Computer Science 364 | Summer, 2003
St. George Campus | University of Toronto

Homework Assignment #3
Due: July 23, 2003, by 6pm
(in the lecture)

On the cover page of your assignment, you must write ***and sign*** *the following statement: "I have read and understood the homework collaboration policy described on the course webpage." Without such a signed statement your homework will not be marked.*

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1. For each of the problems below, state whether you think that the problem is NP-complete. Justify your answer by either proving it to be NP-complete, or giving good reason to suspect that it is not.

[5] (a) **8-Independent-Set**
Instance: A graph G.
Question: Does G have an independent set of size 8?

[5] (b) **Even-Sat**
Instance: A CNF formula S, where every clause has an even number of literals.
Question: Is S satisfiable?

[5] (c) **k-Clique 2k-IS**
Instance: Graph G and positive integer k.
Question: Does G contain both a clique of size k and an independent set of size $2k$?

2. Prove that the following two problems are NP-complete. For the hardness part, use reductions from NP-complete problems in the notes.

[10] (a) **Hamiltonian Path**
Instance: Graph G
Question: Does G have a simple path passing through all the vertices exactly once?

[10] (b) **5-Partition**
Instance: Set of m nonnegative numbers $a_1 \dots a_m$.
Question: Is there a way to partition $a_1 \dots a_m$ into five disjoint sets S_1, S_2, S_3, S_4 and S_5 such that $\Sigma_{a_i \in S_1} a_i = \Sigma_{a_i \in S_2} a_i = \Sigma_{a_i \in S_3} a_i = \Sigma_{a_i \in S_4} a_i = \Sigma_{a_i \in S_5} a_i$?

[10] 3. **Powers of 9**
Let $L = \{\langle M \rangle |\ M$ accepts a string w with length $|w| = 9^k$ for some $k\ \}$. Prove that L is semi-decidable, but not decidable.

[10] 4. **Opposite Turing Machines**
Consider the language $L = \{\langle M_1, M_2 \rangle |\ M_1$ and M_2 are Turing Machines and there is an input $w \in \Sigma^*$ such that M_1 accepts w and M_2 rejects $w\}$. Prove that L is semi-decidable but not decidable.

[10] 5. **INF-substring**
Let INF-SS $= \{\langle M \rangle |\ M$ accepts infinitely many strings with substring '"computability" $\}$. Prove that INF-SS is not semi-decidable and not co-semi-decidable. Hint: use reduction from INF.