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UNIVERSITY OF TORONTO
Faculty of Arts and Science

DECEMBER 2005 EXAMINATIONS

CSC 363H1 F
Instructor: F. Pitt

Duration — 3 hours

Examination Aids: One 8.5" × 11" sheet of paper, *hand*written on both sides.

Student Number: |_|_|_|_|_|_|_|_|_|

Last (Family) Name(s): ________________________________

First (Given) Name(s): ________________________________

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*Do **not** turn this page until you have received the signal to start.*
(In the meantime, please fill out the identification section above, and read the instructions below *carefully*.)

This final examination consists of 6 questions on 9 pages (including this one), printed on one side of the paper. *When you receive the signal to start, please make sure that your copy of the examination is complete and write your student number at the bottom of every page, where indicated.*

Answer each question directly on the examination paper, in the space provided, and use the reverse side of the pages for rough work. If you need more space for one of your solutions, use the reverse side of a page and *indicate clearly the part of your work that should be marked.*

In your answers, you may use without proof any result or theorem covered during the course in lectures, tutorials, assignments, or term tests. You must justify all other facts required for your solution.

If you are unable to answer a question (or part of a question), you will get 20% of the marks for the question (or part of the question) if you state clearly that you do not know how to answer. Note that you will *not* get those marks if your answer is completely blank or contains contradictory statements (such as "I don't know" followed or preceded by parts of a solution that have not been crossed off).

MARKING GUIDE

1: ______/15

2: ______/10

3: ______/10

4: ______/20

5: ______/13

6: ______/12

BONUS MARKS: ______/ 5

TOTAL: ______/80

Good Luck!

Question 1. [15 MARKS]

Part (a) [5 MARKS]

Describe the difference(s) between "A is recognizable but undecidable" and "A is unrecognizable".
(*Hint:* Discuss various types of strings related to A.)

Part (b) [5 MARKS]

For any language L, if $L \leq_m A_{TM}$ and $A_{TM} \leq_m L$, what can we conclude about L and $\overline{L}$? Give the strongest answers possible and justify.

Part (c) [5 MARKS]

Are there countably or uncountably many co-recognizable languages? Justify.

Question 2. [10 MARKS]

Part (a) [8 MARKS]

Prove that the following language is recognizable.

$$S_{TM} = \{ \langle M \rangle \mid M \text{ is a TM that uses more than } |x|^2 \text{ tape cells on some input } x \}$$

Part (b) [2 MARKS]

True of False: Rice's Theorem can be used to conclude that S_{TM} is undecidable? ____________
Justify briefly.

Question 3. [10 marks]

Prove that the following language is unrecognizable. (Your answer will be marked on its structure as well as its content.)

$$D_{TM} = \{ \langle M \rangle \mid M \text{ is a decider} \}$$

Question 4. [20 MARKS]

Part (a) [5 MARKS]

Show that the following language belongs to P.

$$SQ = \{\ x \mid x \text{ is a binary integer and } x = k^2 \text{ for some } k \in \mathbb{N}\ \}$$

Part (b) [5 MARKS]

Does the following language belong to P? Justify.

$$363\textsc{Sum} = \{\ \langle S, t\rangle \mid S \text{ is a set of positive integers and } t \text{ is a positive integer such that there is a subset of 363 integers from } S \text{ whose sum is exactly } t\ \}$$

Question 4. (CONTINUED)

Part (c) [5 MARKS]

Can a language in *P* also be *NP*-complete? If so, what are the consequences, and why? If not, why not?

Part (d) [5 MARKS]

Let A be a language that is *NP*-complete and B be a language that is *NP*-hard but not *NP*-complete. Does $A \leq_p B$? Does $B \leq_p A$? Justify both answers.

Question 5. [13 MARKS]

Give a detailed proof that the following language is *NP*-complete. (Your answer will be marked on its structure as well as its content.)

$$\text{SAMESUM} = \{ \langle S, T \rangle \mid S \text{ and } T \text{ are sets of positive integers such that some nonempty subset of } S \text{ has the same sum as some nonempty subset of } T \}$$

Question 6. [12 MARKS]

Recall the Hamiltonian cycle problem.

HAMCYCLE (Decision): Given an undirected graph G, does G contain a simple cycle through every vertex?

HAMCYCLE (Search): Given an undirected graph G, return a simple cycle through every vertex, if one exists (return a special value NIL otherwise).

Prove that Hamiltonian cycle is polytime self-reducible, including a brief argument that your algorithm is correct and runs within the correct time bound.

Bonus. [5 MARKS]

WARNING! This question is difficult, it is not worth many marks, and it will be marked very harshly—credit will only be given for significant progress towards a correct answer. Please do not attempt this question until you have completed the rest of the examination.

Prove that every infinite recognizable language contains an infinite decidable subset.

Total Marks = 80