Computer Science 364 — Fall, 2003
St. George Campus — University of Toronto

Homework Assignment #3
Due: Nov 26, 2003, by 2pm
(in tutorial)

On the cover page of your assignment, you must write ***and sign*** *the following statement: "I have read and understood the homework collaboration policy described on the course webpage." Without such a signed statement your homework will not be marked.*

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1. For each of the problems below, state whether you think that the problem is NP-complete. Justify your answer by either proving it to be NP-complete, or giving good reason to suspect that it is not. You do not need to show that they are in NP.

[10] (a) **Double-Sat**
Instance: A 3CNF formula ϕ.
Question: Does ϕ have at least two satisfying assignments?

[10] (b) **10-cycle**
Instance: An undirected graph G.
Question: Does G have a cycle of length exactly 10?

[10] (c) **1/5-clique**
Instance: An undirected graph G.
Question: Does G contain a clique of size $\lfloor n/5 \rfloor$, where n is the number of vertices?

[10] (d) **s-t Hamiltonian Path**
Instance: Graph G, vertices s and t
Question: Does G have a simple path from s to t passing through all the vertices exactly once?

[10] (e) **4-Partition**
Instance: Set of m nonnegative numbers $a_1 \ldots a_m$.
Question: Is there a way to partition $a_1 \ldots a_m$ into four disjoint sets S_1, S_2, S_3 and S_4 such that $\Sigma_{a_i \in S_1} a_i = \Sigma_{a_i \in S_2} a_i = \Sigma_{a_i \in S_3} a_i = \Sigma_{a_i \in S_4} a_i$ and $S_1 \cup S_2 \cup S_3 \cup S_4 = \{1, \ldots, m\}$.

[10] 2. **Multiples of 20**
Let $L = \{\langle M \rangle |\ M$ accepts a string w with length $|w| = 20k$ for some $k\ \}$. Prove that L is semi-decidable, but not decidable.

[10] 3. **Turing Machine moving left**
Let $L = \{\langle M, w \rangle | M$ on w attempts to move left when its head is on the leftmost tape cell$\}$. That is, pair $\langle M, w \rangle$ is in L if M ever attempts to move its head left while on leftmost cell during a computation on w. Prove that this language is semi-decidable but not decidable.

[10] 4. **INF even and blank**
Let INF-EVEN $= \{\langle M \rangle | M$ accepts infinitely many strings encoding even numbers, and does not accept blank $\}$. Prove that INF-EVEN is not semi-decidable and not co-semi-decidable.