This assignment is due at the **start** of the lecture on Monday, 16 November 2009.

1. [5 marks]

   I mentioned in class that $\|x\|_\infty \le \|x\|_1$ for all vectors $x \in \mathbb{R}^n$.

   Does a similar result hold for matrices?

   That is, does $\|A\|_\infty \le \|A\|_1$ hold for all matrices $A \in \mathbb{R}^{n\times n}$?

   Justify your answer.

2. [5 marks]

   We showed in class that, if $Ax = b$ and $A\hat{x} = \hat{b}$, where x, b, $\hat{x}$ and $\hat{b} \in \mathbb{R}^n$ and $A \in \mathbb{R}^{n\times n}$ is a nonsingular matrix, then

   $$\frac{\|x - \hat{x}\|}{\|x\|} \le \mathrm{cond}(A)\frac{\|b - \hat{b}\|}{\|b\|} \tag{1}$$

   I mentioned in class that you can also show that

   $$\frac{1}{\mathrm{cond}(A)}\frac{\|b - \hat{b}\|}{\|b\|} \le \frac{\|x - \hat{x}\|}{\|x\|} \tag{2}$$

   Show that (2) holds.

   Note that all the vector norms in (1) and (2) are the same and that the matrix norm associated with cond is the matrix norm *subordinate* to that vector norm. (See page 55 of your textbook for the definition of a matrix norm *subordinate* to a vector norm if you don't recall the definition.)

3. [12 marks: 2 marks for each part]

   Suppose that both sides of a system of linear equations $Ax = b$, where A is a nonsingular matrix, are multiplied by a nonzero real number α.

   (a) Does this change the true solution x of $Ax = b$?

   Justify your answer.

   (b) Does this change the residual $r = b - A\hat{x}$, where $\hat{x}$ is an approximate solution of $Ax = b$?

   Justify your answer.

(c) Does this change the relative residual $\|r\|/\|b\|$, where r is the residual from part (b) and b is right side of $Ax = b$?
Justify your answer.

(d) Does this change the conditioning of the system $Ax = b$?
Justify your answer.

(e) What conclusions can be drawn from parts (a)-(d) above about assessing the quality of a computed solution $\hat{x}$ of $Ax = b$?

(f) Assuming that A has an LU factorization, how does multiplying A by a nonzero real number α change the LU factorization of A?
That is, if $\hat{A} = \alpha A$, $LU = A$ and $\hat{L}\hat{U} = \hat{A}$, where L and $\hat{L}$ are unit lower triangular matrices and U and $\hat{U}$ are upper triangular matrices, how are L and $\hat{L}$ related and how are U and $\hat{U}$ related?
Justify your answer.

4. [4 marks: 2 marks for each part]

Suppose that both sides of a system of linear equations $Ax = b$, where A is a nonsingular matrix, are pre-multiplied by a nonsingular diagonal matrix D of the same dimensions as A.

(a) Does this change the true solution x of $Ax = b$?
Justify your answer.

(b) Can this affect the conditioning of the system?
Justify your answer.

5. [10 marks: 5 marks for each part]

(a) Compute the LU factorization (without pivoting) of the matrix

$$A = \begin{pmatrix} 2 & 6 & -2 \\ 1 & 7 & -3 \\ -1 & -1 & 2 \end{pmatrix}.$$

That is, compute a unit lower triangular matrix L with $|L_{ij}| \leq 1$ for all $i > j$ and an upper triangular matrix U such that $A = LU$.

(b) Use the LU factorization from part (a) to solve $Ax = b$ for

$$b = \begin{pmatrix} 2 \\ 3 \\ 2 \end{pmatrix}.$$