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## Quiz #3 Sample Solutions

a. One way to solve this question is to compare the truth tables of each statement (alternatively, we could check the Venn Diagrams). Here are the truth tables for each statement:

A	B	C	(S0)	(S1)	(S2)	(S3)
Т	Т	Т	Т	Т	Т	Т
Т	Т	$\mathbf{F}$	F	F	F	$\mathbf{F}$
Т	$\mathbf{F}$	Т	Т	Т	Т	Т
Т	$\mathbf{F}$	$\mathbf{F}$	Т	F	Т	Т
$\mathbf{F}$	Т	Т	Т	Т	Т	Т
$\mathbf{F}$	Т	$\mathbf{F}$	$\mathbf{F}$	F	F	$\mathbf{F}$
$\mathbf{F}$	$\mathbf{F}$	Т	Т	Т	Т	Т
$\mathbf{F}$	$\mathbf{F}$	$\mathbf{F}$	$\mathbf{F}$	Т	F	$\mathbf{F}$

The only columns that are identical to (S0) are (S2) and (S3), thus these two statements are equivalent to (S0). The truth value of (S1) differs from (S0) when A is true and both B and C are false, and when all of A, B and C are false.

b. First we'll pick a statement equivalent to (S0) that has less implications to get rid of: we'll choose (S2). We use the fact the  $P \Rightarrow Q$  is true exactly when either P is false or when Q is true (that is,  $\neg P \lor Q$ ) to replace the implication. Thus, one solution is:

$$C \lor (A \land \neg B)$$

Alternatively, we can choose (S3) and use the fact that the implication  $A \Rightarrow B$  is false exactly when A is true and B is false, thus replacing the negated implication to get:

$$(A \land \neg B) \lor C$$

Other solutions exist.