

This assignment is due at the **start** of your lecture/tutorial on Friday, 28 October 2011.

1. [5 marks]

The Euclidean norm (also called the 2-norm) of a real n -vector x is defined by

$$\|x\|_2 = \sqrt{\sum_{i=1}^n x_i^2}. \quad (1)$$

How can you reformulate the expression on the right side of (1) to avoid overflow and harmful underflow in this computation?

Justify your answer.

2. [15 marks: 5 marks for each part]

(a) Write a MatLab function `exp1` to approximate e^x by summing the series

$$1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} \cdots$$

from left to right until the accumulated sum stops changing.

Test your program by computing `exp1(x)` for $x = -25, -24, -23, \dots, +25$ (i.e., `x = -25 : +25` in MatLab).

For each value of x , compute the relative error

$$\frac{\text{exp1}(x) - \exp(x)}{\exp(x)}$$

where `exp(x)` is the MatLab function that approximates e^x .

For the purpose of this question, assume `exp(x) = e^x`.

Format your output neatly.

(b) For what values of x does your function produce accurate approximations to e^x and for what values of x does your function produce poor approximations to e^x ? Explain why your function performs well in the cases where it produces accurate approximations to e^x and also explain why your function performs poorly in the cases where it produces poor approximations to e^x .

Don't just say that it performs poorly because there is rounding error. There is rounding error in your computations for all values of x (except possibly $x = 0$). However, in some cases, the rounding errors are insignificant and you obtain a good approximation to e^x , while, in other cases, the rounding errors are significant and you obtain a poor approximation to e^x . Explain why.

- (c) Make a small change to your function `exp1` so that it produces accurate approximations to e^x for all $x = -25, -24, -23, \dots, +25$. Call your new function `exp2`. For each value of x , compute the relative error

$$\frac{\text{exp2}(x) - \exp(x)}{\exp(x)}$$

where `exp(x)` is the MatLab function that approximates e^x .

Format your output neatly.

Hint: note $e^x = 1/e^{-x}$.

3. [5 marks]

Assume x and y are vectors in \mathbb{R}^2 . Is it possible to have $\|x\|_1 > \|y\|_1$ and $\|x\|_\infty < \|y\|_\infty$?

If so, give an example.

If not, explain why not.

4. [7 marks]

Consider the matrix

$$A = \begin{pmatrix} 5 & 2 \\ 1 & 2 \end{pmatrix}.$$

Compute $\text{cond}_1(A)$, $\text{cond}_2(A)$ and $\text{cond}_\infty(A)$.

Show all your calculations.

5. [5 marks]

Let A be an $n \times n$ nonsingular real matrix and let α be a nonzero real number. Show that

$$\text{cond}(\alpha A) = \text{cond}(A)$$

That is, scaling a nonsingular real matrix by a nonzero real number does not affect its condition number.