Computer Science 364 | Winter, 2004
St. George Campus | University of Toronto

Homework Assignment #3
Due: Mar 31, 2004, by 6pm
(in tutorial)

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1. For each of the problems below, state whether you think that the problem is NP-complete. Justify your answer by either proving it to be NP-complete (the "in NP" part can be a couple of sentences long), or giving a good reason to suspect that it is not. You can assume that the following problems are NP-complete: 3SAT, Partition, Clique, and HamCycle (not HamPath).

[10] (a) **Fixed-size partition**
Instance: Set of n nonnegative numbers $a_1 \dots a_n$.
Question: Is there a set $S \subseteq \{a_1 \dots a_n\}$ such that $\Sigma_{a_i \in S} a_i = \Sigma_{a_i \notin S} a_i$, and, in addition to that, $|S| = n/3$.

[10] (b) **n-5 clique**
Instance: An undirected graph G.
Question: Does G contain a clique of size $n-5$, where n is the number of vertices?

[10] (c) **Half Hamiltonian Path**
Instance: Graph G
Question: Does G have a simple path passing through exactly half of the vertices?

[15] (d) **Sat(3)**
Instance: A 3CNF formula ϕ which has no more than 3 occurrences of every variable.
Question: Is ϕ satisfiable?
Hint: you might find it easier to solve this problem for Sat(5) first (that is, 3CNF with 5 occurrences of every variable in the formula), and then adapt your solution to Sat(3).

[10] 2. **Even length strings**
Let $L = \{\langle M \rangle | \ M \text{ accepts a string } w \text{ with length } |w| = 2k \text{ for some } k\}$. That is, $\langle M \rangle \in L$ iff there is a string with even length that M accepts. Prove that L is semi-decidable and that L is not decidable.

[10] 3. **Turing Machine in state q_5**
Let $L = \{\langle M, w \rangle | M \text{ gets to state } q_5 \text{ during computation on } w\}$. That is, pair $\langle M, w \rangle$ is in L if M ever executes a transition of the form $(q_i, c) \mapsto (q_5, c', L/R)$ during its computation on w. Prove that this language is semi-decidable but not decidable.

[15] 4. **INF-POWERS**
Let INF-POWERS $= \{\langle M \rangle | M$ accepts infinitely many binary strings encoding powers of 2 and does not accept empty string or any string starting with $0\}$. Prove that INF-POWERS is not semi-decidable and not co-semi-decidable by using only one reduction.