PLEASE HAND IN

UNIVERSITY OF TORONTO
Faculty of Arts and Science

APRIL–MAY 2006 EXAMINATIONS

CSC 363H1 S
Instructor(s): F. Pitt

Duration — 3 hours

PLEASE HAND IN

Examination Aids: One 8.5" × 11" sheet of paper, *hand*written on both sides.

Student Number: |_|_|_|_|_|_|_|_|_|

Last (Family) Name(s): ______________________________

First (Given) Name(s): ______________________________

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*Do **not** turn this page until you have received the signal to start.*
(In the meantime, please fill out the identification section above, and read the instructions below *carefully*.)

This final examination consists of 6 questions on 10 pages (including this one), printed on one side of the paper. *When you receive the signal to start, please make sure that your copy of the examination is complete and write your student number where indicated at the bottom of every page* (except page 1).

Answer each question directly on the examination paper, in the space provided, and use the reverse side of the pages for rough work. If you need more space for one of your solutions, use the reverse side of a page and *indicate clearly the part of your work that should be marked.*

In your answers, you may use without proof any result or theorem covered in lectures, tutorials, assignments, tests, or in the textbook, as long as you give a clear statement of the result(s) you are using. You must justify all other facts required for your solution.

If you are unable to answer a question (or part of a question), you will get 20% of the marks for the question (or part of the question) if you state clearly that you do not know how to answer. Note that you will *not* get those marks if your answer is completely blank or contains contradictory statements (such as "I don't know" followed or preceded by parts of a solution that have not been crossed off).

MARKING GUIDE

1: ______/20

2: ______/13

3: ______/12

4: ______/12

5: ______/13

6: ______/20

TOTAL: ______/90

Good Luck!

Question 1. [20 MARKS]

Part (a) [2 MARKS]

State the difference(s) between $\leq_m$ and $\leq_p$.

Part (b) [3 MARKS]

True or False: "$A_{TM} \in coNP$"? Justify briefly.

Part (c) [3 MARKS]

Give a specific example of two languages $L_1 \subseteq L_2$ such that L_1 is undecidable and L_2 is decidable,

Part (d) [3 MARKS]

True or False: "Every language $L \subseteq \{0,1\}^*$ is decidable because it is easy for a TM to check that its input contains only 0s and 1s"? Justify briefly.

Question 1. (continued)

Part (e) [2 marks]

If a TM M loops on input w, then... (check one)

- ☐ ...it is always the case that $w \in L(M)$.
- ☐ ...it is always the case that $w \notin L(M)$.
- ☐ ...sometimes $w \in L(M)$ and sometimes $w \notin L(M)$.

Part (f) [3 marks]

Are there countably or uncountably many languages in *PSPACE*? Justify briefly.

Part (g) [4 marks]

Are there countably or uncountably many languages L with the property that every string in L contains at least five 1s? Justify briefly.

Question 2. [13 MARKS]

State whether the language "More Than Four" (MTF) defined below is decidable, undecidable but recognizable, or unrecognizable, then prove your claim. Your answer will be marked on its structure as well as its content, so write your solution carefully, following the format presented in class.

$$\text{MTF} = \left\{ \langle M \rangle : |L(M)| > 4, \textit{ i.e.}, M \text{ accepts more than 4 different input strings} \right\}$$

Question 3. [12 MARKS]

State whether the language "Exactly Four" (EF) defined below is decidable, undecidable but recognizable, or unrecognizable, then prove your claim. Your answer will be marked on its structure as well as its content, so write your solution carefully, following the format presented in class.

$$\text{EF} = \{\, \langle M \rangle : |L(M)| = 4,\ \textit{i.e.},\ M \text{ accepts exactly 4 different input strings} \,\}$$

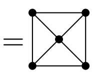
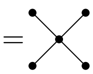
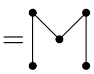
Question 4. [12 MARKS]

Prove that the "Low Degree Spanning Tree" language defined below is NP-complete. A significant portion of your mark will be based on the structure of your answer, so write your proof carefully.

$$\text{LDST} = \{\, \langle G, k \rangle : G \text{ is an undirected graph that contains a spanning tree with degree bound } k \,\}$$

Recall that a "spanning tree" in a graph G is any connected acyclic subset of the edges of G. A spanning tree has "degree bound" k if every vertex has degree no more than k (recall that the "degree" of a vertex is the number of edges connected to that vertex).

For example, in the picture below, T_1 and T_2 are two different spanning trees of G; T_1 has degree bound at least 4 (because the middle vertex has degree 4 and every other vertex is a leaf with degree 1), while T_2 has degree bound 2 (because every vertex has degree 2 except the leaves).

$G =$  $T_1 =$  $T_2 =$ 

(HINTS: $\text{HP} = \{\, \langle G \rangle : G \text{ is an undirected graph that contains some Hamiltonian path} \,\}$ is NP-complete. Consider a small value of k.)

Question 5. [13 marks]

The "Minimum Degree Spanning Tree" search problem is defined as follows:

> MDST: Given an undirected graph G, find a spanning tree of G with the smallest possible degree bound.

(See the previous question for definitions of "spanning tree" and "degree bound", as well as a description of the related decision problem.)
Show that the MDST problem is polytime self-reducible. Follow the format presented in class.

Question 6. [20 MARKS]

For each language below, circle the smallest class that the language belongs to and justify your answer briefly (using one or two short sentences, write down the main idea of a proof.)

For example, a reasonable answer for the language "Long Path" (LP) $= \{ \langle G, s, t, k \rangle : G$ is an undirected graph that contains a simple path with at least k edges from s to $t \}$ would be to circle "*NP*" with justification:

> LP $\in$ *NP* (it takes polytime to verify that a certificate encodes a simple path with k edges from s to t) and LP $\notin$ *P* because LP is *NP*-complete (HAMPATH $\leq_p$ LP by letting $k = n - 1$ because a path is Hamiltonian iff it is simple and contains $n - 1$ edges).

Part (a) [4 MARKS]

"Independent Set Gap" (ISG) $= \{ \langle G, k, \ell \rangle : G$ is an undirected graph that contains an independent set of size $\leq k$ but no independent set of size $\geq \ell \}$ belongs to:

L *P* *NP* *coNP* *PSPACE* I DON'T KNOW

Justification:

Question 6. (CONTINUED)

Part (b) [4 MARKS]

"No Right Move" (NRM) = $\{ \langle M, w\rangle : M$ is a TM with tape alphabet $\Gamma = \{0, 1, \sqcup\}$ such that M never moves right during its computation on input $w \}$ belongs to:

L *P* *NP* *coNP* *PSPACE* I DON'T KNOW

Justification:

Part (c) [4 MARKS]

"Tautology" (TAUT) = $\{ \langle F\rangle : F$ is a propositional formula that is a tautology, *i.e.*, every assignment of truth values to the variables of F makes F true $\}$ belongs to:

L *P* *NP* *coNP* *PSPACE* I DON'T KNOW

Justification:

Question 6. (CONTINUED)

Part (d) [4 MARKS]

"Small Weight Cycle" (SWC) = $\{ \langle G \rangle : G$ is an undirected graph with weights $w(e)$ for each edge e and G contains a simple cycle whose total weight is exactly 100 $\}$ belongs to:

L *P* *NP* *coNP* *PSPACE* I DON'T KNOW

Justification:

Part (e) [4 MARKS]

"Small Cycle" (SC) = $\{ \langle G \rangle : G$ is an undirected graph that contains a simple cycle over 100 of its vertices $\}$ belongs to:

L *P* *NP* *coNP* *PSPACE* I DON'T KNOW

Justification:

Total Marks = 90