Duration: **50 minutes**
Aids Allowed: **NONE** (in particular, no calculator)

Student Number: |_|_|_|_|_|_|_|_|_|

Last (Family) Name(s): ____

First (Given) Name(s): ____

Tutorial Section: (circle one)	BA 2165	BA 2159	BA 2175
	Steve	Alex	Anatoliy

Do ***not*** *turn this page until you have received the signal to start.*
(In the meantime, please fill out the identification section above,
and read the instructions below *carefully*.)

This term test consists of 3 questions on 5 pages (including this one), printed on one side of the paper. *When you receive the signal to start, please make sure that your copy of the test is complete and write your student number where indicated at the bottom of every page* (except page 1).

Answer each question directly on the test paper, in the space provided, and use the reverse side of the pages for rough work. If you need more space for one of your solutions, use the reverse side of a page and *indicate clearly the part of your work that should be marked.*

Remember that, when we ask for a proof using our structured proof form, you may omit the conclusion lines and combine basic steps as long as the structure of your proof is clear. Also, unless otherwise stated, you do not need to include justifications for steps that a reader would readily believe.

Good Luck!

MARKING GUIDE

1: ______/12

2: ______/16

3: ______/12

TOTAL: ______/40

Question 1. [12 MARKS]

Consider the following beginning to a proof:

Assume $x \in D$
  $Q(x)$
  Assume $P(x)$
    Let $y = f(x)$. Then $y \in D$.
    $A(x, y)$
    $\neg Q(y)$

Which of the following statements may we legitimately conclude following the above argument? For each statement that *may* legitimately be concluded, give the rule(s) of inference that allows us to conclude the statement. For each statement that may *not* be legitimately concluded, give a short justification explaining why the statement may be false.

(i) $\forall z \in D, \neg Q(z)$ Circle one choice: valid conclusion   invalid conclusion

Justification:

(ii) $\exists v \in D, A(x, v)$ Circle one choice: valid conclusion   invalid conclusion

Justification:

(iii) $P(x) \Rightarrow Q(x)$ Circle one choice: valid conclusion   invalid conclusion

Justification:

(iv) $\exists w \in D, P(w)$ Circle one choice: valid conclusion   invalid conclusion

Justification:

Question 2. [16 MARKS]

Consider the following statement about sequences of natural numbers:

(S) $(\forall i \in \mathbb{N}, a_i \geq i \wedge \forall j \in \mathbb{N}, a_j \leq j^2) \Rightarrow (\exists k \in \mathbb{N}, \forall m \in \mathbb{N}, a_{m+1} \leq a_k + a_m)$

Part (a) [7 MARKS]

Write a sequence of natural numbers for which (S) is true. Then, using our structured proof format, prove that (S) is true for your given sequence.

Part (b) [9 MARKS]

Recall that statement (S) says:

(S) $(\forall i \in \mathbb{N}, a_i \geq i \wedge \forall j \in \mathbb{N}, a_j \leq j^2) \Rightarrow (\exists k \in \mathbb{N}, \forall m \in \mathbb{N}, a_{m+1} \leq a_k + a_m)$

Write a sequence of natural numbers for which (S) is false. Then, using our structured proof format, prove that (S) is false for your given sequence.

Question 3. [12 MARKS]

Recall that $g \in O(f)$ iff $\exists c \in \mathbb{R}^+, \exists B \in \mathbb{N}, \forall n \in \mathbb{N}, n \geq B \Rightarrow g(n) \leq cf(n)$.

Using our structured proof format, prove or disprove that $g \in O(f)$,

where $g(n) = \begin{cases} 4n^2 - 3n + 6, & \text{if } n \text{ is even} \\ 100, & \text{if } n \text{ is odd} \end{cases}$ and $f(n) = \begin{cases} n^2 - 4, & \text{if } n \text{ is even} \\ 2n, & \text{if } n \text{ is odd} \end{cases}$

Total Marks = 40