Due: By 12:00 noon on Thursday, October 12. **Worth**: 7%
You must complete and sign an assignment cover page, and attach it (with a staple) to the front of your assignment. Assignments should be handed into the drop box in BA 2220.

1. Let A denote the sentence "x is odd" and let B denote the sentence "x^2 is odd". Write the sentence "$A \Rightarrow B$" in natural English:
   (a) Using the words "if" and "then".
   (b) Using the word "implies".
   (c) Using the words "only if".
   (d) Using the words "is necessary for".
   (e) Using the words "is sufficient for".

2. Let $\mathbb{N} = \{0, 1, 2, \dots\}$. For each statement below, rewrite the statement in natural English and clearly state whether it is true or false. Justify your answer briefly, using an example or counter-example when appropriate.
   (a) $\forall c \in \mathbb{N}, \exists d \in \mathbb{N}, c < d$
   (b) $\forall d \in \mathbb{N}, \exists c \in \mathbb{N}, c < d$
   (c) $\exists c \in \mathbb{N}, \forall d \in \mathbb{N}, c < d$
   (d) $\exists d \in \mathbb{N}, \forall c \in \mathbb{N}, c < d$

3. Consider the statement (D) "There is a class clown." This can be interpreted as meaning there is *at least one* class clown or as meaning there is *exactly one* class clown.
   Express each of these two interpretations of (D) symbolically.

4. Consider a two-player card game. Each player holds some number of cards (his/her hand), and play alternates between players. Each turn consists of playing one card from his/her hand. In this game, only some cards can legally be played immediately following another card (for example, the suits or ranks of the cards might have to match).

   Let C be the set of cards. Express the statements (a)-(c) symbolically, defining appropriate predicates, but only using the domain C.
   (a) "Player 1 always holds a card that may be played first."
   (b) "Player 2 can always play a card second."
   (HINT: Use a binary predicate $\ell(b, a)$ meaning it's legal to play card b after card a was played.)
   (c) "There is always a card that when played by Player 2 second, Player 1 cannot play a card third."
   (d) Express your answer to (c) in English that closely resembles the logical structure of your answer.

5. Consider the following statement about natural numbers:

   (S) If a divides b, then a divides b^2.

   (a) Write (S) symbolically. Do not use a predicate to stand for "divides".
   (b) Prove (S) using the same approach used in lecture, and provide a short justification of each step of your proof. Clearly state and justify any properties you use.