University of Toronto, St. George Campus | CSC 165H1 F
Department of Computer Science | Fall 2007

Quiz #7 Sample Solutions

We want to prove the following statement:

$$\sqrt{8n+4} \;\in\; O(n + 3\log_2 n)$$

We construct the standard proof structure and fill it in, choosing values for c and B when needed.

> Let $c = 3$. Then $c \in \mathbb{R}^+$.
> Let $B = 2$. Then $B \in \mathbb{N}$.
> Assume $n \in \mathbb{N}$.
>> Assume $n \geq B$.
>>> Then $\sqrt{8n+4} \leq \sqrt{8n^2+4} \leq \sqrt{8n^2+n^2} = 3n$ (since $n \geq 2$).
>>> Also, $c(n + 3\log_2 n) \geq c(n) = 3n$ (dropping a positive term [$\log_2 n > 0$ since $n > 1$]).
>>> Thus $\sqrt{8n+4} \leq 3n \leq c(n + 3\log_2 n)$ (comparing inequalities).
>>
>> So $n \geq B \Rightarrow \sqrt{8n+4} \leq c(n + 3\log_2 n)$.
>
> Since n is an arbitrary element of $\mathbb{N}$, $\forall n \in \mathbb{N}$, $n \geq B \Rightarrow \sqrt{8n+4} \leq c(n + 3\log_2 n)$.
> Since B is a natural number, $\exists B \in \mathbb{N}, \forall n \in \mathbb{N}, n \geq B \Rightarrow \sqrt{8n+4} \leq c(n + 3\log_2 n)$.
> Since c is a real positive number, $\exists c \in \mathbb{R}^+, \exists B \in \mathbb{N}, \forall n \in \mathbb{N}, n \geq B \Rightarrow \sqrt{8n+4} \leq c(n + 3\log_2 n)$.
> By definition, $\sqrt{8n+4} \;\in\; O(n + 3\log_2 n)$.

Note that these are not the only values for c and B that work. Anything larger would work equally well in this proof, and some smaller values might work (coupled with the right argument).