

Due: By 12:00 noon on Thursday, November 8.

Worth: 10%

You must complete and sign an assignment cover page, and attach it (with a staple) to the front of your assignment. Assignments should be handed into the drop box in BA 2220.

1. (a) Suppose n is an integer. Express the statement “ n is either even or odd” symbolically.
 (b) Use our structured proof form and the statement from part (a) to prove that, for every integer n , $n^2 - n$ is even.
 (c) Use our structured proof form to prove that $\frac{n(n+1)}{2}$ is an integer whenever n is an integer. Be sure to state what you are proving precisely in our symbolic language.
2. Consider the sequence of integers defined by:

$$(A) \quad a_n = \begin{cases} 1 & \text{if } n \text{ is a multiple of 2;} \\ 1 & \text{if } n \text{ is one more than a multiple of 4;} \\ 0 & \text{if } n \text{ is three more than a multiple of 4.} \end{cases}$$

Determine whether each of the following statements is true or false for sequence (A), then give a direct proof or disproof of the statement for (A) using our structured proof form.

- (a) $\forall i \in \mathbb{N}, (a_i \leq a_{i+1} \leq a_{i+2}) \vee (a_i \geq a_{i+1} \geq a_{i+2})$.
- (b) $\forall i \in \mathbb{N}, a_i + a_{i+2} + a_{i+4} \leq 4$.
- (c) $\exists i \in \mathbb{N}, \forall j \in \mathbb{N}, \forall k \in \mathbb{N}, (j = 3k) \Rightarrow (a_j = a_{i+k})$.

3. Consider the following three statements about political parties and their supporters:

- (S1) If two distinct citizens support the same party, then they share the same views.
- (S2) Citizens sharing the same views do not agree on every subject.
- (S3) Some (distinct) citizens agree on each subject.

- (a) Write (S1), (S2) and (S3) in our symbolic language, using the following predicates:

$S(x, p)$: citizen x supports party p

$V(x, y)$: citizens x and y share the same views

$A(x, y, s)$: citizens x and y agree on subject s

$x \neq y$: citizen x is distinct from citizen y

- (b) Using our structured proof form and your symbolic statements from part (a), prove the following statements:

- (i). There is no party supported by all citizens.
- (ii). If there is a party with a supporter, then there are at least two people.

Hint: Use (i) to help prove (ii).