PLEASE HAND IN

UNIVERSITY OF TORONTO
Faculty of Arts and Science

AUGUST 2006 EXAMINATIONS

CSC 363H1 Y
Instructor(s): R. Krueger

Duration — 3 hours

PLEASE HAND IN

Examination Aids: One 8.5" $\times$ 11" sheet of paper, *hand*written on both sides.

Student Number: |_|_|_|_|_|_|_|_|_|

Last (Family) Name(s): ______________________________

First (Given) Name(s): ______________________________

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*Do **not** turn this page until you have received the signal to start.*
(In the meantime, please fill out the identification section above, and read the instructions below *carefully*.)

This final examination consists of 6 questions on 10 pages (including this one), printed on one side of the paper. *When you receive the signal to start, please make sure that your copy of the examination is complete and write your student number where indicated at the bottom of every page* (except page 1).

Answer each question directly on the examination paper, in the space provided, and use the reverse side of the pages for rough work. If you need more space for one of your solutions, use the reverse side of a page and *indicate clearly the part of your work that should be marked.*

In your answers, you may use without proof any result or theorem covered in lectures, tutorials, assignments, tests, or in the textbook, as long as you give a clear statement of the result(s) you are using. You must justify all other facts required for your solution.

If you are unable to answer a question (or part of a question), you will get 20% of the marks for the question (or part of the question) if you state clearly that you do not know how to answer. Note that you will *not* get those marks if your answer is completely blank or contains contradictory statements (such as "I don't know" followed or preceded by parts of a solution that have not been crossed off).

Marking Guide

# 1:	______/20
# 2:	______/10
# 3:	______/13
# 4:	______/13
# 5:	______/ 9
# 6:	______/20
TOTAL:	______/85

Good Luck!

Question 1. [20 MARKS]

Part (a) [2 MARKS]

State the difference(s) between $\leq_m$ and $\leq_p$.

Part (b) [3 MARKS]

Give a precise definition of *coNP*.

Part (c) [3 MARKS]

Suppose a language L has the property $L \leq_m A_{TM}$. What can we conclude (if anything) about L?

Part (d) [3 MARKS]

True or False: there are uncountably many languages in *NP*. Justify briefly.

Question 1. (CONTINUED)

Part (e) [3 MARKS]

True or False: SAT $\leq_m A_{TM}$. Justify briefly.

Part (f) [2 MARKS]

Give a precise definition of *PSPACE*-hard.

Part (g) [4 MARKS]

Show that any *PSPACE*-hard problem is also *NP*-hard.

Question 2. [10 marks]

Part (a) [8 marks]

Prove that the following language is recognizable.

$$S_{TM} = \big\{\langle M\rangle : M \text{ is a TM with a work tape that uses at least } |w| \text{ work tape cells on some input } w\big\}.$$

Part (b) [2 marks]

True or False: Rice's Theorem can be used to conclude that S_{TM} is undecidable? ____________
Justify briefly.

Question 3. [13 MARKS]

State whether the following language is decidable, undecidable but recognizable, or unrecognizable, then prove your claim. Your answer will be marked on its structure as well as its content, so write your solution carefully, following the format presented in class.

$$\text{CRASH} = \big\{\langle P\rangle : P \text{ is a program (say, written in C) and } P \text{ will crash on some input } w\big\}.$$

Question 4. [13 MARKS]

Prove that the following language is *NP*-complete. A significant portion of your mark will be based on the structure of your answer, so write your proof carefully.

$$\text{CRASH-IFS} = \big\{ \langle P \rangle : P \text{ is a C program that contains only } \texttt{if} \text{ control statements (no loops)} \\ \text{and } P \text{ will crash on some input } w \text{ with } |w| \leq |\langle P \rangle| \big\}.$$

HINT: You may use the fact that SAT is *NP*-complete.

Question 5. [9 MARKS]

Consider the following problem:

$$\text{PARTITION} = \{\langle S\rangle : S = \{s_1, \ldots, s_n\} \text{ is a set of positive integers such that } S \text{ can be partitioned into two sets } A \text{ and } \overline{A} = S - A \text{ with } \textstyle\sum_{x\in A} x = \sum_{x\in\overline{A}} x\}.$$

Part (a) [2 MARKS]

Show that PARTITION $\in NP$.

Part (b) [7 MARKS]

Recall that: SUBSETSUM $= \{ \langle S, t\rangle : S$ is a set of positive integers $\{x_1, \ldots, x_n\}$ and t is a positive integer such that there is some subset of S whose sum is exactly t, *i.e.*, $\exists S' \subseteq S$, $\sum_{x\in S'} x = t \}$.

Prove that SUBSETSUM $\leq_p$ PARTITION using the following reduction:
$f(\langle S, t\rangle) = \langle S \cup \{2W - t, W + t\}\rangle$, where $W = \sum_{x\in S} x$.

Question 6. [20 MARKS]

For each language below, circle the smallest class that (we can prove) the language belongs to and justify your answer briefly (using one or two short sentences, write down the main idea of a proof.)

For example, a reasonable answer for the language "Long Path" (LP) $= \{ \langle G, s, t, k\rangle : G$ is an undirected graph that contains a simple path with at least k edges from s to $t \}$ would be to circle "*NP*" with justification:

> LP $\in$ *NP* (it takes polytime to verify that a certificate encodes a simple path with k edges from s to t) and LP $\notin$ *P* because LP is *NP*-complete (HAMPATH $\leq_p$ LP by letting $k = n - 1$ because a path is Hamiltonian iff it is simple and contains $n - 1$ edges).

Part (a) [4 MARKS]

"Maximum Independent Set" MIS $= \{ \langle G, k\rangle : G$ is an undirected graph that contains an independent set of size k but no independent set of size $k + 1 \}$ belongs to:

L *P* *NP* *coNP* *PSPACE* NONE OF THESE I DON'T KNOW

Justification:

Question 6. (CONTINUED)

Part (b) [4 MARKS]

"Addition" ADD $= \{ \langle x, y, z \rangle : x, y, z > 0$ are binary integers and $x + y = z \}$ belongs to:

L *P* *NP* *coNP* *PSPACE* NONE OF THESE I DON'T KNOW

Justification:

Part (c) [4 MARKS]

NOCYCLE $= \{ \langle G, k \rangle : G$ is an undirected graph that contains **no** simple cycle with exactly k vertices$\}$ belongs to:

L *P* *NP* *coNP* *PSPACE* NONE OF THESE I DON'T KNOW

Justification:

Question 6. (continued)

Part (d) [4 marks]

"2-colourable" 2Col $= \{ \langle G \rangle : G$ is an undirected graph that is 2-colourable, *i.e.*, red and blue can be assigned to vertices such that adjacent vertices are given different colours$\}$ belongs to:

L *P* *NP* *coNP* *PSPACE* none of these i don't know

Justification:

Part (e) [4 marks]

"Long cycle" LC $= \{ \langle G, k \rangle : G$ is an undirected graph that contains a cycle of length at least $k\}$ belongs to:

L *P* *NP* *coNP* *PSPACE* none of these i don't know

Justification:

Total Marks = 85