Due: By 7:00pm on Thursday, August 10. **Worth**: 10%

1. [15 marks]

   Consider the following problems:

   > TSP-D (Traveling Salesman Problem—Decision): Given an integer bound B and a directed graph $G = (V, E)$ with integer weights $w(e)$ for each edge $e \in E$ (the weights can be positive or negative or zero), is there a simple cycle including every vertex of G (a "circuit") with total weight no more than B?
   >
   > TSP-S (Traveling Salesman Problem—Search): Given a directed graph $G = (V, E)$ with integer weights $w(e)$ for each edge $e \in E$ (the weights can be positive or negative or zero), find a circuit in G with the smallest possible total weight.

   Show that the Traveling Salesman Problem is polytime self-reducible. Write your solution carefully, following the format presented in class.

2. [15 marks]

   Amazons is a two-player game that combines elements from Chess and Go. In the standard game, Amazons is played on a 10×10 board where each player has 4 amazons. The white player goes first, and play alternates. On each turn, the player must move one amazon (movement is like a chess queen in any of 8 directions) to a new square, and fires an arrow (also like a chess queen) from that amazon to a vacant square. This square is "burned" off the board. Amazons and arrows cannot move onto or across burned squares, nor move through an amazon. There are no captures. The first player who cannot make a legal move loses.

   This game can easily be generalized to any board size $n \times n$. A "position" in such a game is a representation of the $n \times n$ board where each location is either empty, burned, or contains one of the two players' amazons, together with an indication of which player moves next.

   Prove that AMAZONS $\in$ *PSPACE*, where

   $$\text{AMAZONS} = \big\{\langle B\rangle : B \text{ is a position (as defined above) in an } n \times n \text{ game of Amazons where the white player has a winning strategy}\big\}.$$

3. [15 marks]

   Let DOUBLED $= \big\{ww : w \in \Sigma^*\big\}$. Prove that DOUBLED $\in L$.

4. [15 marks]

   An undirected graph is *bipartite* if its vertices may be divided into two sets so that all edges are between a vertex in one set and a vertex in the other set.

   (a) Show that a graph is bipartite iff it does not contain an odd cycle (a cycle on an odd number of vertices).

   (b) Let BIPARTITE $= \big\{\langle G\rangle : G \text{ is a bipartite graph}\big\}$. Prove that BIPARTITE $\in$ *NL*.