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UNIVERSITY OF TORONTO
AT MISSISSAUGA
Faculty of Arts and Science

DECEMBER 2005 EXAMINATIONS

CSC 363H5 F
Instructor: A. Magen

Duration — 3 hours

Examination Aids: One 8.5" × 11" sheet of paper, *hand*written on both sides.

Student Number: |_|_|_|_|_|_|_|_|_|_|

Last (Family) Name(s): _____

First (Given) Name(s): _____

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*Do **not** turn this page until you have received the signal to start.*
(In the meantime, please fill out the identification section above, and read the instructions below *carefully*.)

This final examination consists of 6 questions on 9 pages (including this one), printed on one side of the paper. *When you receive the signal to start, please make sure that your copy of the examination is complete and write your student number at the bottom of every page, where indicated.*

Answer each question directly on the examination paper, in the space provided, and use the reverse side of the pages for rough work. If you need more space for one of your solutions, use the reverse side of a page and *indicate clearly the part of your work that should be marked.*

In your answers, you may use without proof any result or theorem covered during the course in lectures, tutorials, assignments, or term tests. You must justify all other facts required for your solution.

If you are unable to answer a question (or part of a question), you will get 20% of the marks for the question (or part of the question) if you state clearly that you do not know how to answer. Note that you will *not* get those marks if your answer is completely blank or contains contradictory statements (such as "I don't know" followed or preceded by parts of a solution that have not been crossed off).

MARKING GUIDE

# 1:	_____/15
# 2:	_____/10
# 3:	_____/10
# 4:	_____/20
# 5:	_____/13
# 6:	_____/12
BONUS MARKS:	_____/ 5
TOTAL:	_____/80

Good Luck!

Question 1. [15 MARKS]

Part (a) [5 MARKS]

Define what it means for a language to be recognizable but undecidable. Give a specific example of such a language.

Part (b) [5 MARKS]

Prove that $\{00, 11\} \leq_m A$ for all languages A such that $A \neq \varnothing$ and $A \neq \Sigma^*$.
(*Hint:* $A \neq \varnothing$ and $A \neq \Sigma^*$ implies that there are two "types" of strings related to A.)

Part (c) [5 MARKS]

Are there countably or uncountably many languages that contain an odd number of strings? Justify.

Question 2. [10 MARKS]

Part (a) [8 MARKS]

Prove that the following language is recognizable, where state q of a TM M is "unused" if for all input strings x, M never enters state q during its computation on x.

$$S_{TM} = \{ \langle M \rangle \mid M \text{ is a TM with } \textbf{no} \text{ unused state} \}$$

Part (b) [2 MARKS]

True or False: Rice's Theorem can be used to conclude that S_{TM} is undecidable? _____

Justify briefly.

Question 3. [10 MARKS]

Prove that the following language is unrecognizable. (Your answer will be marked on its structure as well as its content.)

$$D_{TM} = \{ \langle M \rangle \mid M \text{ is a TM that } \textbf{rejects} \text{ every string of odd length } \}$$

Question 4. [20 MARKS]

Part (a) [5 MARKS]

Does the following algorithm run in polytime or not, for positive integers x, y? Justify briefly.

```
div(x, y):    # return ⌊x/y⌋
    d = 0
    while x ≥ y:   d = d + 1;   x = x − y
    return d
```

Part (b) [5 MARKS]

Does the following language belong to P? Justify.

$$363\text{COVER} = \{ \langle G \rangle \mid G \text{ is an undirected graph that contains a vertex cover of size 363} \}$$

Question 4. (CONTINUED)

Part (c) [5 MARKS]

Prove that any *PSPACE*-hard language is also *NP*-hard.

Part (d) [5 MARKS]

Explain the error in the following "proof" that $NP = coNP$:

> Let $A \in NP$, *i.e.*, there is a NTM M_A that decides A in time $\mathcal{O}(n^k)$ for some constant k. Create M'_A to be the same as M_A except that q_{accept} and q_{reject} are interchanged. Then, M'_A decides $\overline{A}$ in time $\mathcal{O}(n^k)$ so $\overline{A} \in NP$. Hence, $NP \subseteq coNP$ and the same argument shows $coNP \subseteq NP$.

Question 5. [13 MARKS]

Give a detailed proof that the following language is *NP*-complete. (Your answer will be marked on its structure as well as its content.)

$$\textsc{LargerClique} = \{ \langle G, H \rangle \mid G \text{ and } H \text{ are undirected graphs such that the largest clique in } G \text{ is at least as large as the largest clique in } H \}$$

Question 6. [12 MARKS]

Recall the subset sum problem.

SUBSUM (Decision): Given a set of positive integers S and a positive integer t, is there a subset of S whose sum is exactly t?

SUBSUM (Search): Given a set of positive integers S and a positive integer t, return a subset of S whose sum is exactly t, if one exists (return a special value NIL otherwise).

Prove that subset sum is polytime self-reducible, including a brief argument that your algorithm is correct and runs within the correct time bound.

Bonus. [5 MARKS]

WARNING! This question is difficult, it is not worth many marks, and it will be marked very harshly—credit will only be given for significant progress towards a correct answer. Please do not attempt this question until you have completed the rest of the examination.

Prove that every infinite recognizable language contains an infinite decidable subset.

Total Marks = 80