Computer Science 364 Winter, 2004
St. George Campus University of Toronto

Homework Assignment #2
Due: Mar 1, 2004

[20] 1. **Forming committees**
In a certain department, there are n faculty members. The department needs to organize m different committees (such as undergraduate admission committee, etc). Each faculty member belongs to a group, sometimes several (such as Theory, Graphics, AI); there are k such groups and each group i has a list C_i for $1 \leq i \leq k$ of faculty members belonging to this group. For fairness, there should not be more than one person from each group on every committee.

[15] (a) Given the n, m, k and C_i for $1 \leq i \leq n$, determine whether it is possible for each committee to have *representatives of at least 5 groups, where one person can represent several groups.* Hint: design a flow network that represents this problem. Think of each category as a "layer" in your network.

[5] (b) How can you find an assignment of faculty to committees given this flow network?

[20] 2. **Left-k-jump Turing machine**
Consider a variant of a Turing machine in which instead of moving one cell left, there is an operation of moving k cells left; that is, instead of L there is J, which means "jump k cells left". The goal of this exercise is to show that this is equivalent to the usual definition of a Turing machine.Suppose that you are trying to simulate a regular Turing machine $M = (\Sigma, \Gamma, Q, \delta)$ by a left-k-jump Turing machine $M' = (\Sigma, \Gamma, Q', \delta')$.

[5] (a) Describe in words how you would simulate one left step of a regular Turing machine on a left-k-jump Turing machine.

[10] (b) More formally, given one transition $\delta(q_i, c) = (q_j, c', L)$ of M, describe the corresponding transitions in δ' of M'.

[5] (c) Suppose that $|Q| = n$ and δ contains m transitions, out of which m' are with L. How many states and transitions are there in M'?

[20] 3. **Search-to-decision reduction**
The search problem LargestSubsetSum is defined as follows. Given a set of natural numbers $\{a_1, \ldots, a_n\}$ and a natural number t, find a set $S \subseteq \{a_1, \ldots, a_n\}$ such that $\Sigma_{a_j \in S} a_j = t$ and, in addition, $|S|$ is maximal. For example, if the input is $\{1, 2, 3, 5\}, t = 6$, then the maximal size S is $\{1, 2, 3\}$.

[7] (a) Define a corresponding decision problem LSSD that can be used to solve LargestSubsetSum (note that it is **not** the same as SubsetSum).

[5] (b) State how to find the *number of elements* in the solution to LargestSubsetSum by reduction to LSSD that you just defined. Your reduction should make no more than $O(\log n)$ calls to LSSD, where n is the number of elements $a_1 \ldots a_n$.

[8] (c) Now state how to find a solution S of maximal size, again by reduction to LSSD. You can use your answer from the previous subquestion.

[20] 4. **Concatenation of NP languages.**
Suppose that both L_1 and L_2 are in NP. Define a concatenation of L_1 and L_2 as $L_3 = \{x_1 x_2 | x_1 \in L_1 \text{ and } x_2 \in L_2\}$ (that is, strings in L_3 are of the form a string from L_1 concatenated with a string of L_2.) Show using the formal definition of NP that L_3 is in NP as well.