Due: By 12:00 noon on Thursday, December 7. **Worth**: 7%
You must complete and sign an assignment cover page, and attach it (with a staple) to the front of your assignment. Assignments should be handed into the drop box in BA 2220.

1. Consider the following program, where A is an array of n integers:

```
m = A[0] + 1;
for (i = 0; i < A.length; i++) {
    if (A[i] < m) {
       m = A[i];
       s = A[i];
       for (j = i+1; j < A.length; j++)
         s = s + A[j];
    }
}
```

   (a) Describe an array of length n that is a worst-case input for this algorithm (in terms of number of lines executed). Explain why your answer is a worst-case input.

   (b) Let $T(n)$ be the worst-case number of lines executed by this program over all arrays of length n. Prove that $T(n) \in \Theta(n^2)$.

2. Consider the normalized floating point system $\mathcal{F}$ with $\beta = 2$, $t = 6$, $e_{\max} = 7$, $e_{\min} = -8$ that uses round-to-nearest.

   (a) How many real numbers are representable exactly in $\mathcal{F}$? Justify your answer.

   (b) Give the decimal (base 10) representation of the largest and smallest positive real numbers representable in $\mathcal{F}$.

   (c) Give an example of a decimal number that will cause overflow in $\mathcal{F}$. Explain.

   (d) Give an example of a decimal number that will cause underflow in $\mathcal{F}$. Explain.

   (e) Suppose a real number x is represented in $\mathcal{F}$ by $x' = 1.01101 \times 2^1$. What range of decimal values could x have been? Justify your answer.

3. Computing the expression $x^2 - 4$ is susceptible to error for certain values of x.

   (a) Explain what kind of error can occur here and for what values of x it occurs. Illustrate your claim with an example.

   (b) Reformulate the expression to avoid this error. Explain why this error is avoided.

4. Suppose we need to compute the expression $(x - 1)^3$ for $x = 0.85$ in the normalized floating point system with $\beta = 10$, $t = 3$, $e_{\max} = 2$, $e_{\min} = -3$ that uses round-to-nearest.

   (a) Evaluate $x^3 + -3x^2 + 3x + -1$ by performing the addition operations in this system from left to right. (Remember order of operations says to do the multiplications first!) Show your work.

   (b) Evaluate $x^3 + -3x^2 + 3x + -1$ by performing the addition operations in this system from right to left. Show your work.

   (c) Which computation is more stable for this value of x? Why?