1. Here is the solution to the Sudoku puzzle: (bold entries were originally given)

4	6	**5**	9	7	2	3	**1**	**8**
3	1	**7**	5	8	4	6	**2**	9
2	8	9	**6**	**1**	**3**	4	7	5
5	**9**	**6**	3	2	1	7	8	4
1	4	3	**7**	6	**8**	5	9	**2**
7	2	8	4	5	9	**1**	**3**	6
9	7	1	**2**	**4**	**5**	8	6	3
6	**3**	4	8	9	7	**2**	5	1
8	**5**	2	1	3	6	**9**	4	**7**

2. [9 marks]

   (a) Let P stand for "The Maple Leafs make the playoffs," S stand for "The Maple Leafs stay healthy," and H stand for "The fans are happy." Then our translations are simply:
   (S1): $P \Rightarrow S$
   (S2): $P \Rightarrow H$

   (b) Contrapositive of (S1): $\neg S \Rightarrow \neg P$, or, in English, "If the Maple Leafs don't stay healthy then the Maple Leafs won't make the playoffs."
   Contrapositive of (S2): $\neg H \Rightarrow \neg P$, or, in English, "If the fans aren't happy then the Maple Leafs didn't make the playoffs." [Note: this is very different from the statement "The fans won't be happy if the Maple Leafs don't make the playoffs."]

   (c) Converse of (S1): $S \Rightarrow P$, or, in English, "If the Maple Leafs stay healthy, then they will make the playoffs."
   Converse of (S2): $H \Rightarrow P$, or, in English, "The fans will be happy only if the Maple Leafs make the playoffs."

   (d) We don't know whether the Maple Leafs made the playoffs or not. They could have stayed healthy and missed the playoffs. The only way we know for sure that the fans are happy is if the Maple Leafs made the playoffs. The fans might be happy that the Leafs won their final game of the season. Or maybe the fans are really mad that the Leafs missed the playoffs again or played poorly. So we can't conclude whether the fans are happy or not.

Comments: Many students over-complicated this question. A predicate doesn't have to take an argument; that is, it can be 0-ary. However, if you do define a predicate with an argument, you should say what role the argument has in the definition. Also, notice that $\forall x \in A, P(x)$, is the same as $\forall \mathit{mapleLeafs} \in A, P(\mathit{mapleLeafs})$, since x and $\mathit{mapleLeafs}$ are just dummy variables.

3. [15 marks]
   We define the domain P = set of pets, and the predicates $S(x)$ meaning pet x is soft, $C(x)$ meaning pet x is cuddly, $L(x)$ meaning pet x costs less than 40, and $M(x)$ meaning pet x costs more than 75.

   (a) $\forall x \in P, L(x) \Rightarrow S(x) \wedge C(x)$
   True, by checking the price of all pets in the table and verifying that the bunny and the cat are both soft and cuddly.

   (b) $\exists x \in P, \neg S(x)$
   True, an example is the aardvark, which isn't soft.

   (c) $\forall x \in P, C(x) \Rightarrow S(x)$
   True, by checking the cuddliness of all pets in the table and verifying that the bunny and the cat are soft.

   (d) $\forall x \in P, \neg(S(x) \wedge M(x))$
   False, counterexamples include the dobermann and the emu.

   (e) $\forall x \in P, C(x) \wedge M(x) \Rightarrow \neg S(x)$
   True, by checking the cuddliness and price of all the pets in the table, we find no pets are both cuddly and cost more than 75, thus no counterexamples exist.

   Comments: Many students are confused about the difference between sets and predicates. Sets are just a collection of objects, like the natural numbers. Predicates are boolean-valued functions. Something can't be both a set and a predicate. If you define a set, don't use it as a predicate, and vice versa. For example, don't define a set Q then use it in a boolean formula as in $\forall x \in A, Q(x) \wedge P(x)$. Instead, define a related predicate (perhaps $q(x)$) to stand for the statement "x is in set Q."

4. [6 marks]

   This is a statement about students in the class, and being a class clown is a property of students. Hence, our domain is S, the set of students in the class, and our predicate is $C(x)$, meaning student x is a class clown.

   "There is *at least one* class clown": This is simply an existential statement, saying a class clown exists. We can formalize this as $\exists x \in S, C(x)$.

   "There is *exactly one* class clown": This is slightly harder to formalize. The only tools we have are saying something is true about somebody in the domain (existential quantifier) or saying something is true about everybody in the domain (universal quantification). We need to restate this in English that is closer to our symbolic language. One way to restate this is as "there is somebody who is a class clown and nobody else is a class clown" (check that this really means there is exactly one class clown). One formalization might be $\exists x \in S, C(x) \wedge \forall y \in S, C(y) \Rightarrow y = x$. Other answers exist.

   [Side note: Since this is long and cumbersome to write, some people abbreviate this as $\exists! x \in S, C(x)$, meaning there is a unique $x \in S$ for which $C(x)$ holds. We'll avoid this notation in this course.]

   Comments: Some students interpreted "class clown" as "class" being a property of some clowns (classy clowns perhaps?). Though not the intended interpretation, this formalization leads to answers very similar to the above solution (and thus equally acceptable).