

Duration: **50 minutes**
Aids Allowed: **NONE** (in particular, no calculator)

Student Number: _____

Last (Family) Name(s): _____

First (Given) Name(s): _____

Tutorial Section:
(circle one)

LM 157
Peter

SS 2105
Hania / Matthew

SS 2128
Costis

*Do **not** turn this page until you have received the signal to start.*
(In the meantime, please fill out the identification section above,
and read the instructions below *carefully*.)

This term test consists of 4 questions on 5 pages (including this one), printed on one side of the paper. *When you receive the signal to start, please make sure that your copy of the test is complete and write your student number where indicated at the bottom of every page (except page 1).*

Answer each question directly on the test paper, in the space provided, and use the reverse side of the pages for rough work. If you need more space for one of your solutions, use the reverse side of a page and *indicate clearly the part of your work that should be marked.*

MARKING GUIDE

1: _____/ 6

2: _____/ 4

3: _____/ 8

4: _____/ 8

TOTAL: _____/26

Good Luck!

Question 1. [6 MARKS]

Consider the following statement:

(S) If A, then B and C.

Part (a) [1 MARK]

What, if anything, can be concluded from (S) if A is true?

Part (b) [1 MARK]

What, if anything, can be concluded from (S) if B is true and C is true?

Part (c) [2 MARKS]

Express symbolically a statement equivalent to (S), but without using implication ($\Rightarrow$).

Part (d) [2 MARKS]

Express symbolically a statement equivalent to (S), but without using conjunction ($\wedge$).

Question 2. [4 MARKS]

For each of the following statements, draw a Venn diagram with overlapping circles for A, B and C, making 8 regions. Shade in all the regions where (S) is true, and put an X in each region where (S) is false.

If A, then B and C.

A or B are true, when C is true.

Question 3. [8 MARKS]

Let F be a set of friends, and let $P(x, y)$ mean “ x pays for y .”

Express each of the following statements symbolically, matching the English form as closely as possible.

Part (a) [2 MARKS]

Everybody pays for at least one person.

Part (b) [2 MARKS]

Everyone has somebody who pays for him.

Part (c) [2 MARKS]

Somebody pays for him or herself.

Part (d) [2 MARKS]

Nobody pays for somebody else.

Question 4. [8 MARKS]

Consider the following statement about sequences of natural numbers $a_0, a_1, a_2, \dots$ (recall that $\mathbb{N} = \{0, 1, 2, \dots\}$):

$$(T) \forall i \in \mathbb{N}, \exists k \in \mathbb{N}, (a_i = a_k \implies a_i < k)$$

Part (a) [2 MARKS]

Write the negation of (T) symbolically, moving negations inside as much as possible.

Part (b) [4 MARKS]

For the following sequences, state whether (T) is true or false. Justify your claim, using an example or counterexample when appropriate.

1, 2, 3, 4, 5, 6, 7, $\dots$

1, 2, 1, 1, 2, 1, 1, 1, 2, $\dots$

Part (c) [2 MARKS]

Give the structure a direct proof of (T) would look like. (Do not try to actually prove (T).)

Total Marks = 26

Student #: _____

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END OF TERM TEST # 1