University of Toronto, St. George Campus — CSC 165H1 F
Department of Computer Science — Fall 2006

Quiz #4 Sample Solutions

We want to prove the following statement:

$$3n\sqrt{4+3n} \;\in\; O(n^2+2n)$$

We construct the standard proof structure and fill it in, choosing values for c and B when needed.

Let $c = 6$. Then $c \in \mathbb{R}^+$.
Let $B = 2$. Then $B \in \mathbb{N}$.
Let $n \in \mathbb{N}$. Assume $n \geq B$.
Then $\sqrt{4+3n} \leq \sqrt{4+3n^2} \leq \sqrt{n^2+3n^2} = 2n$ (since $n \geq 2$).
So $3n\sqrt{4+3n} \leq 3n \cdot 2n = 6n^2$.
Then $c(n^2+2n) \geq c(n^2) = 6n^2$ (dropping a positive term).
Thus $3n\sqrt{4+3n} \leq 6n^2 \leq c(n^2+2n)$ (comparing inequalities).
So $n \geq B \Rightarrow 3n\sqrt{4+3n} \leq c(n^2+2n)$.
Since n is an arbitrary element of $\mathbb{N}$, $\forall n \in \mathbb{N}$, $n \geq B \Rightarrow 3n\sqrt{4+3n} \leq c(n^2+2n)$.
Since B is a natural number, $\exists B \in \mathbb{N}, \forall n \in \mathbb{N}, n \geq B \Rightarrow 3n\sqrt{4+3n} \leq c(n^2+2n)$.
Since c is a real positive number, $\exists c \in \mathbb{R}^+, \exists B \in \mathbb{N}, \forall n \in \mathbb{N}, n \geq B \Rightarrow 3n\sqrt{4+3n} \leq c(n^2+2n)$.
By definition, $3n\sqrt{4+3n} \;\in\; O(n^2+2n)$.

Note that these are not the only values for c and B that work. Anything larger would work equally well in this proof, and some smaller values might work (coupled with the right argument).