Computer Science 263 September 25, 2007
St. George Campus University of Toronto

Homework Assignment #1
Due: October 4, 2007, by 10:10 am

1. *Please complete and attach (with a staple) an assignment cover page to the front of your assignment. You may work alone or with one other student. If you work in a group, write both your names on the cover sheet and submit only one copy of your homework.*
2. *If you do not know the answer to a question, and you write "I (We) do not know the answer to this question", you will receive 20% of the marks of that question. If you just leave a question blank with no such statement, you get 0 marks for that question.*
3. *Unless we explicitly state otherwise, you should justify your answers. Your paper will be marked based on the correctness and completeness of your answers, and the clarity, precision and conciseness of your presentation.*

Question 1. (10 marks) In class we studied *binary* heaps, i.e., heaps that store the elements in nearly-complete *binary* trees. This question is about *d-ary* heaps, i.e., heaps that store the elements in nearly-complete *d-ary* trees (where each node has at most d children, every level is full except for the bottom level, and all the nodes at the bottom level are as far to the left as possible), for some $d \geq 2$. Here we focus on MAX heaps, where the priority of each node in the d-ary tree is greater or equal to the priority of its children (if any).

a. (3 marks) Explain how to implement a d-ary heap as an array A with an associated $Heapsize$ variable. Specifically, describe
(i) how to map each element in the tree into the array, and
(ii) in which array location each element's parent and children nodes (if any) are stored.

b. (3 marks) Suppose the heap contains exactly n elements.
(i) Precisely which elements of array A represent leaf nodes of the tree?
(ii) What is the exact height of the tree? Do not use asymptotic notation.

c. (4 marks) The EXTRACT-MAX(A) operation removes a key with highest priority from the d-ary Describe an efficient algorithm for EXTRACT-MAX(A) and give its worst-case running time (in terms of n and d) using asymptotic notation.

Question 2. (15 marks) A small chain of stores recently introduced a rewards program, giving out reward cards to n of their customers. The head office has been tracking all purchases made with a reward card, and wants to reward its top customers by sending a \$50 gift certificate to the top $\lfloor \log_2 n \rfloor$ customers, and a 20%-off coupon to the next $\lfloor \sqrt{n} \rfloor$ customers. They want you to improve on their $O(n \log n)$ algorithm of sorting all n customers and then scanning the list to find the two groups of customers. Describe an $O(n)$ algorithm to find the top $\lfloor \log_2 n \rfloor$ customers and the next $\lfloor \sqrt{n} \rfloor$ top customers. Justify its correctness and worst-case running time.

Question 3. (15 marks) Suppose we have a max-binomial heap (*alias* max-binomial queue) containing n elements. Consider the following operation on this binomial heap:

- DECREASE-KEY(Q, x, k), where x is a pointer to an element in the binomial heap Q and $k < key[x]$: Change the priority of the element pointed to by x to k and restore the heap ordering property.

a. (5 marks) Suppose we use the following algorithm to implement this operation:

DECREASE-KEY-A(Q, x, k)
    remove x from the binomial heap
    insert a new node containing the data of x and new key k into the binomial heap

Give and justify an upper bound on the worst-case running time of this algorithm.

b. (10 marks) Suppose instead we use the following algorithm to implement this operation:

DECREASE-KEY-B(Q, x, k)
    set $key[x]$ to be k
    let y be a child of x with maximum key
    while $key[x] < key[y]$ do
        swap nodes x and y
        let y be a child of x with maximum key

Prove a tight asymptotic bound on the worst-case running time of this algorithm (i.e., prove Θ).

Question 4. (20 marks) This question is a programming assignment. To see its description follow the link given in the "Assignments" section of the course web page.