Duration: **50 minutes**
Aids Allowed: **NONE** (in particular, no calculator)

Student Number: |_|_|_|_|_|_|_|_|_|

Last (Family) Name(s): ____________________

First (Given) Name(s): ____________________

Tutorial Section: **(circle one)**

BA 1210 Anthony To

BA 1220 Periklis Papakonstantinou

Do ***not*** *turn this page until you have received the signal to start.*
(In the meantime, please fill out the identification section above, and read the instructions below *carefully*.)

This term test consists of 3 questions on 4 pages (including this one), printed on one side of the paper. *When you receive the signal to start, please make sure that your copy of the test is complete and write your student number where indicated at the bottom of every page* (except page 1).

Answer each question directly on the test paper, in the space provided, and use the reverse side of the pages for rough work. If you need more space for one of your solutions, use the reverse side of a page and *indicate clearly the part of your work that should be marked.*

In your answers, you may use without proof any result or theorem covered in lectures, tutorials, assignments, or in the textbook, as long as you give a clear statement of the result(s)/theorem(s) you are using. You must justify all other facts required for your solutions.

If you are unable to answer a question (or part of a question), you will get 20% of the marks for the question (or part of the question) if you state clearly that you do not know how to answer. Note that you will *not* get those marks if your answer contains contradictory statements (such as "I don't know" followed or preceded by parts of a solution that have not been crossed off).

MARKING GUIDE

\# 1: ______/10

\# 2: ______/10

\# 3: ______/10

TOTAL: ______/30

Good Luck!

Question 1. [10 MARKS]

A *multi-headed Turing machine* (MHTM) is like a regular Turing machine except that it has k independent heads (working on the same, single tape), for some fixed $k > 1$. Initially, all heads are on the leftmost symbol of the input. For each transition, the current state and the symbols read by each head determine the next state, the symbols written by each head, and the direction of movement for each head—if multiple heads attempt to write different symbols on the same tape square, we adopt the convention that the symbol written is the one dictated by the latest numbered head (for example, if heads 2, 3 and 5 attempt to write symbols to a particular cell, the symbol written is the symbol dictated by head 5).

Prove or disprove that multi-headed Turing machines are equivalent to regular Turing machines.

Question 2. [10 MARKS]

Show that the language

$$L = \big\{ \langle M \rangle \in \{0,1\}^* : M \text{ is a TM such that } L(M) \text{ contains at least one string that contains a } 0 \big\}$$

is recognizable. (It is sufficient to give a detailed high-level answer.)

Question 3. [10 MARKS]

Show that the language

$$L = \{\langle M \rangle \in \{0,1\}^* : M \text{ is a TM such that } L(M) \text{ contains at least one string that contains a } 0\}$$

is undecidable. (Your answer will be marked on its structure as well as its content.)

Total Marks = 30