Omega(q) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{q^2}{2\sigma^2}} \ ,$$

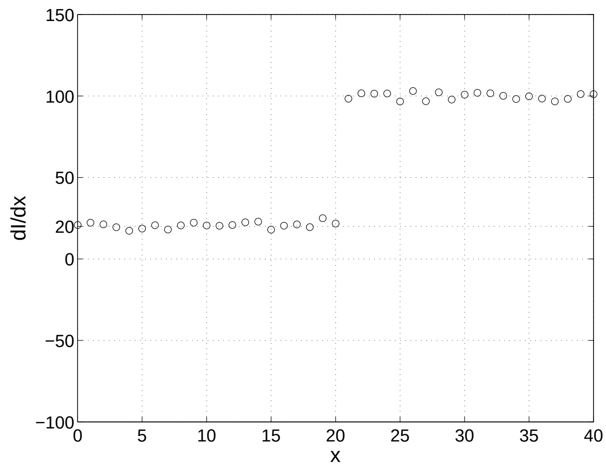
with $\sigma = 1$ (plotted below). Observe that the function is maximized at $q = 0$ and is almost zero when q is outside the range $[-2\sigma, 2\sigma]$.



(b1) [5 Marks] Plot the estimated intensity $I(x)$ on the graph below for $x \in [5, 35]$ and indicate the x values where important transitions in the shape of $I(x)$ will occur. For reference, the original pixel intensities are shown as well.



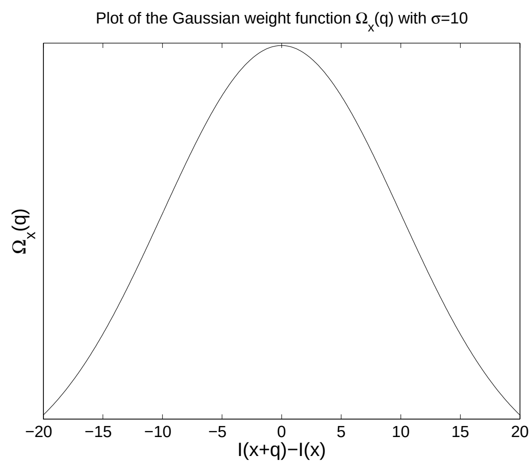
(b2) [5 Marks] Plot the estimated intensity derivative $\frac{d}{dx}I(x)$ for $x \in [5, 35]$. Indicate the x values where important transitions in the shape of $\frac{d}{dx}I(x)$ will occur and indicate the (approximate) value of $\frac{d}{dx}I(x)$ at those locations. For reference, the original pixel intensities are shown as well.



(c) [5 Marks] Finally, suppose that we do our estimation with a Gaussian weight function that *changes* from window to window and depends on pixel *intensities* within the window. Specifically, for the window centered at pixel x, we use the weight function

$$\Omega_x(q) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(I(x+q)-I(x))^2}{2\sigma^2}} ,$$

with $\sigma = 10$ (plotted below).



Plot the estimated intensity $I(x)$ on the graph below for $x \in [5, 35]$ and indicate the x values where important transitions in the shape of $I(x)$ will occur. For reference, the original pixel intensities are shown as well.